System Identification
&
Parameter Estimation
(Wb2301)

Course 2006-2007

Course manual

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Status:
This document is a kind of manual to the course system identification and parameter estimation (SIPE). It gives an overview, per lecture, of the goals and indicates what text is relevant to read. The content will be updated after each lecture.

Contents

Lecture 1: Introduction

The first lecture gives an overview of the course and a general introduction to system identification and parameter estimation.
System identification, in this course, tries to elucidate the dynamic relation between time-signals and to parameterize this relation in a mathematical model (where the model is based on differential equations). In this course the emphasis is on system identification in frequency domain. Key element of this approach is the Fourier transform. Major advantage of the frequency domain approach is that no a priori knowledge is required of the system (order and sources of noise).
Every recorded time signal will be contaminated with noise. Noise is, by nature, a random process and consequently measured signals are stochastic. In stochastic theory not the individual realization is important but its statistical properties, e.g. mean, standard deviation, and also the probability density functions. The ergodicity concept states that one, sufficiently long, realization is representative for many other realizations. This implicates that it is sufficient to capture one recording of a signal to assess its (statistical) properties (in stead of multiple recordings). Cross-products and cross-covariance functions are measures to estimate the relation between two (stochastic) signals in time-domain. The interpretation of these functions will be discussed in lecture 2.

Topics:

- System identification: signals vs. systems
- Parameter estimation: what is a model?
- Deterministic and stochastic signals
- Stochastic theory and ergodicity
- Probability density function
- Fourier transform

Selected readings:

- Schoukens and Pintelon, Chapter 1
- Optional: reader Signaalanalyse, Chapter 2, 5, 6, 7

Lecture 2: Open loop identification

This lecture gives the basics about system identification in frequency domain.
In time domain, open loop systems can be investigated using transient deterministic signals or stochastic signals (e.g. white noise). The output of an open loop system is the convolution of the system's impulse response function with its input. The Fourier transform of the impulse response function is called the transfer function or frequency response function (FRF). Advantage of the FRF is that it appears as an (algebraic) multiplication with the Fourier transform of the input signal. Thus, a convolution in time domain becomes a multiplication in frequency domain.
Spectral density is defined as the Fourier transform of the covariance function. An estimator of the system's FRF is obtained by dividing the cross spectral

density by the auto spectral density of the input. In this case the input signal does not have to be white noise. Frequency response functions are graphically plotted in a Bode diagram, presenting gain and phase per frequency. To get an indication of the amount of noise at the system's output the coherence function is used. The coherence varies between 0 and 1, where one indicates that there is no noise and thus that the system behaves linear while lower values indicate the presence of noise, either from external noise sources or from nonlinearities within the system.

Topics:

- Cross-products and cross-covariance functions
- Time vs. frequency domain identification
- Estimate frequency response function (FRF) of open loop systems
- Coherence

Selected readings:

- Schoukens and Pintelon, Chapter 2
- Optional: reader Signaalanalyse Chapters 8, 9, 10

Lecture 3: Open- and closed loop identification

This lecture gives a background on estimators in general and describes an estimator for systems that operate within a closed loop configuration.
An estimator gives an estimate for a certain 'true' variable or function. Estimators are contaminated with errors which can be random (variance of the estimator) and/or structural (bias of the estimator). An accurate estimator has negligible bias and low variance. With a consistent estimator the variance of the estimator reduces with the number of samples used for averaging.
Raw (non-averaged) spectral estimators are not consistent, i.e. with increasing observation time the resolution in frequency domain increases but the variance of this estimator remains equal. Furthermore using the raw estimators of the spectral density the estimator for the coherence is always 1. The variance of the estimator for the spectral density can be reduced by averaging over adjacent frequencies, but at the cost of frequency resolution! Using averaged spectral densities results in a better estimate for the FRF and coherence. Note that the coherence is always overestimated.

Topics:

- Theory on estimators
- Properties of spectral estimators and coherence
- Estimate frequency response function (FRF) of closed loop systems
- Discuss the pitfalls when identifying closed loop systems

Selected readings:

- Optional: reader Systeemidentificatie A, Chapter 2

Lecture 4: Time domain models

Continuous systems can be approximated well by discrete time models. Discrete time models are in fact the regression coefficients of a discrete impulse response function. Having *N* discrete signal values, discrete models normally require far less regression coefficients ($n<N$) compared to FRFs (N). Major advantage of time domain models is that in the time domain noise can be separated from signals. Compared to the frequency domain where noise is mixed with the signal that requires a posteriori averaging, in the discrete time domain noise models are estimated a priori. Immediately, this a priori knowledge of the noise models is the main drawback since it requires some knowledge of the system's structure which is often not known beforehand.
In this lecture, different time domain models are presented. Besides the models are all linear input-output models, they are not all linear in their parameters (i.e. regression coefficients). Linearity in the parameters means a linear contribution of the parameters to the model error, which is typically the difference of the modeled output and the system's output. Advantage of linearity in the parameters is that the model parameters can be obtained algebraically from the input and output signals only. Linearity in the parameters depends on the chosen noise model. E.g. ARX is linear in its parameters and ARMAX is nonlinear in its parameters.
Two discrete closed loop estimators are presented (two stage and coprime factorization), both utilizing two open loop estimation steps but each in a different way.

Topics

- AR and MA basic model structures
- Combinations of AR and MA system and noise models
- Linearity in the parameters
- Quadratic error criterion
- Optimal model order selection
- Closed loop estimators

Selected readings:

- Not yet decided

Lecture 5: Discrete and continuous systems

The distortional effect of sampling and signal reconstruction is presented. Sampling introduces additional high frequencies in the sampled signals while reconstruction acts as a low-pass filter. Both effects can be diminished by the choice of a sufficiently high sample frequency and the use of anti aliasing filters.
The conversion from the continuous to the discrete time domain is explained. Conversion of models between the domains is performed by using the z-transformation.

Topics:

- Sampling, reconstruction

- Continuous Fourier spectrum of a sampled signal: spectral repetition
- Discrete Fourier transform of a discrete signal of finite time
- FRFs of discrete systems: spectral repetition also follows directly from the z transform

Selected readings:

- Optional: reader Signaalanalyse Chapters 3, 4

Lecture 6: Perturbation signal properties

In most cases system identification is a battle against noise. One should try to decrease the power of the noise and/or increase the power of the signal. Several methods exist to boost the power of the signal and such improve the signal-to-noise ratio (SNR). However random signals always introduce leakage, an effect of the observation time and resulting discrete frequency resolution. Multisine signals are composed of multiple sines. These deterministic signal do not introduce leakage and as the power is distribute over a limited number of frequencies the power per frequency can be high. With cresting, a technique to minimize the ratio between the outliers of the time and the standard deviation of the signal, the power can even be further increased. And the effect of the input signal on the system identification procedure is discussed.

Topics:

- Aliasing
- Leakage
- Signal-to-noise ratio (SNR)
- Multisine signals
- Cresting of multisine signals

Selected readings:

- Schoukens and Pintelon, Chapter 4

Lecture 7: Multi-variable systems

Multivariable systems

Topics:

- Multivariable systems

Selected readings:

- De Vlugt et al. Journal of Neuroscience Methods 122 (2003) 123-140