Exercise Solid-State Physics (ET2908 and 8027) 2009-20010Q1: dr. R. Ishihara, DIMES-TC00.0044, r.ishihara@tudelft.nl

Exercise-sheet 2.1a

I. Principles of Quantum Mechanics

(I.1) Calculate the de Broglie wavelength for (a) An electron with kinetic energy of 1eV and (b) A 2000 kg truck traveling at 20m/s.

(I.2) An electron and a photon have the same energy. At what value of energy (in eV) will the wavelength of the photon be 10 times that of electron?

(I.3) The uncertainty in position is 12 Å for a particle of mass 5×10^{-29}kg. Determine the minimum uncertainty in the momentum of the particle.

II. Wave

(II.1) Express a standing wave with a cosine function using any of the following constants; the amplitude A, the wavelength λ and the period T.

(II.2) Express a traveling wave at the origin with a cosine function using any of the following constants; the amplitude A, the wavelength λ and the period T.

(II.3) Express a traveling wave in the positive direction with a cosine function using any of the following constants; the amplitude A, the wavelength λ and the period T.

(II.4) Express the traveling wave obtained in the above question with an exponential function.

(II.5) Express the velocity of the traveling wave using any of the following constants; the amplitude A, the wavelength λ and the period T.

III. General Solution of Wave Function

The time independent Schrödinger's wave function is given by the following type of the differential equation:

$$\frac{\partial^2 \psi(x)}{\partial x^2} - c^2 \psi(x) = 0, \tag{1}$$

where c is a constant. General solutions of this type of equation are as follows.

When $-c^2 > 0$ (the c is an imaginary),

$$\psi(x) = Ae^{j|c|x} + Be^{-j|c|x} \tag{2}$$

When $-c^2 < 0$ (the c is a real),

$$\psi(x) = Ae^{cx} + Be^{-cx}. \tag{3}$$

(III.1) Show that, when $-c^2 > 0$,

$$\psi(x) = C\sin(|c|x) + D\cos(|c|x) \tag{4}$$

is also the general solution of the wave function.

(III.2) In time-independent Schrödinger equation, which solution must be used in case of (a) $E > V_0$ and (b) $E < V_0$? (Assume $V(x) = V_0$.)

(III.3) Is a time-independent quantum wave function below traveling towards the positive or negative direction? Explain why.

$$\psi(x) = Ae^{jkx}, \tag{5}$$

where the k is a real and positive number.

IV. Applications of Schrödinger's wave equation

(IV.1) An electron in free space is described by a plane wave given by $\Psi(x,t) = Ae^{j(kx-\omega t)}$ where $k = 1.5 \times 10^9 \mathrm{m}^{-1}$ and $\omega = 1.5 \times 10^{13}$ rad/s.

(a) Determine the phase velocity of the plane wave.

(b) Calculate the wavelength, momentum and kinetic energy (in eV) of the electron.