Exercise-sheet 3

I. Normalization of Schrödinger's wave function

(I.1) A time-independent wave function of a particle in potential box is given by

$$\psi(x) = A \sin \frac{n\pi}{a} x = \frac{A(e^{in\pi x/a} - e^{-in\pi x/a})}{2i} \qquad (n = 1, 2, 3..., 0 \leq x \leq a) \tag{1}$$

(a) Normalize the wave function to determine the constant A.
The following formula could be used.

$$\sin^2 \frac{1}{2} x = \frac{1 - \cos x}{2} \tag{2}$$

(b) What would be the time-dependent wave function?

II. time-independent Schrödinger equation

(II.1) A one dimensional infinite potential well with a width of 12Å contains an electron. (a) Calculate the first two energy levels that the electron may occupy. (b) If an electron drops from the second energy level to the first, what is the wavelength of the photon that might be emitted?

(II.2) Consider the particle in the infinite potential well as shown in Figure.1.
(a) Using a boundary condition, derive an expression of the wave functions.
(b) Using the normalization, determine a constant of the amplitude of the wave function. The following formula could be used;

$$\cos^2 x = \frac{\cos 2x + 1}{2} \tag{3}$$

(II.3) For the 1D step potential function shown in Figure 2, assume that the total energy of electrons $E > V_0$ and that the electrons are incident from the +x direction traveling in the -x direction.
(a) Write the general solutions of time-independent Schrödinger's equation for each region.
(b) By applying boundary conditions, derive expressions for the transmission T and reflection coefficients R, respectively. (R can be obtained by

$$R = \frac{A_2 A_2^*}{B_2 B_2^*} \tag{4}$$

where A_2 and B_2 is the amplitude for the reflected and the incident electrons, respectively.)

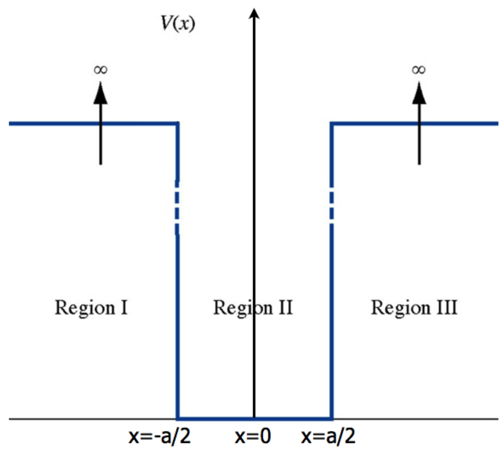

Figure 1: Potential function

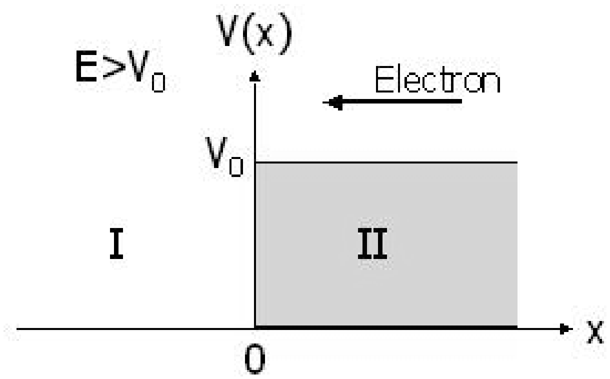

Figure 2: 1D step potential