Exercise Solid-State Physics (ET2908 and 8027) 2009-20010Q1: dr. R. Ishihara, DIMES-TC00.0044, r.ishihara@tudelft.nl

# Exercise-sheet 4

## I. The potential barrier

**(I.1)** Estimate the tunneling probability of an electron tunneling through a rectangular barrier with a barrier height $V_0$ of 1eV and a barrier width of 15Å. The electron energy is 0.20 eV.

## II. The hydrogen atoms

**(II.1)**

(a) Calculate the energy of the electron in the hydrogen atom (in units of eV) for the first four allowed energy levels.

(b) When an electron jumps from a higher energy level to a lower energy level, a photon is emitted. Calculate a wavelength of the emitted photon for an electron jumping from $n$=3 to $n$=2.

**(II.2)** Let us now consider the time-independent Schrödinger equation for the two-dimensional hydrogen atom model:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\psi(\vec{r}), \tag{1}$$

where $\vec{r} = (x, y)$ and $\nabla^2 = (\partial/\partial x)^2 + (\partial/\partial y)^2$. $V(\vec{r})$ represents the coulomb potential of the proton and is given by

$$V(\vec{r}) = V(r) = -\frac{e^2}{4\pi\epsilon_0 r}, \tag{2}$$

where $r = \sqrt{x^2 + y^2}$, $e$ is the charge of electron, and $\epsilon_0$ is the permittivity of vacuum. Using cylindrical coordinates $(r, \theta)$ the $\nabla^2$ operator can be expressed as

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \tag{3}$$

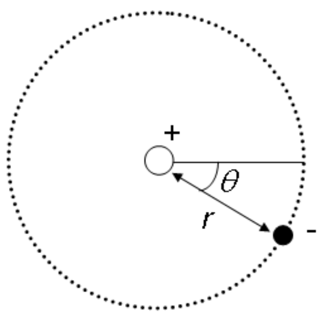


Figure 1: Two-dimensional model of the hydrogen atom.

(a) Substitute Eqs. (2) and (3) into Eq. (1) and derive the Schrödinger equation in cylindrical coordinates.

(b) Let us assume that $\psi$ does not depend on $\theta$. How does this assumption simplify the Schrödinger equation?

(c) Now try a function of the type $\psi(r) = e^{-\eta r}$. For which value of $\eta$ is this a correct solution?

(d) Find the corresponding value for the energy.