## I. Density of States Function

(I.1) Consider free electron in a 2D box with a width of $a$. The allowed wave-numbers $k_{x}$ and $k_{y}$ for $x$ and $y$ directions respectively are expressed as $\frac{\pi n_{x}}{a}$ and $\frac{\pi n_{y}}{a}$ where $n_{x}, n_{y}=1,2,3, \ldots$. The figure ?? shows the schematic view of the 2D array of allowed quantum states in k -space.
(1) Determine the area in k -space occupied by one allowed $k$ point.
(2) Express number of allowed states in the area enclosed by the first quatorant circles with the radius of $k$ and $k+d k$, where $k \gg d k$.
(3) Express density of allowed states in real space per unit energy.


Figure 1: 2D array of allowed quantum states in k-space
(I.2) Determine the total number of energy states in Si (a) between $E_{c}$ and $E_{c}+k T$ at $T=300 K$, and (b) between $E_{v}$ and $E_{v}-k T$ at $T=300 K$.

## II. Statistical Mechanics

(II.1) Assume the Fermi energy level is exactly in the center of the bandgap energy of a semiconductor at $T=300 \mathrm{~K}$.
(a) Calculate the probability that an energy state in the bottom of the conduction band is occupied by an electron for Si and Ge .
(b)Calculate the probability that an energy state in the top of the valence band is empty for Si and Ge.
(II.2) Neaman Problem 3.36

