Solution for Exercise-sheet 7

I. Density of States Function

(I.1)
(1) Area occupied by 1 k value is

$$\underline{\left(\frac{\pi}{a}\right)^2} \tag{1}$$

(2) Number of allowed states per unit area in k-space is then

$$2 \times \left(\frac{a}{\pi}\right)^2, \tag{2}$$

where the spin effect is took into account.

Area enclosed by the first quatorant circles with the radius k and $k + dk$ can be expressed by

$$\frac{1}{4}\pi(k+dk)^2 - \frac{1}{4}\pi k^2 = \frac{1}{4} \cdot 2\pi k \cdot dk \tag{3}$$

Number of allowed states in the area enclosed by the first quatorant circles with the radius k and $k + dk$ can be expressed by

$$\underline{2 \times \left(\frac{a}{\pi}\right)^2 \cdot \frac{1}{4} \cdot 2\pi k \cdot dk = \frac{a^2}{\pi} k \cdot dk} \tag{4}$$

(3) The relation between k and E is

$$k = \frac{1}{\hbar}\sqrt{2mE}. \tag{5}$$

Taking the differential, we obtain

$$dk = \frac{1}{\hbar}\sqrt{2m}\frac{1}{2}\frac{1}{\sqrt{E}}dE = \frac{1}{\hbar}\sqrt{\frac{m}{2E}}dE. \tag{6}$$

Substituting these expressions into the equation 4, we obtain

$$\frac{a^2}{\pi}\frac{1}{\hbar}\sqrt{2mE} \cdot \frac{1}{\hbar}\sqrt{\frac{m}{2E}}dE = \frac{a^2 m}{\pi\hbar^2}dE, \tag{7}$$

which represents number of allowed states in real space per unit energy, between E and $E + \Delta E$. Density of allowed states in real space per unit energy can be obtained by deviding the equation by a^2 and dE, i.e.,

$$\underline{\frac{m}{\pi\hbar^2}}. \tag{8}$$

(I.2) (a)

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_C} \tag{9}$$

Then

$$g_T = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_{E_C}^{E_C+kT}(E - E_C)^{1/2}dE = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\left(\frac{2}{3}\right)(E - E_C)^{3/2}\Bigg|_{E_C}^{E_C+kT} \tag{10}$$

or

$$g_T = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2} \tag{11}$$

which yields

$$g_T = 2.12 \times 10^{25}\text{m}^{-3} = 2.12 \times 10^{19}\text{cm}^{-3} \tag{12}$$

(b) Similarly,

$$g_T = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2} \tag{13}$$

$$g_T = 7.92 \times 10^{24}\text{m}^{-3} = 7.92 \times 10^{18}\text{cm}^{-3} \tag{14}$$

II. Statistical Mechanics

(II.1) (a)

At $E = E_{\text{midgap}}$,

$$f(E) = \frac{1}{1 + \exp\frac{E-E_F}{kT}} = \frac{1}{1 + \exp\frac{E_{\text{g}}/2}{kT}} \tag{15}$$

Si:$E_{\text{g}} = 1.12eV$,

$$f(E) = \frac{1}{1 + \exp\frac{1.12}{2(0.0259)}} = \underline{4.07 \times 10^{-10}} \tag{16}$$

Ge:$E_{\text{g}} = 0.66eV$,

$$f(E) = \frac{1}{1 + \exp\frac{0.66}{2(0.0259)}} = \underline{2.93 \times 10^{-6}} \tag{17}$$

(b) The answers are the exactly the same as (a).