Exercise Solid-State Physics (ET2908 and 8027) 2009-20010Q1: dr. R. Ishihara, DIMES-TC00.0044, r.ishihara@tudelft.nl

## Solution for Exercise-sheet 7

## I. Density of States Function

(I.1)
(1) Area occupied by $1 k$ value is

$$
\begin{equation*}
\left(\frac{\pi}{a}\right)^{2} \tag{1}
\end{equation*}
$$

(2) Number of allowed states per unit area in k-space is then

$$
\begin{equation*}
2 \times\left(\frac{a}{\pi}\right)^{2} \tag{2}
\end{equation*}
$$

where the spin effect is took into account.
Area enclosed by the first quatorant circles with the radius $k$ and $k+d k$ can be expressed by

$$
\begin{equation*}
\frac{1}{4} \pi(k+d k)^{2}-\frac{1}{4} \pi k^{2}=\frac{1}{4} \cdot 2 \pi k \cdot d k \tag{3}
\end{equation*}
$$

Number of allowed states in the area enclosed by the first quatorant circles with the radius $k$ and $k+d k$ can be expressed by

$$
\begin{equation*}
2 \times\left(\frac{a}{\pi}\right)^{2} \cdot \frac{1}{4} \cdot 2 \pi k \cdot d k=\frac{a^{2}}{\pi} k \cdot d k \tag{4}
\end{equation*}
$$

(3) The relation between $k$ and $E$ is

$$
\begin{equation*}
k=\frac{1}{\hbar} \sqrt{2 m E} . \tag{5}
\end{equation*}
$$

Taking the differential, we obtain

$$
\begin{equation*}
d k=\frac{1}{\hbar} \sqrt{2 m} \frac{1}{2} \frac{1}{\sqrt{E}} d E=\frac{1}{\hbar} \sqrt{\frac{m}{2 E}} d E . \tag{6}
\end{equation*}
$$

Substituting these expressions into the equation 4 , we obtain

$$
\begin{equation*}
\frac{a^{2}}{\pi} \frac{1}{\hbar} \sqrt{2 m E} \cdot \frac{1}{\hbar} \sqrt{\frac{m}{2 E}} d E=\frac{a^{2} m}{\pi \hbar^{2}} d E \tag{7}
\end{equation*}
$$

which represents number of allowed states in real space per unit energy, between $E$ and $E+\Delta E$. Density of allowed states in real space per unit energy can be obtained by deviding the equation by $a^{2}$ and $d E$, i.e.,

$$
\begin{equation*}
\frac{m}{\pi \hbar^{2}} . \tag{8}
\end{equation*}
$$

(I.2) (a)

$$
\begin{equation*}
g_{c}(E)=\frac{4 \pi\left(2 m_{n}^{*}\right)^{3 / 2}}{h^{3}} \sqrt{E-E_{C}} \tag{9}
\end{equation*}
$$

Then

$$
\begin{equation*}
g_{T}=\frac{4 \pi\left(2 m_{n}^{*}\right)^{3 / 2}}{h^{3}} \int_{E_{C}}^{E_{C}+k T}\left(E-E_{C}\right)^{1 / 2} d E=\left.\frac{4 \pi\left(2 m_{n}^{*}\right)^{3 / 2}}{h^{3}}\left(\frac{2}{3}\right)\left(E-E_{C}\right)^{3 / 2}\right|_{E_{C}} ^{E_{C}+k T} \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
g_{T}=\frac{4 \pi\left(2 m_{n}^{*}\right)^{3 / 2}}{h^{3}}\left(\frac{2}{3}\right)(k T)^{3 / 2} \tag{11}
\end{equation*}
$$

which yields

$$
\begin{equation*}
g_{T}=2.12 \times 10^{25} \mathrm{~m}^{-3}=2.12 \times 10^{19} \mathrm{~cm}^{-3} \tag{12}
\end{equation*}
$$

(b) Similarly,

$$
\begin{gather*}
g_{T}=\frac{4 \pi\left(2 m_{p}^{*}\right)^{3 / 2}}{h^{3}}\left(\frac{2}{3}\right)(k T)^{3 / 2}  \tag{13}\\
g_{T}=7.92 \times 10^{24} \mathrm{~m}^{-3}=7.92 \times 10^{18} \mathrm{~cm}^{-3}  \tag{14}\\
\text { II. STATISTICAL MECHANICS }
\end{gather*}
$$

(II.1) (a)

At $E=E_{\text {midgap }}$,

$$
\begin{equation*}
f(E)=\frac{1}{1+\exp \frac{E-E_{F}}{k T}}=\frac{1}{1+\exp \frac{E_{\mathrm{g}} / 2}{k T}} \tag{15}
\end{equation*}
$$

$\mathrm{Si}: E_{\mathrm{g}}=1.12 \mathrm{eV}$,

$$
\begin{equation*}
f(E)=\frac{1}{1+\exp \frac{1.12}{2(0.0259)}}=\underline{4.07 \times 10^{-10}} \tag{16}
\end{equation*}
$$

$\mathrm{Ge}: E_{\mathrm{g}}=0.66 e V$,

$$
\begin{equation*}
f(E)=\frac{1}{1+\exp \frac{0.66}{2(0.0259)}}=\underline{2.93 \times 10^{-6}} \tag{17}
\end{equation*}
$$

(b) The answers are the exactly the same as (a).

