Chapter 4

The Semiconductor in Equilibrium
Topics

- Thermal-equilibrium concentration of electron and holes
- Intrinsic carrier concentration
- Intrinsic Fermi-level position
- Dopant atoms and energy levels
- Extrinsic carrier concentration and temperature dependence
- Ionization energy of dopant atoms in silicon
- Fermi level in extrinsic semiconductor
- Degenerate semiconductors
- Fermi level in two systems in contact with each other and at thermal equilibrium.
Why?

- Current is determined by flow rate and density of charge carriers.
- The density of electron and holes are related to the density of states function and the Fermi distribution (or probability) function.
Density of states

\[ g(E) = \frac{4\pi (2m^*)^{3/2}}{h^3} \sqrt{E} \]

Density of quantum states per unit volume at energy \( E \)

For conduction band

\[ g_c(E) = \frac{4\pi (2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \]

For valence band

\[ g_v(E) = \frac{4\pi (2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E} \]
Fermi-Dirac distribution (or probability) function

\[
\frac{N(E)}{g(E)} = f(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}}
\]

The probability that a quantum state at an energy \( E \) will be occupied by an electron

The ratio between filled and total quantum states at any energy \( E \)
Fermi-Dirac distribution (or probability) function

\[ f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \]
Distribution of electron and holes

Number of electrons at \( E \) (in conduction band)

\[
n( E ) = g_c( E ) f_F( E )
\]

Density of states at \( E \)
Fermi-Dirac probability function

Number of holes at \( E \) (in valence band)

\[
p( E ) = g_v( E ) \left[ 1 - f_F( E ) \right]
\]
conduction band

valence band
Electron concentration

\[ n_0 = \int_{\text{Top of conduction band}}^{\text{Bottom of conduction band}} \text{Density of states} \ast \text{Probability function} \, dE \]

\[ n_0 = \int_{E_C}^{\infty} g_c(E) f_F(E) \, dE \]

The equation is valid for both intrinsic and extrinsic semiconductors
\[ n_0 = \int g_c(E) f_F(E) \, dE \]

\[ f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \]

**Condition:**

\[ E - E_F >> kT \]

\[ f_F(E) \approx \exp\left[\frac{-(E - E_F)}{kT}\right] \text{ Boltzmann approximation} \]
Comparison of Fermi-Dirac probability function and Maxwell-Boltzmann approximation

\[ f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \]

Maxwell-Boltzmann approximation and Fermi-Dirac function are within 5% of each other when \( E - E_F \geq 3kT \)
\[ n_0 = \int_{E_c}^{\infty} \frac{4\pi (2m^*_n)^{3/2}}{h^3} \sqrt{E - E_c} \exp\left[ \frac{-(E - E_F)}{kT} \right] dE \]

\[ \eta = \frac{E - E_c}{kT} \]

\[
n_0 = \frac{4\pi (2m^*_n kT)^{3/2}}{h^3} \exp\left[ \frac{-(E_c - E_F)}{kT} \right] \int_0^\infty \eta^{1/2} \exp(-\eta) d\eta
\]

Gamma function:

\[
\frac{1}{2} \sqrt{\pi}
\]
\[ n_0 = 2 \left( \frac{2\pi m^*_n kT}{\hbar^2} \right)^{3/2} \exp \left[ \frac{-(E_c - E_F)}{kT} \right] \]

\[ N_c = 2 \left( \frac{2\pi m^*_n kT}{\hbar^2} \right)^{3/2} \]

\( N_c = \text{effective density of states function in the conduction band} \)

\[ n_0 = N_c \exp \left[ \frac{-(E_c - E_F)}{kT} \right] \]

The equation is valid for both intrinsic and extrinsic semiconductors.
Hole concentration

\[ p_0 = \int g_v(E)[1 - f_F(E)] \, dE \]

\[ 1 - f_F(E) = \frac{1}{1 + \exp\left(\frac{E_F - E}{kT}\right)} \]

\[ 1 - f_F(E) = \frac{1}{1 + \exp\left(\frac{E_F - E}{kT}\right)} \approx \exp\left[\frac{-(E_F - E)}{kT}\right] \]

\[ p_0 = \int_{-\infty}^{E_v} \frac{4\pi (2m^*_p)^{3/2}}{h^3} \sqrt{E_v - E} \exp\left[\frac{-(E_F - E)}{kT}\right] \, dE \]
\[ p_0 = 2 \left( \frac{2\pi m_p^* kT}{h^2} \right)^{3/2} \exp \left[ -\frac{(E_F - E_v)}{kT} \right] \]

\[ N_v = 2 \left( \frac{2\pi m_p^* kT}{h^2} \right)^{3/2} \]

\[ p_0 = N_v \exp \left[ -\frac{(E_F - E_v)}{kT} \right] \]

\( N_v \) = effective density of states function in the valence band
Effective density of states function and effective mass values

<table>
<thead>
<tr>
<th></th>
<th>$N_c (\text{cm}^{-3})$</th>
<th>$N_v (\text{cm}^{-3})$</th>
<th>$m_n^* / m_0$</th>
<th>$m_p^* / m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>$2.8 \times 10^{19}$</td>
<td>$1.04 \times 10^{19}$</td>
<td>1.08</td>
<td>0.56</td>
</tr>
<tr>
<td>Gallium Arsenide</td>
<td>$4.7 \times 10^{17}$</td>
<td>$7.0 \times 10^{18}$</td>
<td>0.067</td>
<td>0.48</td>
</tr>
<tr>
<td>Germanium</td>
<td>$1.04 \times 10^{19}$</td>
<td>$6.0 \times 10^{18}$</td>
<td>0.55</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Intrinsic semiconductor

- Intrinsic electron concentration = Intrinsic hole concentration

\[ n_i = p_i \]

Intrinsic carrier concentration

Why?

- charge carriers due to thermal excitation
- thermally generated electrons and holes always created in pairs.
**INTRINSIC Semiconductor**

Intrinsic Fermi level

\[ n_0 = n_i = N_c \exp \left[ -\frac{(E_c - E_{Fi})}{kT} \right] \]

Intrinsic carrier concentration

\[ p_0 = p_i = n_i = N_v \exp \left[ -\frac{(E_{Fi} - E_v)}{kT} \right] \]

\[ n_i^2 = N_c N_v \exp \left[ -\frac{(E_c - E_{Fi})}{kT} \right] \cdot \exp \left[ -\frac{(E_{Fi} - E_v)}{kT} \right] \]
$$n_i^2 = N_c N_v \exp \left[ \frac{- (E_c - E_v)}{kT} \right] = N_c N_v \exp \left[ \frac{-E_g}{kT} \right]$$

Commonly accepted values of $n_i$ at T=300K

<table>
<thead>
<tr>
<th>Semiconductor</th>
<th>$N_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>$1.5 \times 10^{10}$ cm${}^{-3}$</td>
</tr>
<tr>
<td>Gallium Arsenide</td>
<td>$1.8 \times 10^{6}$ cm${}^{-3}$</td>
</tr>
<tr>
<td>Germanium</td>
<td>$2.4 \times 10^{13}$ cm${}^{-3}$</td>
</tr>
</tbody>
</table>
\[
\log_{10}(n_i) = \frac{1}{2} \log_{10}(N_c N_v) - \frac{E_g}{2kT} \log_{10} e
\]

Plot of \(\log_{10}(n_i)\) vs. \(1/T\) is straight line.

Slope is negative. From the slope \(E_g\) can be calculated
Application of the intrinsic semiconductors

- High Electron Mobility Transistor
- High resistivity substrate for RF circuits
- amorphous-Si Solar Cells

Structure of solar cell
Where is the intrinsic Fermi level?

\( E_{Fi} \) (Intrinsic Fermi level): \( E_F \) at which electron and hole concentration becomes equal
Even in intrinsic semiconductor, Fermi level is not exactly at centre between conduction and valence bands.
The Extrinsic Semiconductor
Intrinsic silicon lattice
Acceptor and Donor Impurities:

- In Si four electrons in the valence shell participate in bonding.
- Atom with more than 4 valence electrons $\rightarrow$ donor impurity
- Less than 4 $\rightarrow$ acceptor impurity.
Donor Impurity:

- At very low temperature, the donor (excess) electron is still bound to the impurity atom.
- However, the donor electron is loosely bound to the impurity atom and can become free with small amount of thermal energy. Impurity atom is then ionized and positively charged.
Donor electron energy level:

- little energy required to move donor electrons from donor states to conduction band.
- positively charged donor ions are fixed but donor electrons in the conduction band can move through the crystal.
Acceptor Impurity:

- One covalent bond is incomplete for Si.
- With little thermal energy, a valence electron can break from another covalent bond and can occupy this position, thus creating a hole at the location of the broken covalent bond.
- The acceptor impurity is then ionized and negatively charged.
Acceptor Energy Level:

• little energy required to move valence electrons to acceptor levels.
• negatively charged acceptor ions are fixed but holes in the valence band can move through the crystal.
Electron concentration vs. temperature in n-type semiconductor

Electron concentration vs. temperature showing partial ionization, extrinsic and intrinsic regions.
Electron concentration vs. temperature in extrinsic semiconductor

- At low temperatures, donor impurities are partially ionized. As temperature increases, the percentage of ionized donor impurities also increases.

- Once all donor impurities are ionized, there is no increase in carrier concentration. Even though intrinsic carrier concentration continues to increase, it is still small compared to extrinsic concentration.

- At high temperatures, intrinsic carrier concentration dominates, and electron concentration continues to increase again.
Ionization energy:

The ionization energy is the energy necessary to remove an electron from the neutral atom.

In case of donor atoms, the ionization energy is the energy necessary to elevate an electron from the donor level to conduction band.
In the next few slides, we will calculate the approximate ionization energy for donor atoms.

We use Bohr atomic model for these calculations. For hydrogen atom, Bohr model and quantum mechanics give similar results.

Donor impurity atom can be visualized as one donor electron orbiting the positively charged donor ion. This condition is similar to that in a hydrogen atom.

However we have to consider permittivity of silicon instead of permittivity of free space.
Angular Momentum Quantization

- Bohr proposed that circumference of electron orbit = integer number of wavelengths $\Rightarrow 2\pi r = n\lambda_n$

Then angular momentum,

$$L = mvr = \frac{hr}{\lambda} = \left[ \frac{hr}{2\pi r} \right] = \frac{nh}{2\pi}$$

angular momentum of electron is quantized.
Ionization energy calculation:

Coulomb attraction force  Centripetal force

\[
\frac{e^2}{4\pi \varepsilon r_n^2} = \frac{m^* v^2}{r_n}
\]

Angular momentum quantization

\[
m^* r_n v = n\hbar
\]

\[
r_n = \frac{n^2 \hbar^2 4\pi \varepsilon}{m^* e^2}
\]
As defined in Chapter 2, Bohr radius is given by:

$$a_0 = \frac{4\pi \varepsilon_0 \hbar^2}{m_0 e^2} = 0.53 \text{ Å}$$

The orbiting electron radius relative to the Bohr radius is:

$$\frac{r_n}{a_0} = n^2 \varepsilon_r \left( \frac{m_0}{m^*} \right)$$

For silicon, \(\varepsilon_r = 11.7\) and \(\frac{m^*}{m_0} = 0.26\). For \(n=1\),

$$\frac{r_1}{a_0} = 45$$
• $r_1/a_0 = 45$ or $r_1 = 23.9 \text{ Å}$

• This radius $\sim 4$ lattice constants of Si.

• Each unit cell contains 8 silicon atoms.

• Donor electron thus loosely bound to the donor atom.

• We will next find the approximate ionization energy.
Total energy $E = T + V$

Kinetic energy $T = \frac{m* e^4}{2(n\hbar)^2(4\pi \epsilon)^2}$

Potential energy $V = \frac{-e^2}{4\pi \epsilon r_n} = \frac{-m* e^4}{(n\hbar)^2(4\pi \epsilon)^2}$

$E = T + V = \frac{-m* e^4}{2(n\hbar)^2(4\pi \epsilon)^2}$

(refer slide 37)
• Ionisation energy of Hydrogen in lowest energy state = -13.6eV
• For Si, it is -25.8meV << band gap.
• Calculations using Bohr model give only the order of magnitude of the ionisation energy. Actual values differ.

### Impurity ionization energies in Silicon and Germanium

<table>
<thead>
<tr>
<th>Impurity</th>
<th>Ionization energy (eV)</th>
<th>Donors</th>
<th>Acceptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>0.045</td>
<td>0.05</td>
<td>0.045</td>
</tr>
<tr>
<td>Ge</td>
<td>0.012</td>
<td>0.0127</td>
<td>0.0104</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>0.045</td>
<td>0.05</td>
<td>0.045</td>
</tr>
<tr>
<td>Arsenic</td>
<td>0.05</td>
<td>0.0127</td>
<td>0.0102</td>
</tr>
<tr>
<td>Beryllium</td>
<td>0.028</td>
<td>0.0307</td>
<td>0.0345</td>
</tr>
<tr>
<td>Zinc</td>
<td>0.0347</td>
<td></td>
<td>0.0404</td>
</tr>
<tr>
<td>Cadmium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silicon</td>
<td>0.0058</td>
<td>0.0058</td>
<td>0.0345</td>
</tr>
<tr>
<td>Germanium</td>
<td>0.0061</td>
<td>0.0061</td>
<td>0.0404</td>
</tr>
</tbody>
</table>

### Impurity ionization energies in gallium arsenide

<table>
<thead>
<tr>
<th>Impurity</th>
<th>Ionization energy (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selenium</td>
<td>0.0059</td>
</tr>
<tr>
<td>Tellurium</td>
<td>0.0058</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.0058</td>
</tr>
<tr>
<td>Germanium</td>
<td>0.0061</td>
</tr>
<tr>
<td>Beryllium</td>
<td>0.028</td>
</tr>
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</tr>
<tr>
<td>Germanium</td>
<td>0.0404</td>
</tr>
</tbody>
</table>
**Extrinsic Semiconductor**

![Diagram showing n-type and p-type semiconductors with majority and minority carrier concentrations.]

- **n-type**
  - Majority carrier: electrons
  - Minority carrier: holes
  - Equation: \( n_0 = N_c \exp \left[ -\frac{(E_c - E_F)}{kT} \right] \)

- **p-type**
  - Majority carrier: holes
  - Minority carrier: electrons
  - Equation: \( p_0 = N_v \exp \left[ -\frac{(E_F - E_v)}{kT} \right] \)
The equation is valid for both intrinsic and extrinsic semiconductors.
Another form (relation between $E_F$ and $E_{Fi}$)

Intrinsic carrier concentration

\[
n_0 = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \exp\left[\frac{(E_F - E_{Fi})}{kT}\right]
\]

\[
n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]
\]

\[
p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]
\]

\[
n_0p_0 = n_i^2
\]
Where is the Fermi level?

\[ n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] \]

\[ E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) \]

\[ n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \]

\[ E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) \]
Condition for the Boltzmann approximation

\[ E_C - E_F > 3KT \]
If the impurity concentration is very high....

Fermi level will be very close to conduction band or valence band.

No Boltzmann approximation

Use The Fermi-Dirac Integral
\[
f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}
\]

Boltzmann approximation

\[
f_F(E) \approx \exp\left[\frac{-(E - E_F)}{kT}\right]
\]

Only if \(E_C - E_F > 3KT\)
Fermi-Dirac Integral

\[ n_0 = \frac{4\pi}{h^3} (2m_n^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2} dE}{1 + \exp \left( \frac{E - E_F}{kT} \right)} \]

\[ \eta = \frac{E - E_c}{kT} \]

\[ \eta_F = \frac{E_F - E_c}{kT} \]

\[ n_0 = 4\pi \left( \frac{2m_n^* kT}{h^2} \right)^{3/2} \int_0^\infty \frac{\eta^{1/2} d\eta}{1 + \exp (\eta - \eta_F)} \]

\[ F_{1/2}(\eta_F) = \int_0^\infty \frac{\eta^{1/2} d\eta}{1 + \exp (\eta - \eta_F)} \]

If \( \eta_F > 1 \), then \( E_F > E_C \)
Degenerate Semiconductors
If the impurity atoms are very close each other...

• Donor electrons interact with each other
• The single discrete donor energy will split into a band
• The band may overlap the conduction band
• If the concentration exceed $N_c$, $E_F$ lies within the conduction band
Degenerated Semiconductor

\[ N_d > N_c \quad \text{and} \quad N_a > N_v \]

Fermi level in the conduction band: Metallic conduction
Statistics of donors and acceptors
How many electrons still in the donor levels compared to the total number of electrons?

derpends on the temperature and the Fermi level....
Probability function for donor & acceptor levels

The probability function for donor & acceptor levels is given by:

\[
\frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2} \exp\left(\frac{E_d - E_F}{kT}\right)}
\]

This is the same as the Fermi-Dirac probability function except for the pre-exponential coefficient of \(1/2\).

The concentration of ionized donors, \(n_d\), is given by:

\[
 n_d = N_d - N_d^+ 
\]
similar for holes:

\[
p_a = \frac{N_a}{1 + \frac{1}{g} \exp \left( \frac{E_F - E_a}{kT} \right)} = N_a - N_a^-
\]

\(g=\)degeneration factor; 4 for GaAs and Si acceptor levels
Low temperature

Moderate temperature

High temperature
Moderate temperature

\[ E_d - E_F \gg kT \]

\[
\frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2} \exp\left(\frac{E_d - E_F}{kT}\right)}
\]

\[
n_d \approx \frac{N_d}{1 + \frac{1}{2} \exp\left(\frac{E_d - E_F}{kT}\right)} = 2N_d \exp\left[-\frac{(E_d - E_F)}{kT}\right]
\]

If \( E_d - E_F \gg kT \), then even \( E_c - E_F \gg kT \)

Then, \( n_0 = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right] \)
Fraction of electrons still in the donor states

\[
\frac{n_d}{n_d + n_0} = \frac{2N_d \exp \left[\frac{-(E_d - E_F)}{kT}\right]}{2N_d \exp \left[\frac{-(E_d - E_F)}{kT}\right] + N_c \exp \left[\frac{-(E_c - E_F)}{kT}\right]}
\]

\[
\frac{n_d}{n_d + n_0} = \frac{1}{1 + \frac{N_c}{2N_d} \exp \left[\frac{-(E_c - E_d)}{kT}\right]}
\]

With Phosphorus doping of \(N_d = 10^{16}\) cm\(^{-3}\), at 
\(T=300K\), \(n_d/(n_d+n_0)=0.41\%\)

Almost complete ionization at Room Temp!
Extremely low temperature (T=0K)

\[ n_d = N_d \]

\[ n_d = \frac{N_d}{1 + \frac{1}{2} \exp \left( \frac{E_d - E_F}{kT} \right)} \]

\[ E_F > E_d \]
High temperature

\[ n \gg N_D \] (because of thermally generated electrons)

\[ N_{D^+} = N_D \]

\[ p = n \gg N_D \] (because of thermally generated holes)

\[ n_0 = n_i = N_c \exp \left[ \frac{-(E_c - E_{F_i})}{kT} \right] \]

- At very high temperature behavior is just like the intrinsic semiconductor
Compensated semiconductor

- Both donor and acceptor impurities in the same region
- If $N_d > N_a \rightarrow$ n-type compensated semiconductor
- If $N_d < N_a \rightarrow$ p-type compensated semiconductor
- If $N_d = N_a \rightarrow$ completely compensated (will behave like intrinsic material)
- Practical semiconductor is always compensated semiconductor.

Eg. Substrate is predoped usually p-type. All other dopings are done on top of this.
Charge neutrality:

\[ n_0 + \frac{N_a}{N_a - p_a} = p_0 + \frac{N_d}{N_d - n_d} \]

**complete ionization**

\[ n_d \text{ and } p_a \text{ are both zero} \]

\[ n_0 + N_a = p_0 + N_d \]
Recall \[ n_0 + N_a = p_0 + N_d \]

Using the relation \[ n_i^2 = n_0 p_0 \]

\[ n_0 + N_a = \frac{n_i^2}{n_0} + N_d \]

\[ n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \]

\[ n_0 \text{ is not simply } N_d \]
Similarly in p-type semiconductor,

\[ p_0 = \frac{N_a - N_d}{2} + \sqrt{\left( \frac{N_a - N_d}{2} \right)^2 + n_i^2} \]

Minority carrier concentration

\[ p_0 = \frac{n_i^2}{n_0} \]

\[ n_0 = \frac{n_i^2}{p_0} \]

(n-type material) \hspace{2cm} (p-type material)
IV. Position of Fermi Energy Level
Where is the Fermi level of an extrinsic semiconductor?

\[ n_0 = N_c \exp \left[ \frac{-(E_c - E_F)}{kT} \right] \]

\[ E_c - E_F = kT \ln \left( \frac{N_c}{n_0} \right) \]

N-type: \( N_d >> n_i \) then \( n_0 \approx N_d \)

\[ E_c - E_F = kT \ln \left( \frac{N_c}{N_d} \right) \]
Where is the Fermi level of a p-type extrinsic semiconductor?

\[ p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] \]

\[
E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right)
\]

P-type: \( N_a >> n_i \) then \( p_0 \approx N_a \)

\[
E_F - E_v = kT \ln\left(\frac{N_v}{N_a}\right)
\]
Position of Fermi level for an (a) n-type and (b) p-type semiconductor.
Different expression for the n-type...

\[ E_F - E_{Fi} = kT \ln \left( \frac{n_0}{n_i} \right) \]

Another expression for the p-type

\[ E_{Fi} - E_F = kT \ln \left( \frac{p_0}{n_i} \right) \]
Variation of $E_F$ with doping concentration:

$$E_F = E_c - kT \ln \left( \frac{N_c}{N_d} \right)$$

$$E_F = E_v + kT \ln \left( \frac{N_v}{N_a} \right)$$
**Variation of $E_F$ with temperature $T$**

\[
E_F - E_{Fi} = kT \ln \left( \frac{n_0}{n_i} \right)
\]

\[
E_{Fi} - E_{F} = kT \ln \left( \frac{p_0}{n_i} \right)
\]

- At higher temperatures, the semiconductor becomes more intrinsic. $n_i$ increases and Fermi level moves towards mid-gap.

- At $T=0$, Fermi level is above $E_d$ in n-type and below $E_a$ in p-type semiconductor.

Variation of Fermi level with temperature for different doping concentrations.
EF must be equal when different systems are in contact and in thermodynamic equilibrium.

Consider a material A, with Fermi level $E_{FA}$. Bands below $E_{FA}$ are full and above are empty.

material B with Fermi level $E_{FB}$.
EF must be equal when different systems are in contact and in thermodynamic equilibrium

- When A and B are brought in contact, electrons will flow from A into lower energy states of B, until thermal equilibrium is reached.

- Thermal equilibrium $\Rightarrow E_F$ same in A & B
Summary

• Electron concentration

\[ n_0 = N_c \exp \left[ \frac{(E_c - E_F)}{kT} \right] \]

• Hole concentration

\[ p_0 = N_v \exp \left[ \frac{(E_F - E_v)}{kT} \right] \]

• Intrinsic carrier concentration:

\[ n_i^2 = n_0 p_0 = N_c N_v \exp \left[ \frac{(E_c - E_v)}{kT} \right] - N_c N_v \exp \left[ \frac{E_g}{kT} \right] \]

• In intrinsic semiconductor, Fermi level is close to but not exactly in the centre between conduction and valence bands.

\[ E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln \left( \frac{m_p^*}{m_n^*} \right) \]
Summary

• In extrinsic semiconductor, Fermi level is close to conduction band (n-type) or valence band (p-type)

• Position of Fermi level in extrinsic semiconductor

\[ E_F - E_{Fi} = kT \ln \left( \frac{n_0}{n_i} \right) \]

• In compensated n-type semiconductor electron concentration is given by

\[ n_0 = \frac{N_d - N_a}{2} \sqrt{\left( \frac{N_d + N_a}{2} \right) \frac{2}{n_i^2} - n_i^2} + \]

• When two different systems are in contact and in thermal equilibrium, \( E_F \) must be the same in both systems.