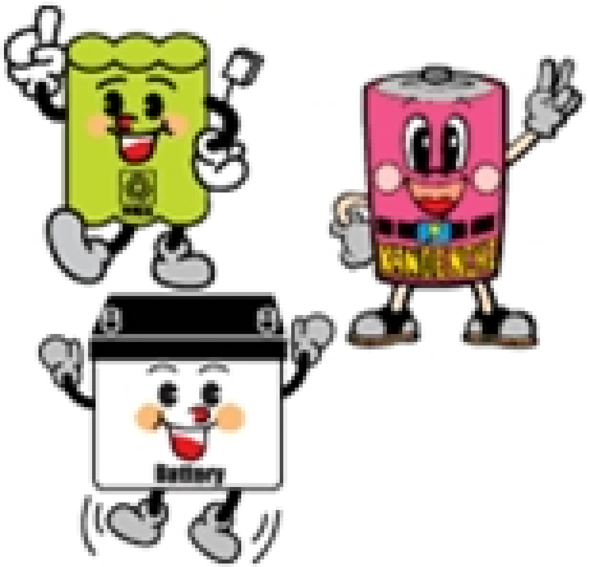
Chapter 5

Carrier Transport Phenomena

Department of Microelectronics and Computer Engineering

TU Delft

We now study the effect of external fields (electric field, magnetic field) on semiconducting material



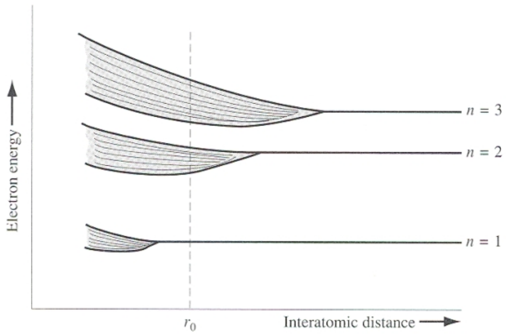
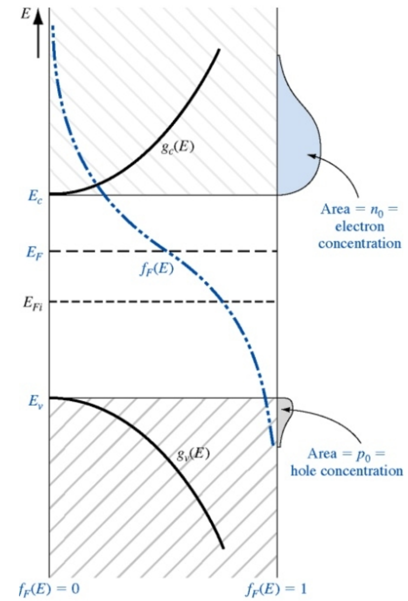


Figure 2.12 Schematic showing the splitting of three energy states into allowed bands of energies



Department of Microelectronics and Computer Engineering

TU Delft

Objective

- Discuss drift and diffusion current densities
- Explain why carriers reach an average drift velocity
- Discuss mechanism of lattice and impurity scattering
- Define mobility, conductivity and resistivity
- Discuss temperature and impurity dependence on mobility and velocity saturation
- State the Einstein relation
- Describe the Hall effect

Department of Microelectronics and Computer Engineering

TU Delft

Drift current

Electric field → force on electrons and holes

↓ Free states in conduction and valence band

net movement of electrons and holes

Net movement of charge due to electric field is called drift.

Department of Microelectronics and Computer Engineering

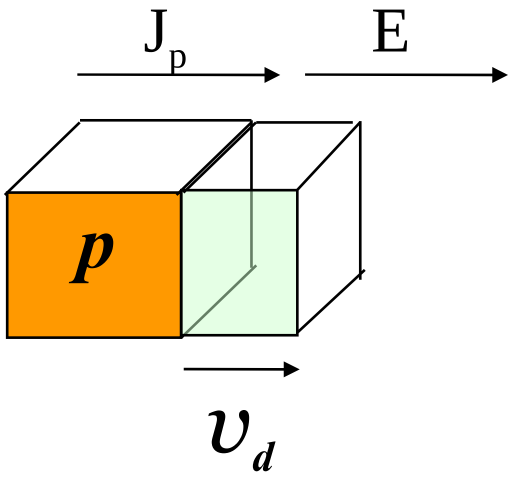
Drift current density

for holes

Average drift velocity

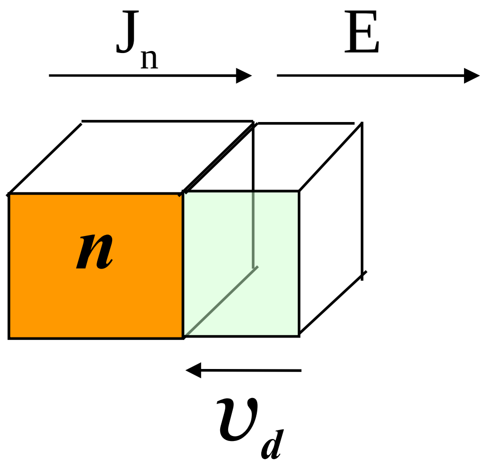
$$J_{p,drf} = e\,p\,v_d \quad A/cm^2$$

(Volume) density of holes



for electrons

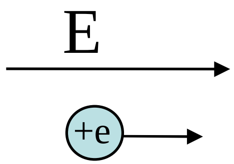
$$J_{n,drf} = -e\,n\,v_d \quad A/cm^2$$



Department of Microelectronics and Computer Engineering

TU Delft

Velocity of the particles



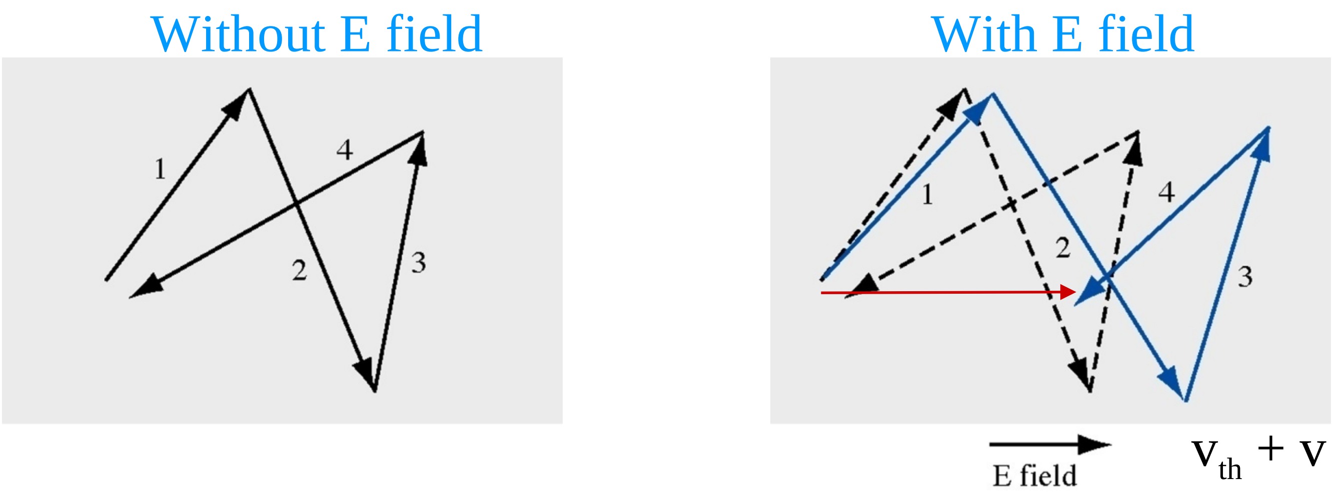
$$F = m_p^* \frac{dv}{dt} = e\mathrm{E} \qquad v = \frac{e\mathrm{E}t}{m_p^*}$$

v → drift velocity of the hole in the electric field

So does velocity monotonically increase with time?

Department of Microelectronics and Computer Engineering

Thermal and drift velocities



(a) (b)

- Even in the absence of E-field the holes have random thermal velocity (v_{th})
- They collide with ionized impurity atoms and thermally vibrating lattice atoms.
- Let τ_{cp} → mean time between collisions.
- with E-field → net drift of holes in the direction of the E-field
- net drift velocity is small perturbation on random thermal velocity.
- so τ_{cp} remains almost unchanged even in the presence of E-field.

Department of Microelectronics and Computer Engineering

Thermal and drift velocities

Without E field | With E field

(a) (b)

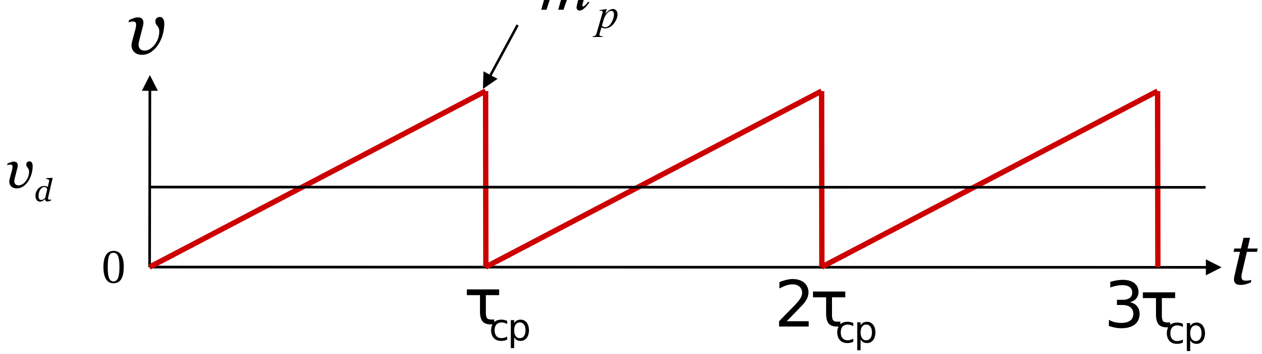
Now, $\upsilon = \dfrac{eEt}{m_p^*}$ $\Rightarrow$ $v_{peak} = \dfrac{e\tau_{cp}}{m_p^*} E$

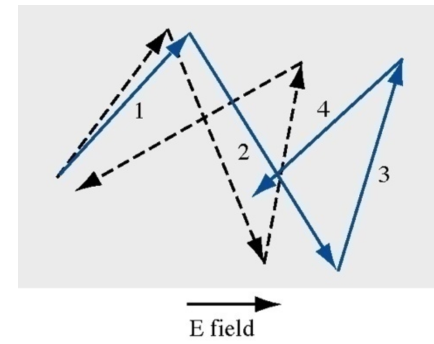
Department of Microelectronics and Computer Engineering

TUDelft

Drift velocity v_d

$$v_{peak} = \frac{e\tau_{cp}}{m_p^*}E$$





(b)

Average drift velocity = $\langle v_d \rangle = \frac{1}{2}\left(\frac{e\tau_{cp}}{m_p^*}\right)E$

Using more accurate model including the effect of statistical distribution,

$$\langle v_d \rangle = \boxed{\left(\frac{e\tau_{cp}}{m_p^*}\right)}E$$

μ_p (mobility)

Department of Microelectronics and Computer Engineering

TU Delft

Mobility:

$$\upsilon_d = \mu E$$

For holes: $\mu_p \rightarrow$ *hole mobility*

$$J_{p|drf} = (ep)\upsilon_{dp} = e\mu_p p E$$

For electrons: $\mu_n \rightarrow$ *electron mobility* $\upsilon_{dn} = -\mu_n E$

$$J_{n|drf} = (-en)(-\mu_n E) = e\mu_n n E$$

Department of Microelectronics and Computer Engineering

TU Delft

Mobility:

$$\mu = v_d / E$$ Unit: cm^2/Vs

	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)
Silicon	1350	480
Gallium Arsenide	8500	400
Germanium	3900	1900

	m_n^*/m_0	m_p^*/m_0
Silicon	1.08	0.56
Gallium Arsenide	0.067	0.48
Germanium	0.55	0.37

$$\mu_n = \frac{e\tau_{cn}}{m_n^*}$$

Department of Microelectronics and Computer Engineering

Scattering:

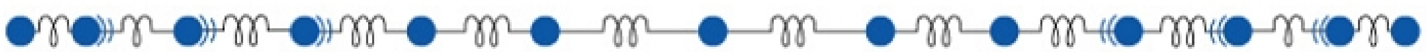
Two main scattering mechanisms -

- Lattice scattering or phonon scattering
- Impurity scattering

Department of Microelectronics and Computer Engineering

Phonon Scattering

Phonons are lattice vibrations (Atoms randomly vibrate about their position @ T>0K)

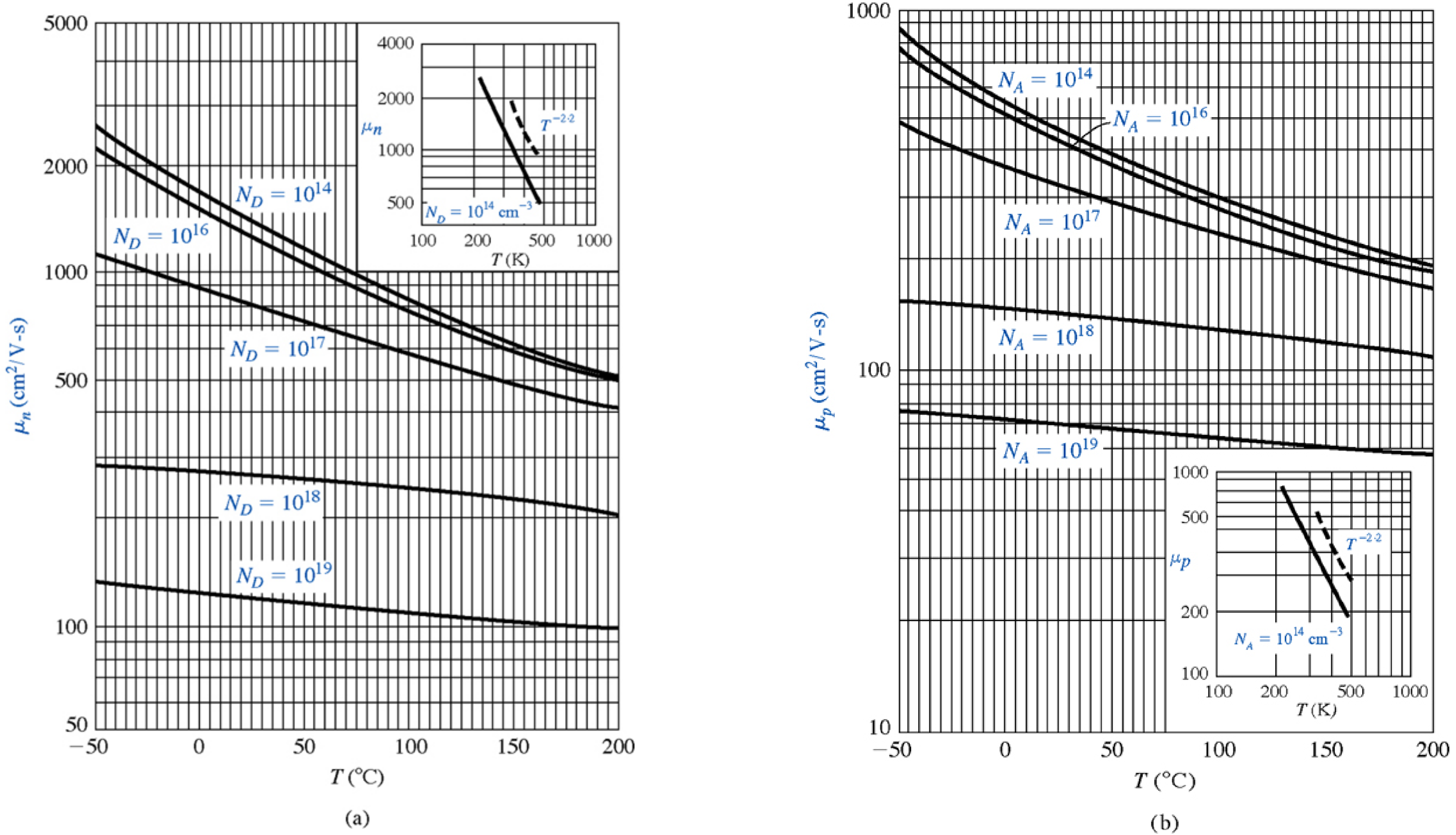


Lattice vibration causes a local volume change and hence lattice constant change. → Bandgap generally widens with a smaller lattice constant. → Disruption of valence and conduction band edges scatters the carriers.

•Mobility due to lattice scattering $\mu_L \propto T^{-\frac{3}{2}}$ (as temp increases vibration of atoms also increases)

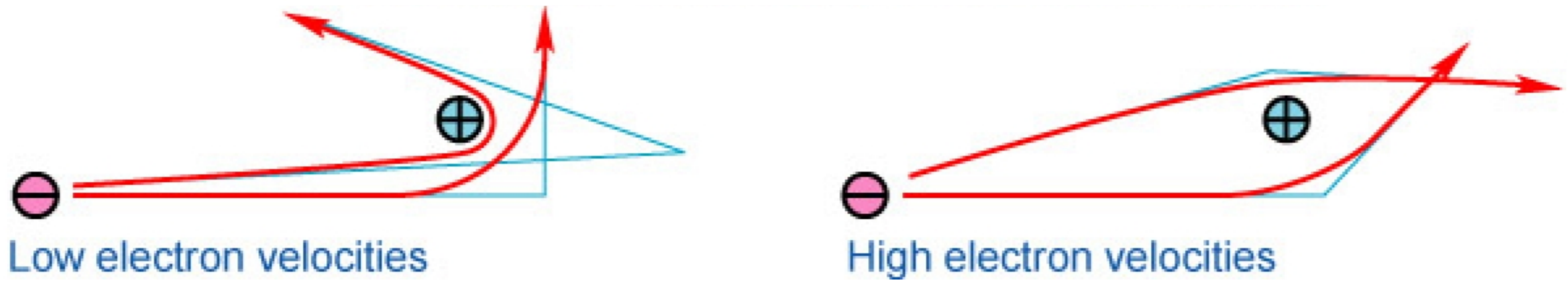
Department of Microelectronics and Computer Engineering

TU Delft



(a) Electron and (b) Hole mobilities in Si vs. T at different doping concentrations. (Inserts show dependence for almost intrinsic Si)

Ionized impurity scattering



Scattering due to coulomb interaction between electrons/holes and ionized impurities.

$$\mu_I \propto \frac{T^{+3/2}}{N_I}$$

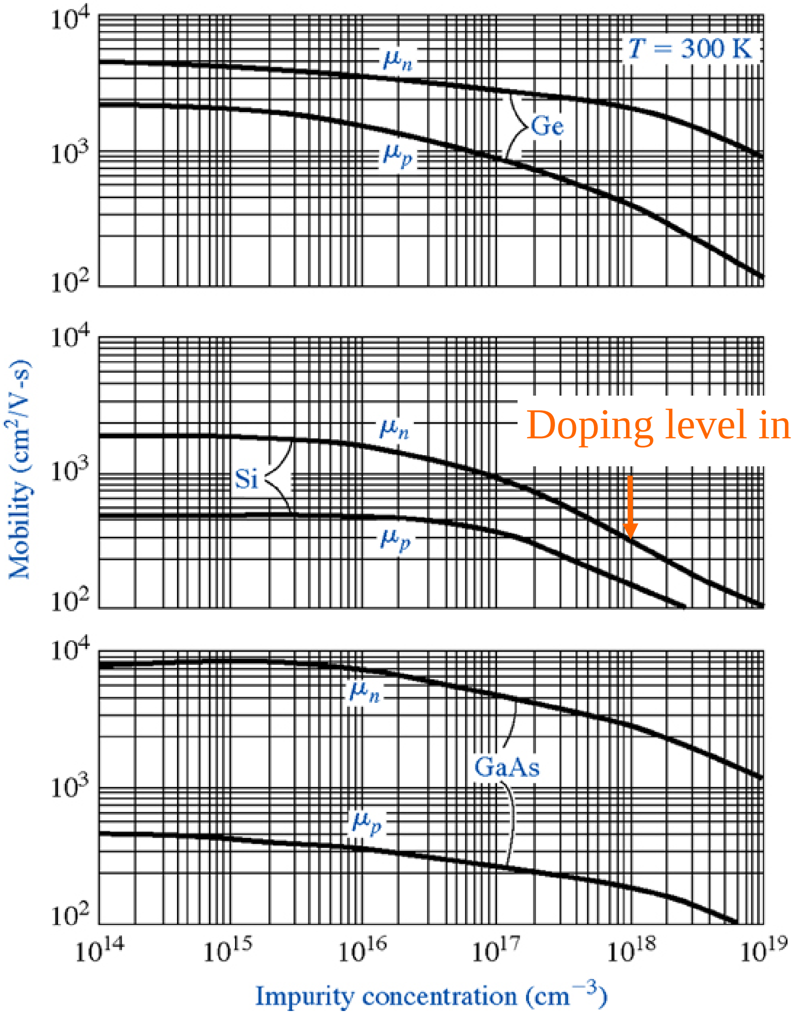
T increases → thermal velocity v_{th} increases, so less time spent for scattering → $\mu_I \propto T^n$

$$N_I = N_d^+ + N_a^-$$

N_I increases → the scattering chance increases → $\mu_I \propto 1/N_I$

Department of Microelectronics and Computer Engineering

TU Delft



High doping is required to overcome short channel effects even though it reduces mobility.

Electron and Hole mobility vs. impurity concentration.

Department of Microelectronics and Computer Engineering

TU Delft

How to combine mobility effects?

τ_I : average time between two collisions with dopant atoms

τ_L : average time between two collisions with "vibrating" lattice points

$\frac{dt}{\tau_I}$: Number of collisions in time dt due to impurity scattering

$\frac{dt}{\tau_L}$: Number of collisions in time dt due to lattice scattering

Total number of collisions in dt: $\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L}$

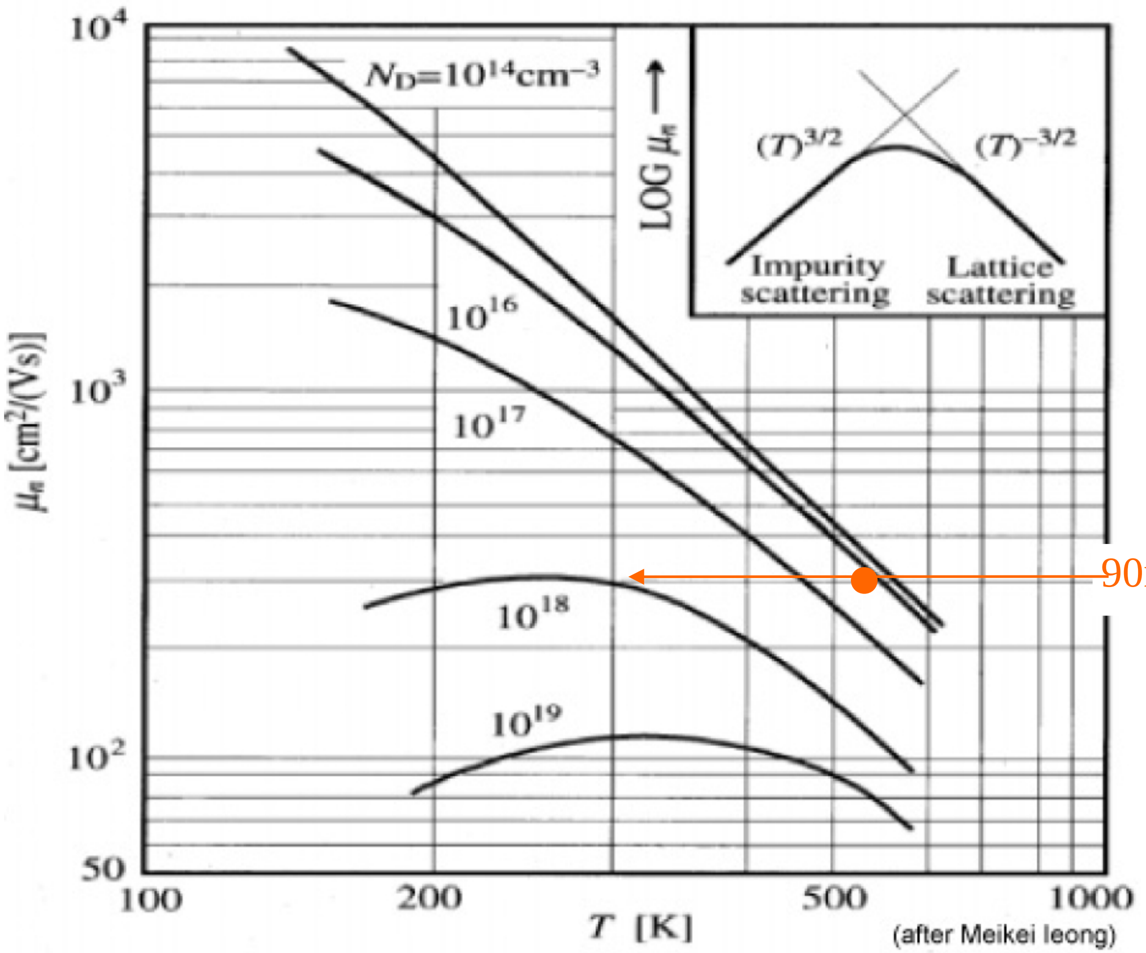
$\mu_n = \frac{e\tau_{cn}}{m_n^*}$

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

Department of Microelectronics and Computer Engineering

TU Delft

Electron mobility of Si vs. T for various Na



At present doping level, cooling does not improve speed much.

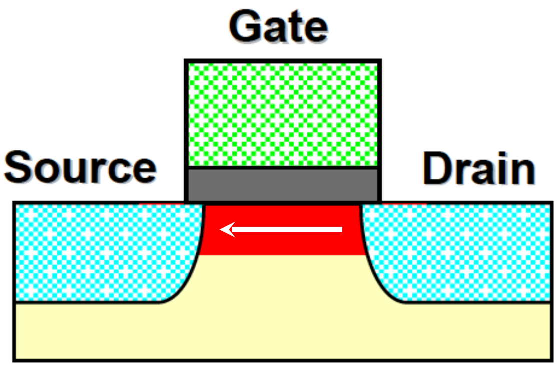
Low doping concentration or high T → The lattice scattering dominates

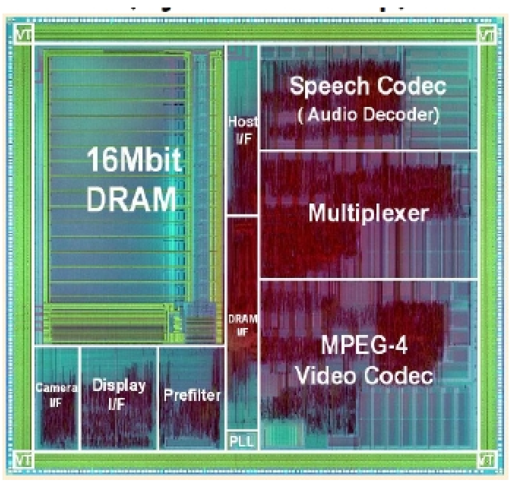
High doping concentration or low T → The impurity scattering dominates

Department of Microelectronics and Computer Engineering

Drift current density

$$J_{n|drf} = e \mu n E \qquad \mu = \frac{e \tau_c}{m^*}$$





- J increases then
  - cut-off frequency increases
  - circuit density increases

How can we increase J?

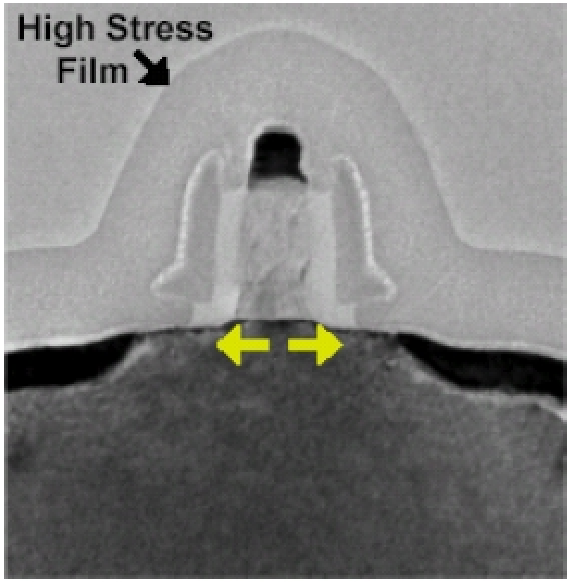
How can we increase J?

- Increase mobility
  - Strained silicon → effective mass↓ , mobility↑
  - GaAs or Ge as semiconducting material
- Increase electric field
  - Shorter gate length

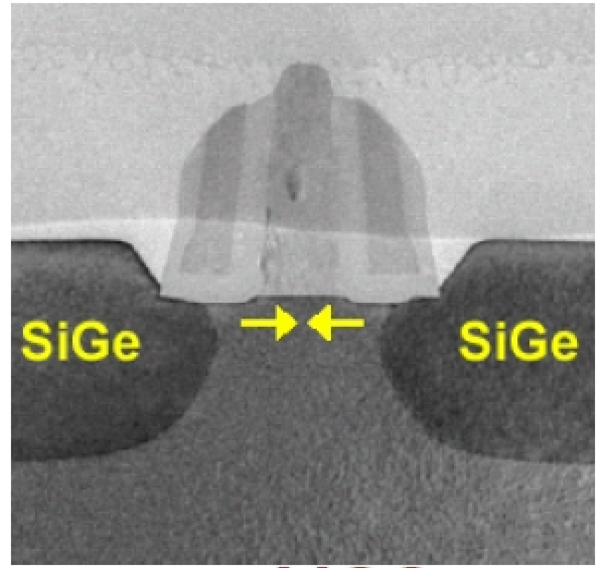
Department of Microelectronics and Computer Engineering

TU Delft

Strained Si



nMOS



pMOS

Intel 90nm process

- Strain decreases the effective mass, increasing the mobility
- already been used for production

Department of Microelectronics and Computer Engineering

TU Delft

Conductivity

$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E} = \sigma \mathrm{E}$$

$$\sigma = e\mu_n n + e\mu_p p$$

$\sigma \rightarrow$ *conductivity*

Units $\rightarrow (\Omega\text{-cm})^{-1}$

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

$\rho \rightarrow$ *resistivity*

Units $\rightarrow \Omega\text{-cm}$

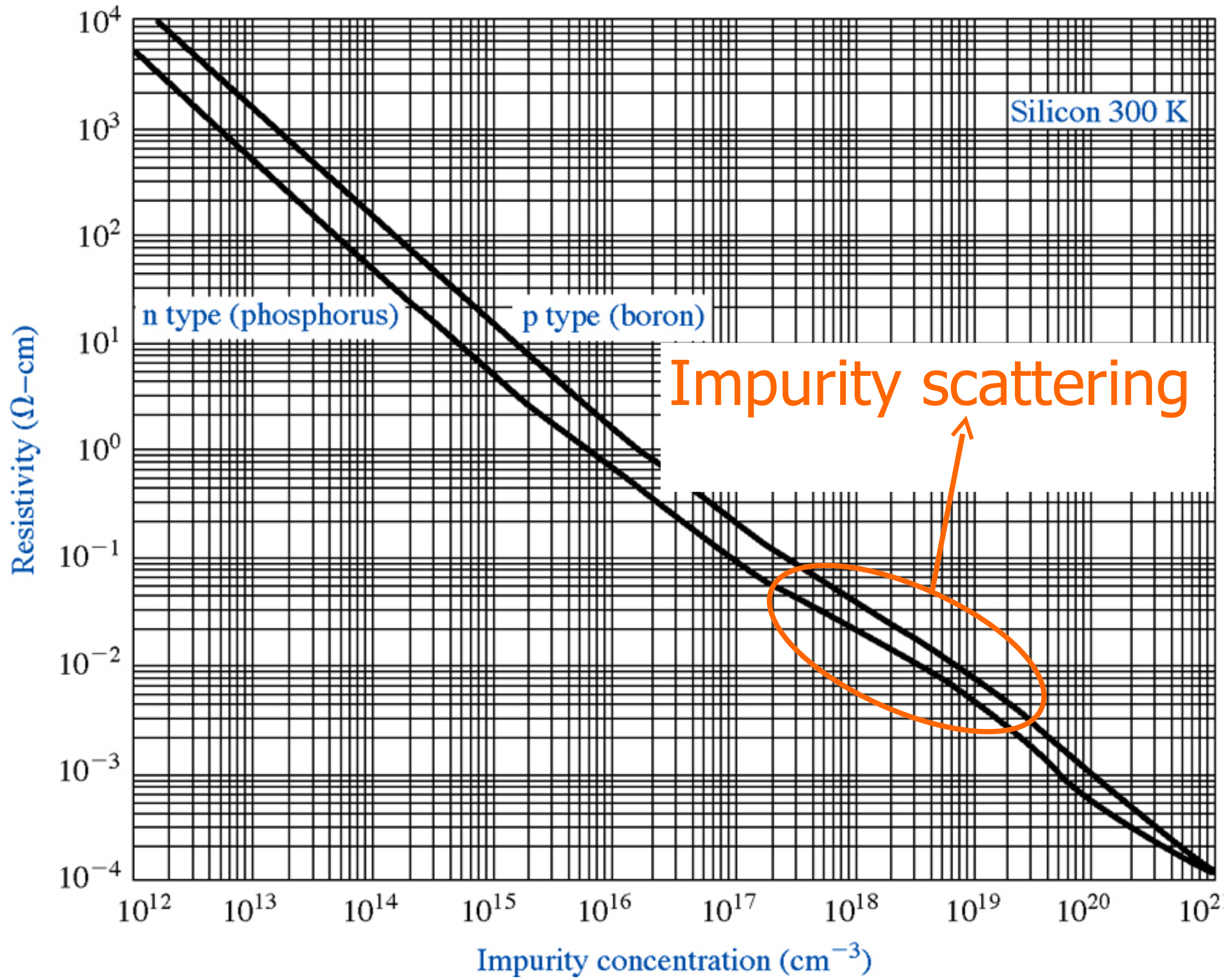
If we assume complete ionization,

$$\sigma = \frac{1}{\rho} = e\mu_n N_d \text{ or } e\mu_p N_a$$

But curve not linear because -

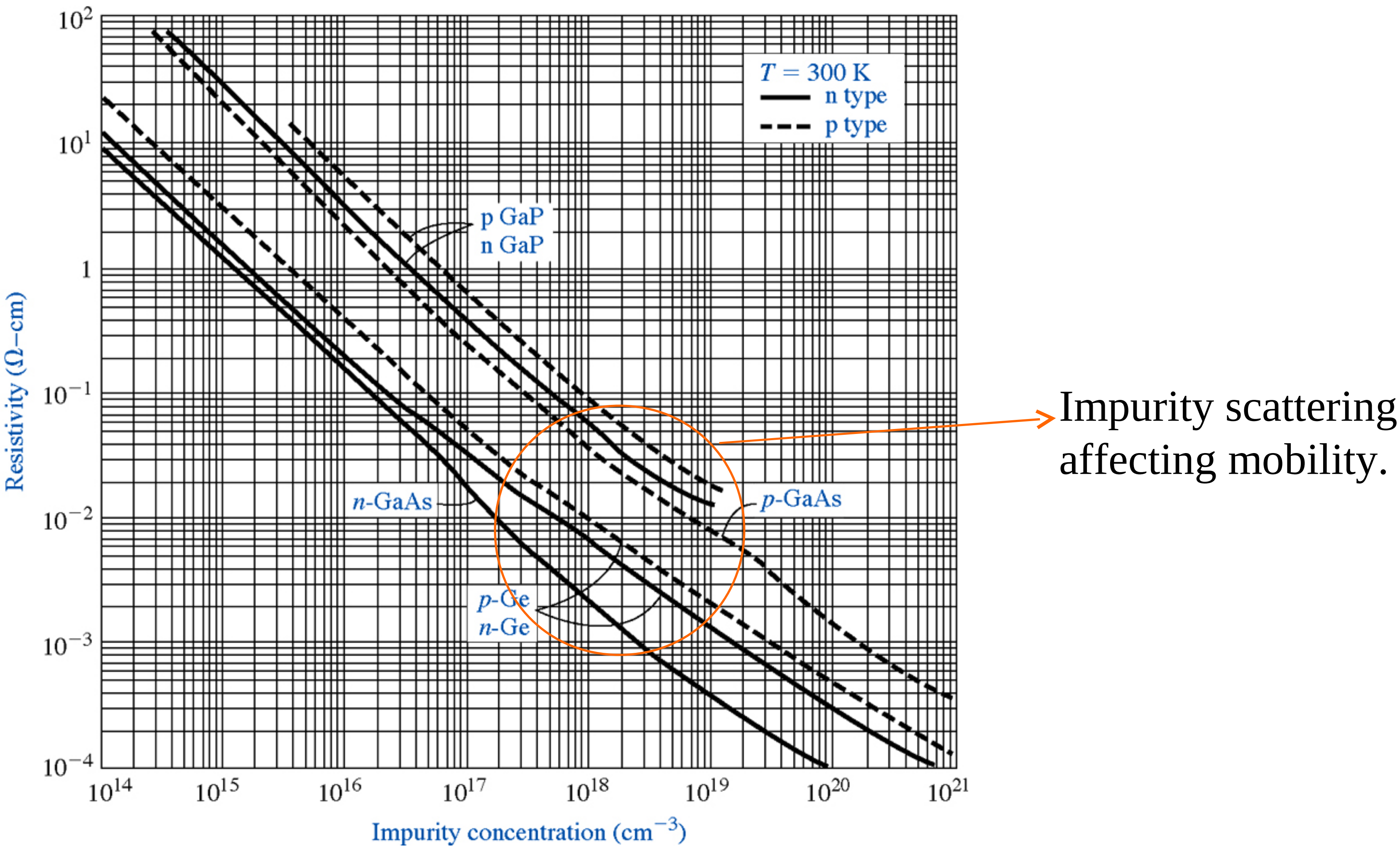
$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

Impurity scattering affects mobility.



Resistivity vs. impurity concentration in Si at T=300K

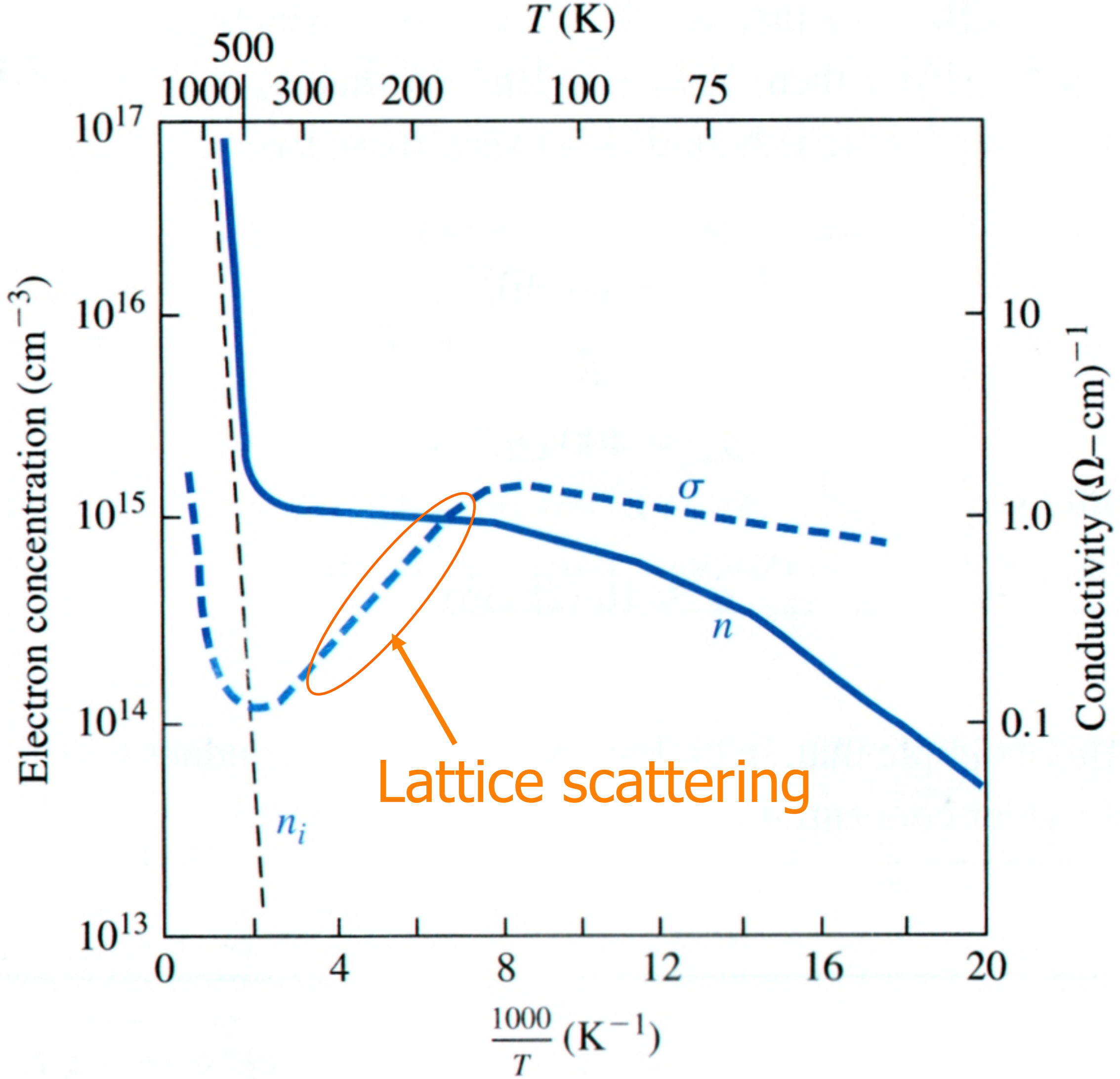
Department of Microelectronics and Computer Engineering



Resistivity vs. impurity concentration for Ge, GaAs and GaP

Department of Microelectronics and Computer Engineering

Electron concentration and conductivity vs. 1/T



Department of Microelectronics and Computer Engineering

TUDelft

Electron concentration and conductivity vs. 1/T

- Assuming a n-type material with donor doping $N_d >> ni$,

$$\sigma = e(\mu_n n + \mu_p p) \approx e\mu_n n$$ n → electron concentration

- If we also assume complete ionization, $\sigma = 1/\rho = e\mu_n N_d$

Mid temp → complete ionization → n constant at N_d but μ reduces with temp due to lattice scattering → so conductivity drops.

- At high temperature, intrinsic carrier concentration increases and dominates both n and σ

- At low temp, due to freeze-out both n and σ reduce

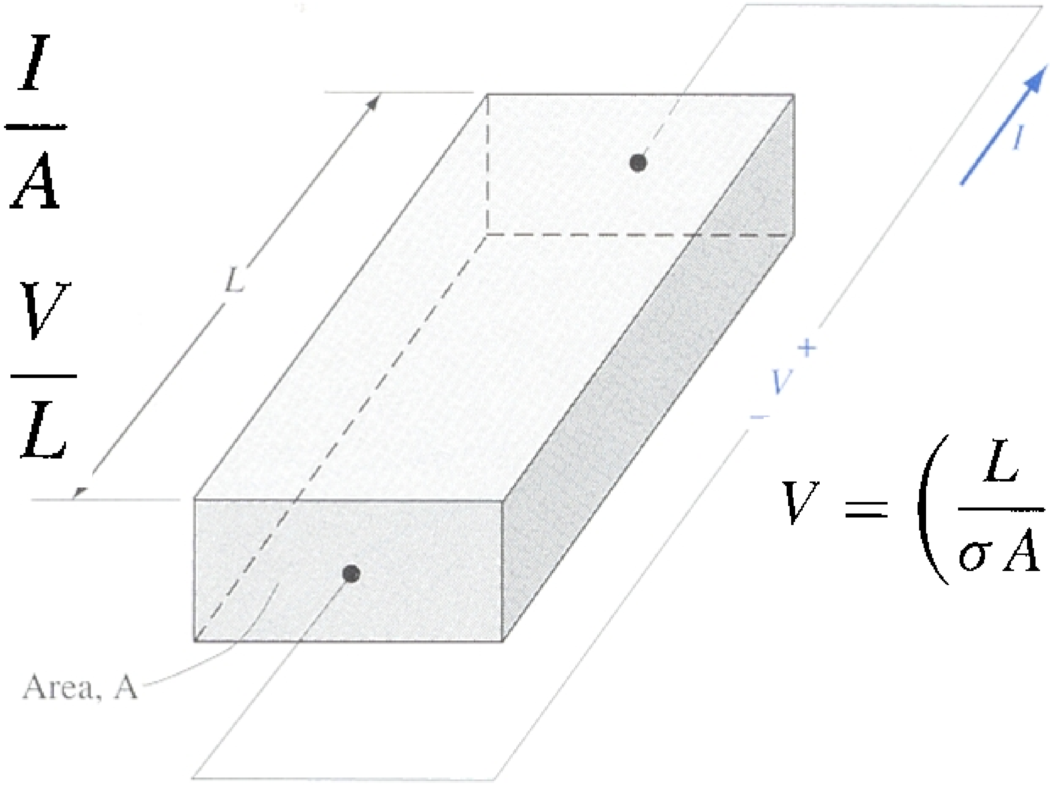
Department of Microelectronics and Computer Engineering

Ohms Law:

$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E} = \sigma \mathrm{E}$$

$$J = \frac{I}{A}$$

$$E = \frac{V}{L}$$



$$\frac{I}{A} = \sigma\left(\frac{V}{L}\right)$$

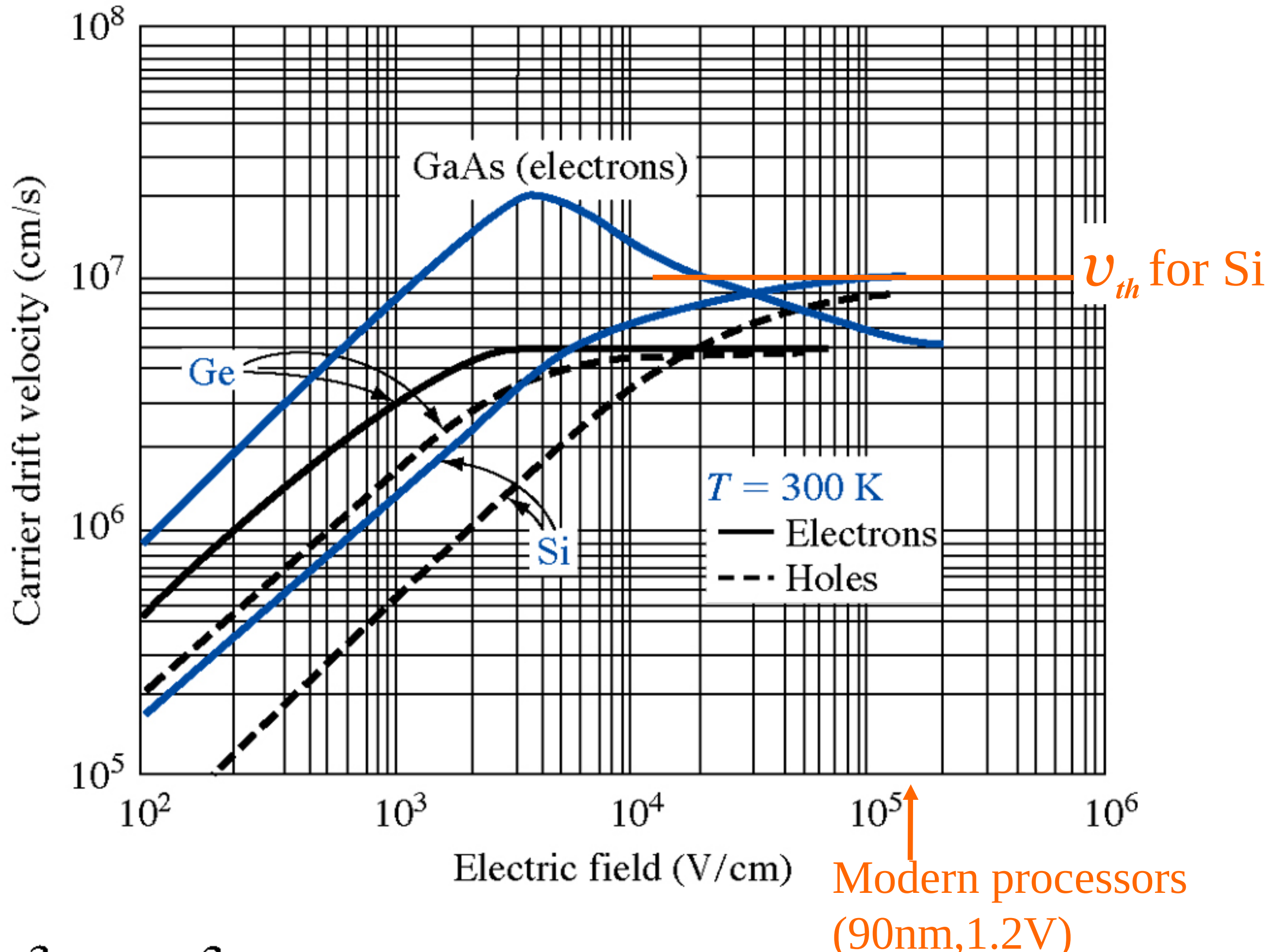
$$V = \left(\frac{L}{\sigma A}\right) I = \left(\frac{\rho L}{A}\right) I = IR$$

Ohm's law

Department of Microelectronics and Computer Engineering

Velocity saturation

$$v_d = \mu E$$



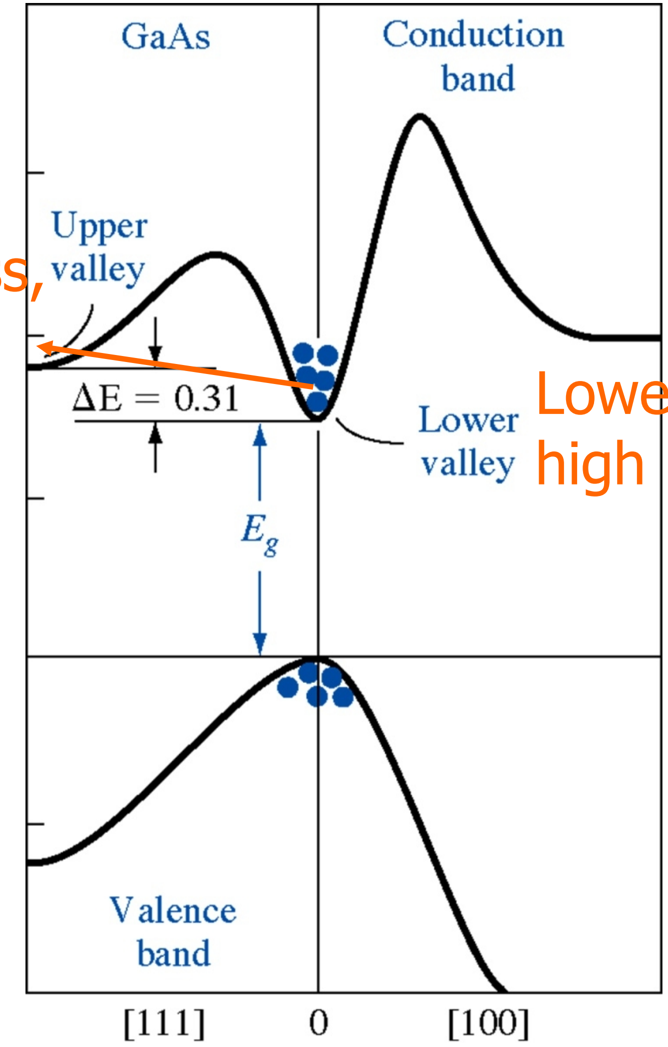
Random thermal energy $= \frac{1}{2} m v_{th}^2 = \frac{3}{2} kT = \frac{3}{2}(0.0259) = 0.03885 \text{ eV} \longrightarrow v_{th} \approx 10^7 \text{cm/s}$

Department of Microelectronics and Computer Engineering

Intervalley transfer mechanism in GaAs

Higher effective mass, low mobility

At higher E → lower mobility → lower current → negative resistance



Lower effective mass, high mobility

Department of Microelectronics and Computer Engineering

Negative resistance

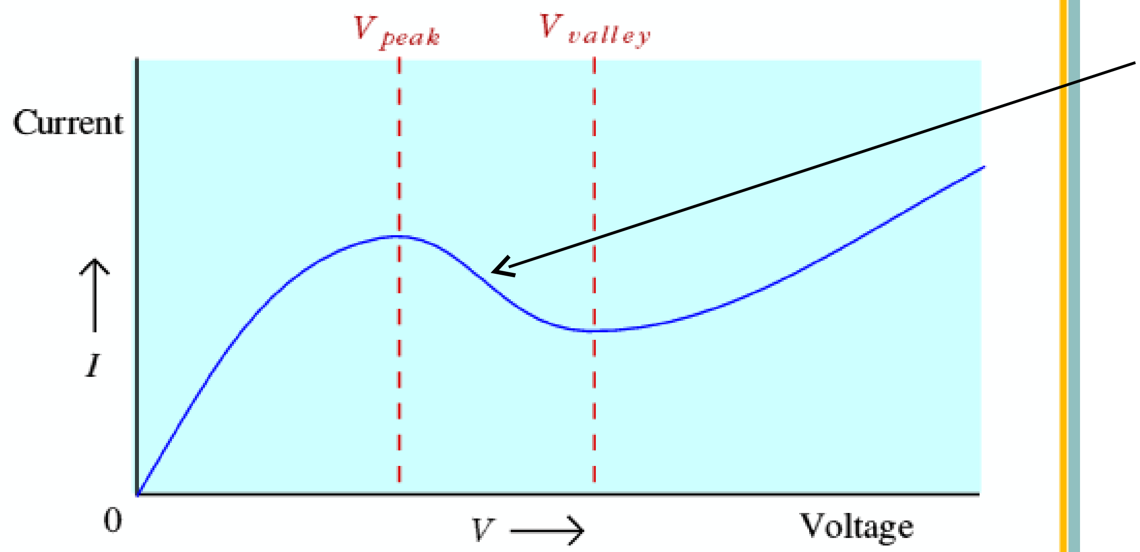


Figure 5.1 I/V curve of a Gunn diode.

Oscillator: GaAs 300GHz, GaN 3THz

Negative resistance region

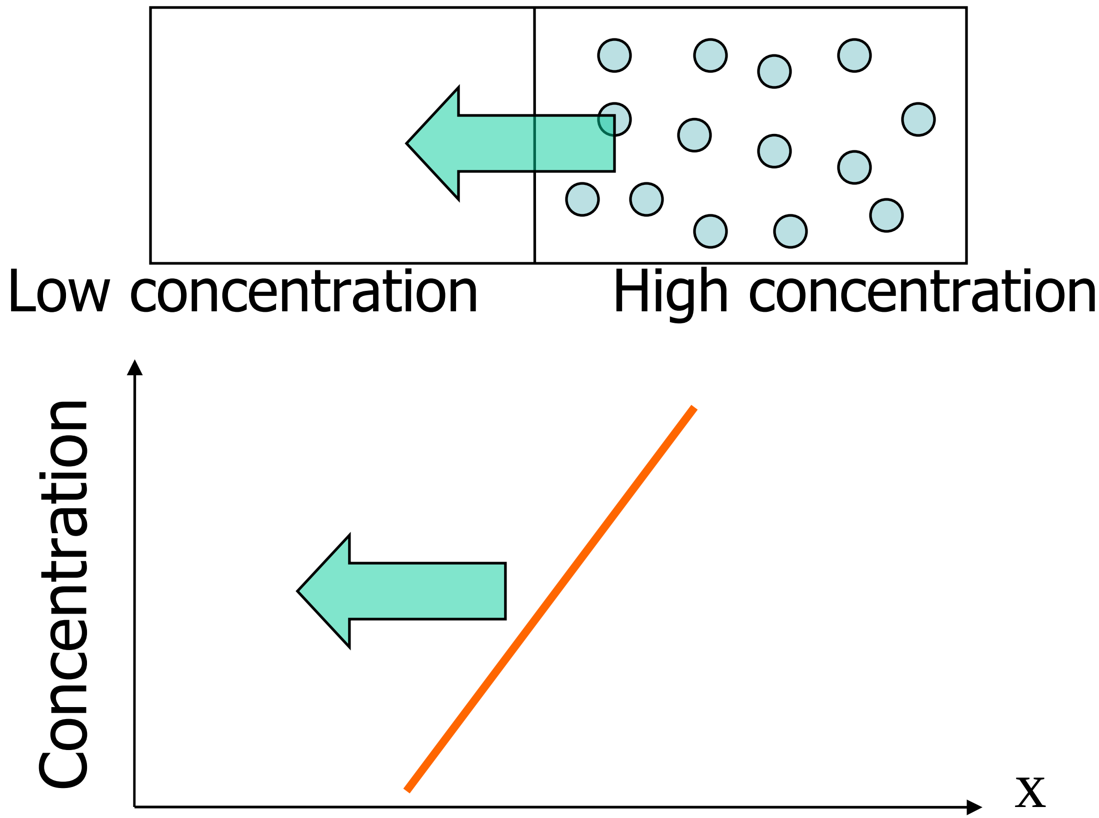
- -ve resistance used in design of oscillators.

- Oscillation frequency depends on transit time in the device.

Department of Microelectronics and Computer Engineering

TU Delft

Carrier Diffusion



Positive slope in x-direction → a flux towards negative-x direction

Department of Microelectronics and Computer Engineering

Diffusion current density

for holes

$$J_{px|dif} = -eD_p \frac{dp}{dx}$$

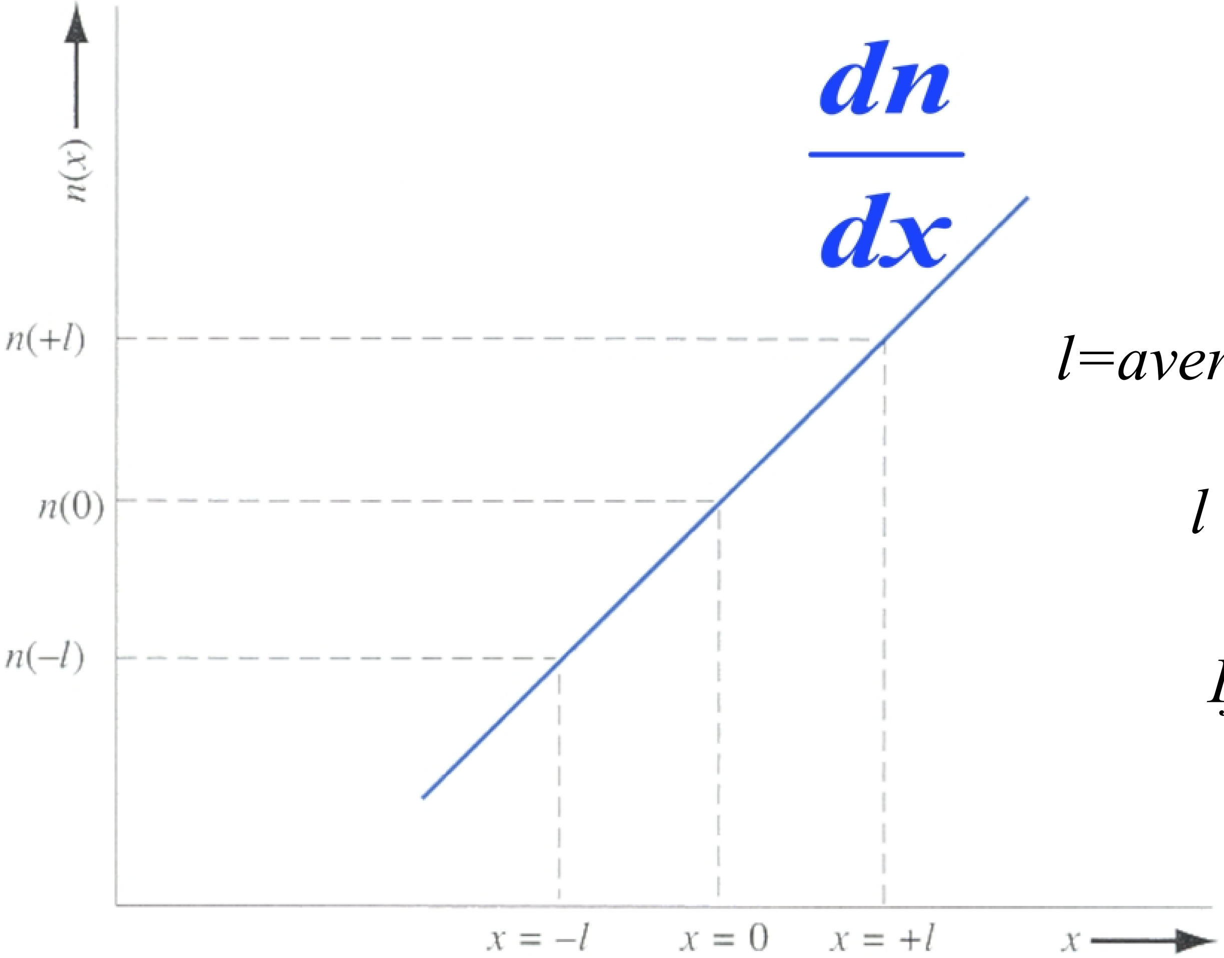
D: diffusion coefficient

for electrons

$$J_{nx|dif} = eD_n \frac{dn}{dx}$$

$$J_{(n,p)x|dif} = eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

Diffusion coefficient (I)



Electron concentration versus distance.

l=average mean free path

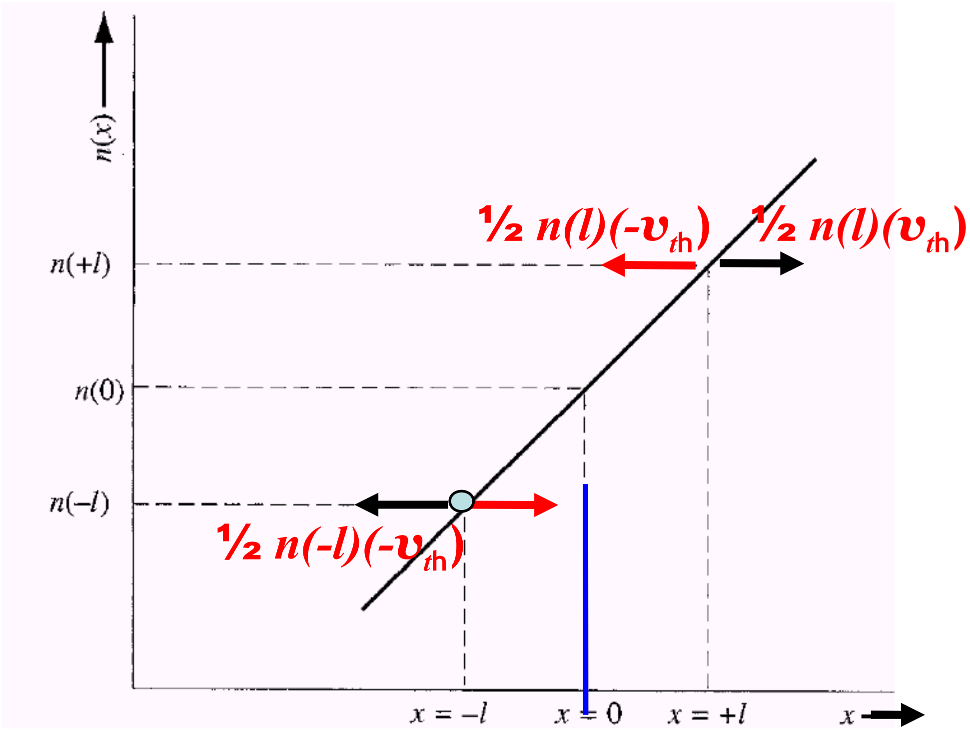
$$l = (v_{th} + v_{dr})\,\tau_{cp}$$

If E=0, vdr=0

$$l = v_{th}\,\tau_{cp}$$

Department of Microelectronics and Computer Engineering

Diffusion coefficient (II)



F_n → net rate of electron flow in the $+x$ direction at $x = 0$

= sum of electron flow in +x direction at x=-l and electron flow in –x direction at x=l

$$F_n = \frac{1}{2}n(-l)v_{th} - \frac{1}{2}n(+l)v_{th} = \frac{1}{2}v_{th}[n(-l) - n(+l)]$$

Department of Microelectronics and Computer Engineering

TU Delft

Diffusion coefficient (III)

$$F_n = \frac{1}{2}n(-l)\upsilon_{th} - \frac{1}{2}n(+l)\upsilon_{th} = \frac{1}{2}\upsilon_{th}[n(-l) - n(+l)]$$

Taylor expansion of $n(-l)$ en $n(+l)$ at $x = 0$:

$$n(+l) = n(0) + l\frac{dn}{dx} + \ldots\ldots$$

$$F_n = \frac{1}{2}\upsilon_{th}\left\{\left[n(0) - l\frac{dn}{dx}\right] - \left[n(0) + l\frac{dn}{dx}\right]\right\}$$

$$F_n = -\upsilon_{th}l\frac{dn}{dx}$$

Department of Microelectronics and Computer Engineering

Diffusion coefficient (IV)

Recall that $F_n = -v_{th} l \frac{dn}{dx}$

The current is $J = -eF_n = +e v_{th} l \frac{dn}{dx}$

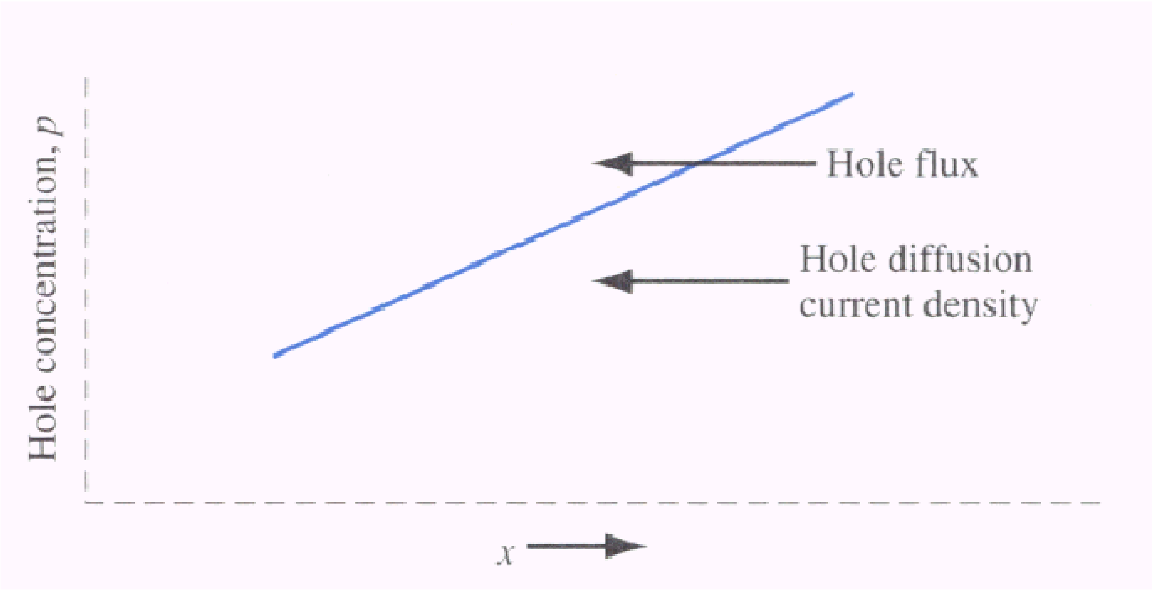
D: diffusion coefficient = $v_{th} l$ (cm²/s)=$v_{th}^2 \tau_{cp}$

$$J_{nx|dif} = eD_n \frac{dn}{dx}$$

Department of Microelectronics and Computer Engineering

TU Delft

For holes:

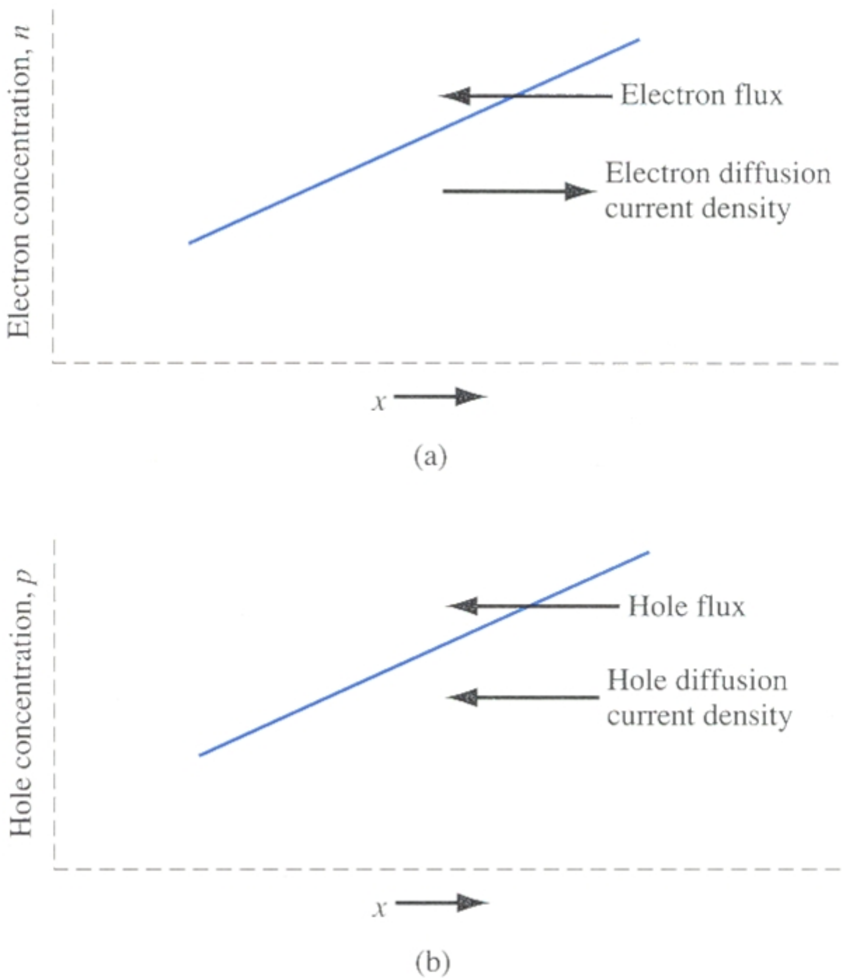


$$J_{px|dif} = -eD_p \frac{dp}{dx}$$

Note

$D_p \rightarrow$ *hole diffusion coefficient*

$J_{px|dif} \rightarrow$ hole diffusion current density for one-dimensional case



$$J_{nx|dif} = eD_n \frac{dn}{dx}$$

$$J_{px|dif} = -eD_p \frac{dp}{dx}$$

Diffusion of (a) electrons and (b) holes in a density gradient

Department of Microelectronics and Computer Engineering

Total Current Density

$$J = en\mu_n \mathrm{E}_x + ep\mu_p \mathrm{E}_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

DRIFT

DIFFUSION

Generalized current Density Equation -

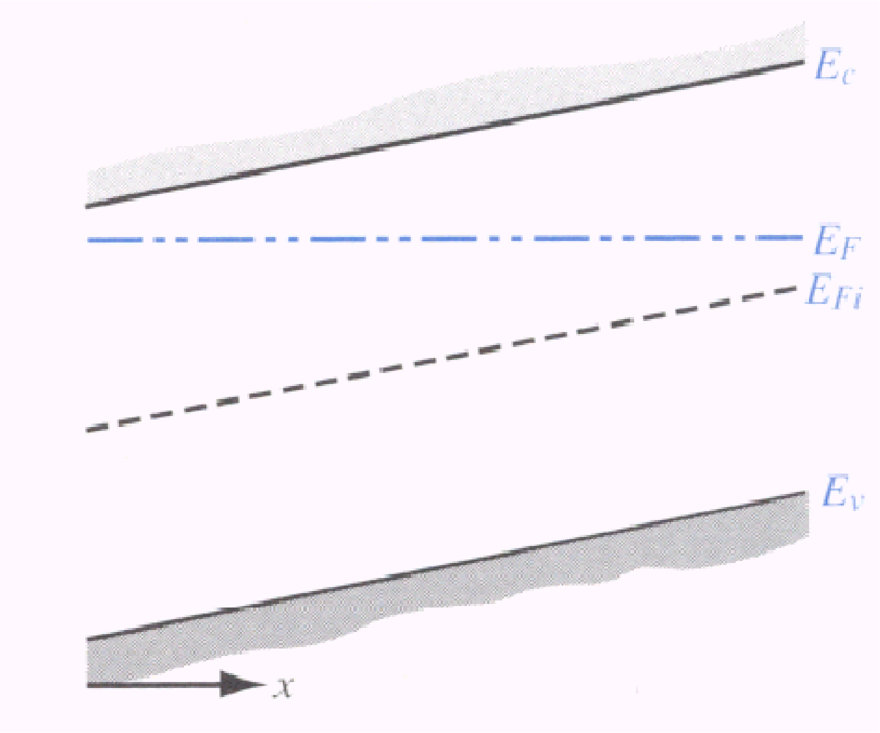
$$J = en\mu_n \mathrm{E} + ep\mu_p \mathrm{E} + eD_n \nabla n - eD_p \nabla p$$

Department of Microelectronics and Computer Engineering

TU Delft

If a semiconductor is non-uniformly doped?

Consider a case where donor concentration increases in x-direction



Diffusion of electrons

↓

Formation of electric field

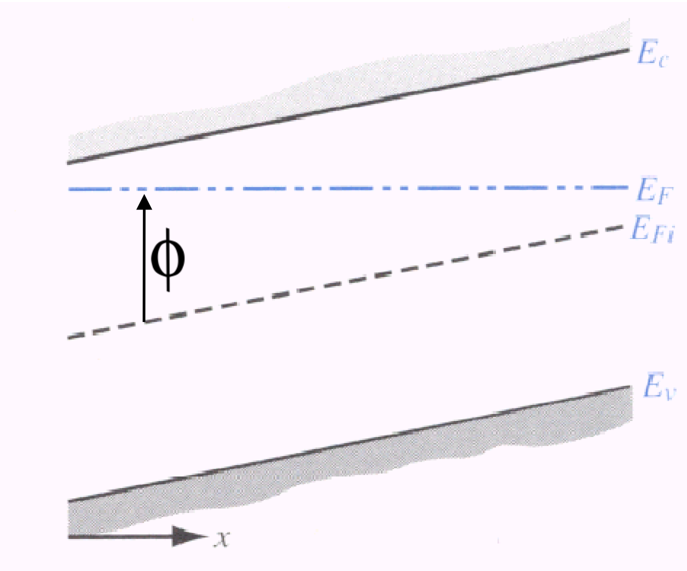
↓

Prevents further diffusion

- Due to electric field → potential difference across the device
- In the region at lower potential → E_F-E_{Fi} higher

Department of Microelectronics and Computer Engineering

Induced Electric Field Ex



Potential: $$\phi = +\frac{1}{e}(E_F - E_{Fi})$$

Electric field

$$E_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

Assuming Electron concentration ≈ Donor concentration

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x)$$

Recall $$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x)$$

Taking log $$E_F - E_{Fi} = kT \ln\left(\frac{N_d(x)}{n_i}\right)$$

Take the derivative with respect to x $$-\frac{dE_{Fi}}{dx} = \frac{kT}{N_d(x)} \frac{dN_d(x)}{dx}$$

Electric field $$E_x = -\left(\frac{kT}{e}\right) \frac{1}{N_d(x)} \frac{dN_d(x)}{dx}$$

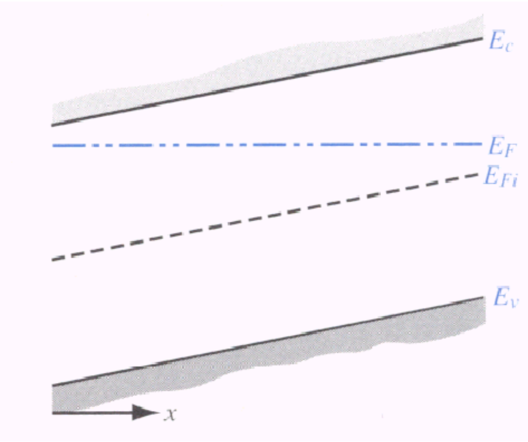
Department of Microelectronics and Computer Engineering

The Einstein Relation

Consider the general current density equation:

$$J_{n,p} = en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

Assume n-graded semiconductor material in thermal equilibrium:



$$J_n = 0 = en\mu_n E_x + eD_n \frac{dn}{dx}$$

Assuming n≈N_d

$$J_n = 0 = e\mu_n N_d(x) E_x + eD_n \frac{dN_d(x)}{dx}$$

Department of Microelectronics and Computer Engineering

TU Delft

Substituting $E_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$

$$0 = -e\mu_n N_d(x)\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n\frac{dN_d(x)}{dx}$$

$$0 = \left[-e\mu_n\left(\frac{kT}{e}\right) + eD_n\right]\frac{dN_d(x)}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{kT}{e}$$

because $\frac{dN_d}{dx} \neq 0$

Department of Microelectronics and Computer Engineering

$$\frac{D_n}{\mu_n} = \frac{kT}{e}$$

Similarly $$\frac{D_p}{\mu_p} = \frac{kT}{e}$$

Einstein relation ⟶ $$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}}$$

Department of Microelectronics and Computer Engineering

TU Delft

The Hall effect

- n- or p-type, carrier concentration and mobility can be experimentally measured.
- Electric and magnetic fields are applied to a semiconductor.

Department of Microelectronics and Computer Engineering

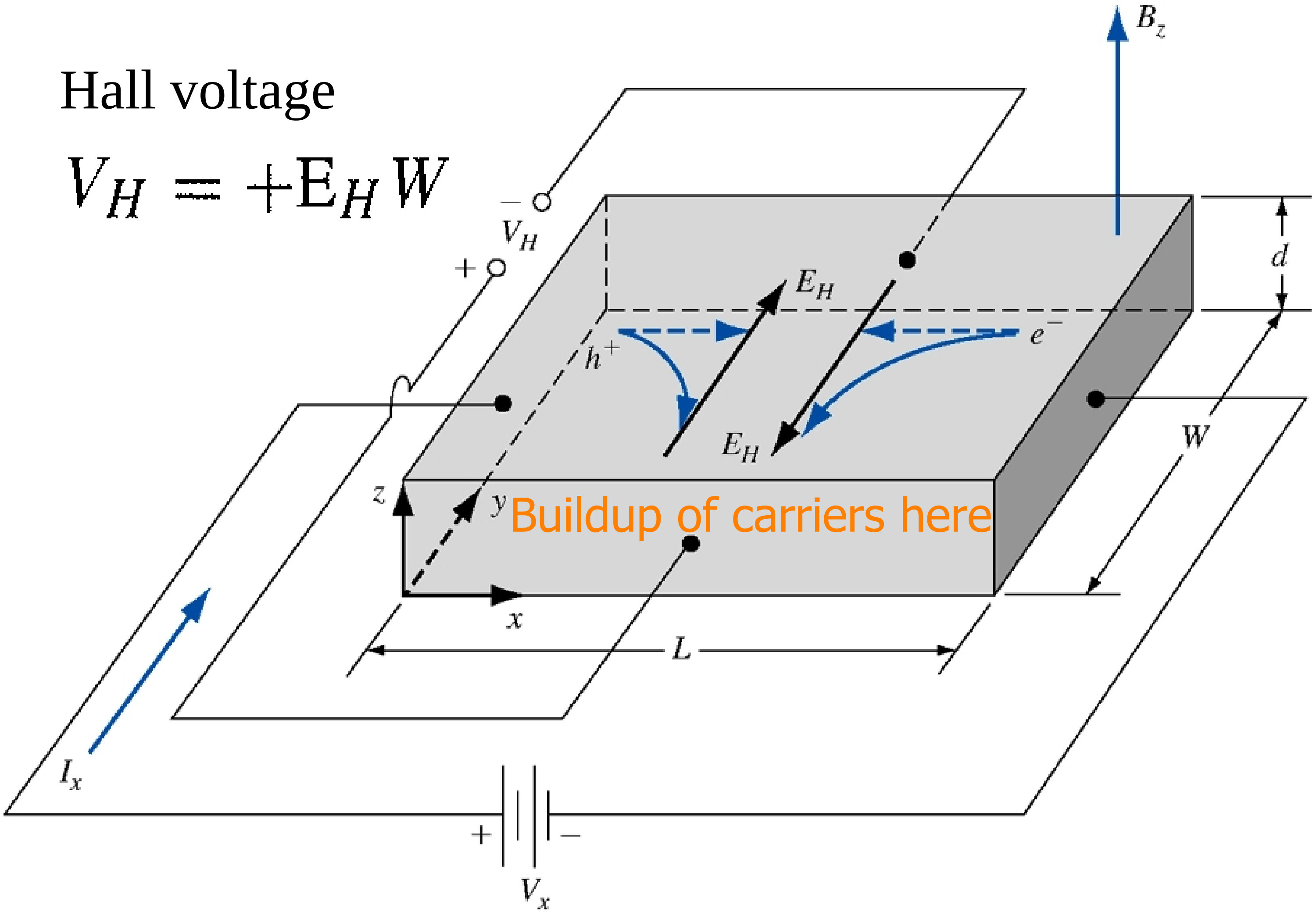
The Hall Effect

(Lorentz force)

$$F = qv_x \times B_z$$

Hall voltage

$$V_H = +E_H W$$



Buildup of carriers here

Department of Microelectronics and Computer Engineering

TU Delft

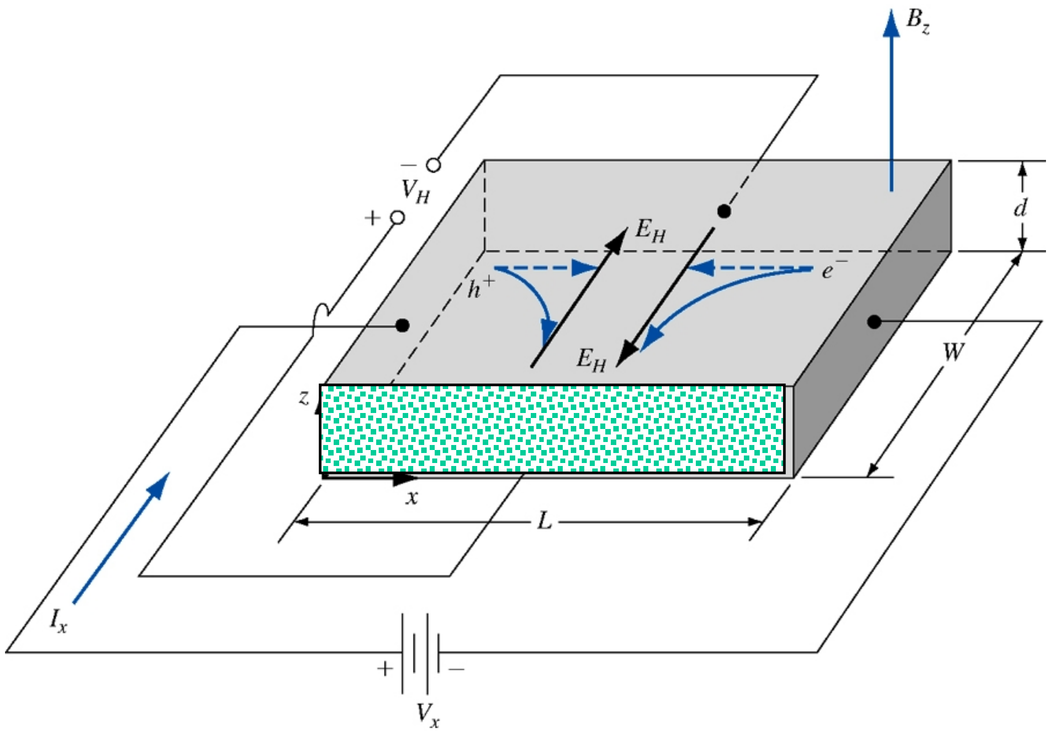
- Due to magnetic field, both electrons and holes experience a force in –y direction.

- In n-type material, there will be a build up of –ve charge at y=0 and in p-type material, there will be a build up of +ve charge.

- The net charge induces a electric force in y-direction opposing the magnetic field force and in steady state they will exactly cancel each other.

Hall Voltage:

$$F = q[E + v \times B] = 0$$



$$qE_y = q v_x B_z \longrightarrow E_y = v_x B_z$$

Hall voltage

$$V_H = E_y W = v_x W B_z$$

Department of Microelectronics and Computer Engineering

How to determine n-type or p-type & doping concentration?

For p-type material: V_H will be +ve

$$v_x = \frac{J_x}{ep} = \frac{I_x}{(ep)(Wd)} \rightarrow V_H = \frac{I_x B_z}{epd} \rightarrow p = \frac{I_x B_z}{edV_H}$$

For n-type material: V_H will be -ve

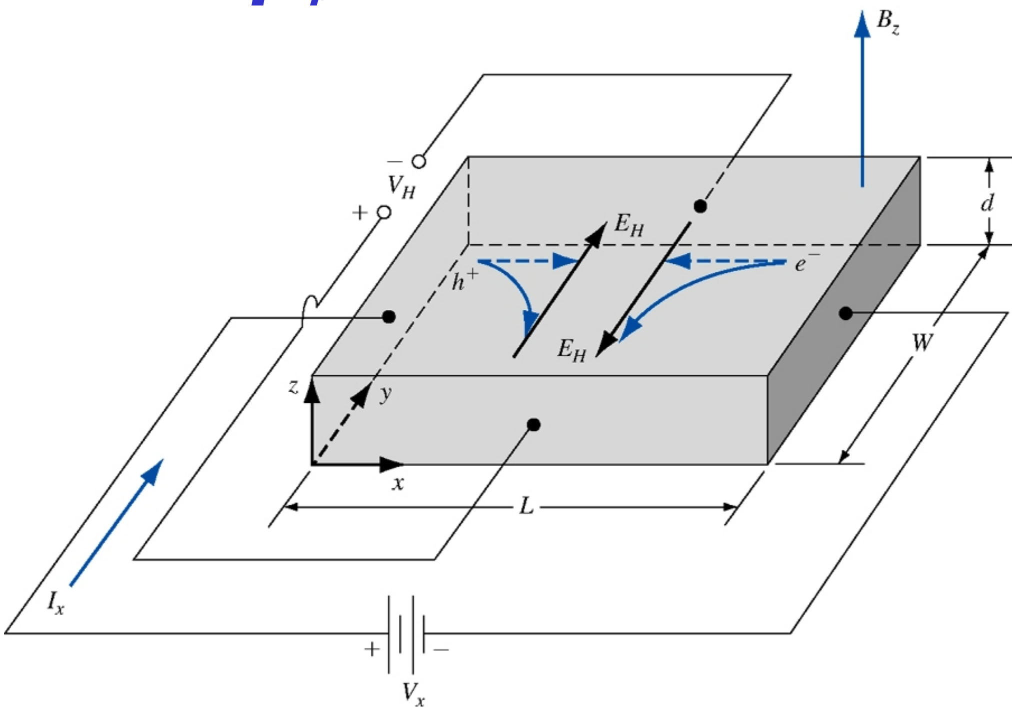
$$V_H = -\frac{I_x B_z}{ned} \longrightarrow n = -\frac{I_x B_z}{edV_H}$$

For n-type material V_H will be -ve → n will still be +ve !

Department of Microelectronics and Computer Engineering

TU Delft

How can you determine μ_n, μ_p ?



$$J_x = ep\mu_p E_x$$

$$\frac{I_x}{Wd} = \frac{ep\mu_p V_x}{L}$$

$$\mu_p = \frac{I_x L}{ep V_x W d}$$

$$\mu_n = \frac{I_x L}{en V_x W d}$$

Summary

- Drift → net movement of charge due to electric field.
  - Drift current = $J_{drf} = e(n\mu_n + p\mu_p)E$

- Mobility (μ) ← lattice and impurity scattering
  - Mobility due to lattice scattering $\mu_L \propto T^{-\frac{3}{2}}$
  - Mobility due to impurity scattering $\mu_I \propto \frac{T^{\frac{3}{2}}}{N_I}$

- Ohms law → $V = \frac{L}{\sigma A} I = \frac{\rho L}{A} I = RI$

- Drift velocity saturates at high electric field → velocity saturation

Summary

- Some semiconductors → mobility μ reduces at high E → negative resistance → used in design of oscillators.

- Diffusion current $J_{diff} = eD_n \frac{dn}{dx} \quad eD_p \frac{dp}{dx}$

- Einstein relation → relation between diffusion coefficient and mobility → $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$

- Hall effect → can be used to determine semiconductor type, doping concentration and mobility