Anchor Theory

Penetration behavior of an anchor in sand.
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OE5661 – Offshore Moorings

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CONTENTS

1 Introduction

In this case study an analyses is made of the penetration behavior of an anchor in sand.

The following points should be taken into account.

- The geometry of the anchor and how to simplify it.
- What happens when the anchor penetrates the soil?
- Which forces will occur during penetration?
- How to solve this mathematically?

In this report the outcome of this research is described and the above mentioned points were taken into account.

First of all, the most common anchors on the market are analyzed and a general anchor geometry will be chosen. This chosen geometry will be simplified to a 2D geometry, which will suite for a first analysis.

After this step the penetration behavior of an anchor will be described in different phases, such a way that it is clear and easy to understand. Forces on the soil layer, fluke, shank and mooring line forces will be defined and analyzed. The forces will be described as a function of the geometries including the relevant variable angles.

Final chapter consist a conclusion and several recommendations.

2 The geometry of the anchor

While searching for the best simplified anchor geometry, knowledge of the most common anchors on the market is needed. Vrijhof anchors will give a good overview of the most common anchors.
Two types of anchors can be considered, horizontal load anchors and vertical load anchors. The vertical load anchor can withstand both horizontal and vertical mooring forces. The horizontal anchor or drag embedment anchor can only resist the horizontal loads.
The drag embedment anchor is mainly used for catenary moorings where the mooring line arrives the seabed horizontally. The vertical load anchor is used in taut leg mooring systems where the mooring line arrives at a certain angle the seabed.

A good starting-point for this case is to analyze the horizontal load anchor, because this anchor is often used and will form an adequate challenge.
To determine a simplified geometry of this horizontal anchor an actual figure of this anchor is needed. In the figures below, a sketch of the selected anchor can be found.

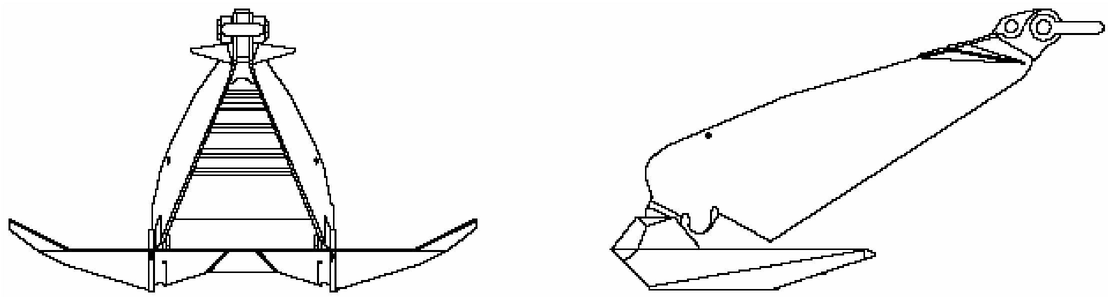


Figure 2.1: Front and side view of the chosen anchor.

The actual blade (fluke) on the anchor that will penetrate the soil is represented by the horizontal part of the above given figure. The shank is represented by the other part of the anchor. On the end of the anchor you will find an anchor shackle.

A first simplification will be made by modeling the anchor as a 2D model. Hereby all the calculations will be made easier, but the geometry is still to complex to determine all the forces. A second simplification can be made by supposing the anchor as two straight lines as can be seen in figure 2.2. This simplification is allowed, because we are working in a conceptual (first) model. When all the forces and the penetration curve of this simple model are known, a much more complicated model can be made.

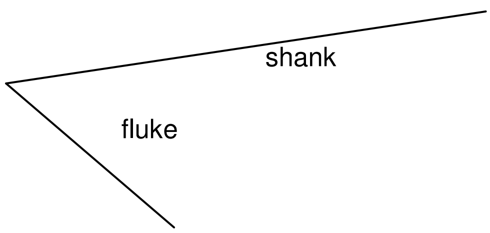


Figure 2.2: The simplified geometry of the anchor

For the rest of the case a few assumptions must be made. First, the type of soil will be sand, this way the cohesion and adhesion effects can be neglected. As a next assumption we consider that the anchor penetrates the soil at a very low velocity. Therefore inertia and water tension can also be neglected.
Several constrains where also made to simplify the 3D force analysis into a 2D analysis. This way several shear zones can be neglected. As a final assumption, the force acting on the point of the fluke will be neglected. This force is low considering a big anchor and will be fully cancelled by the force perpendicular on the shank.

The next chapters will treat further modeling of the penetration behavior of an anchor in sand.

3 Penetration phases of the anchor in sand

Using the simplified geometry, the determination of the forces on the anchor can be studied by using the cutting theory (Miedema) and the strip footing theory as described in Verruijt.

Here, the cutting blade is represented by the fluke of the anchor and will be modeled as a 2D blade. In this situation a 2D force and moment balance can be used.
The penetration of the anchor can be brought back to 4 phases and will be further discussed below.

Phase 1

<u>No penetration</u>

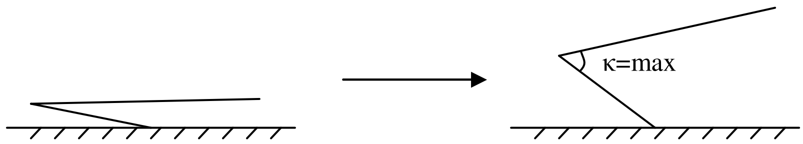


Figure 3.1: Phase 1

In this first situation the anchor lies on the bed of soil and the fluke/shank angle will be considered as a minimum. When pulling on the mooring line the anchor will scratch over the seabed. A bed of soil will be formed in front of the fluke and will give some resistance. Because of this resistance, an angle κ will reach his maximum at a certain point.
At that certain point, the bed of soil in front of the fluke will give his highest resistance and it will become easier to penetrate than scratching over the seabed.
When we make the assumption of a perfect sharp fluke point, the point load can be neglected.

Phase 2

<u>Penetration causes fluke forces.</u>

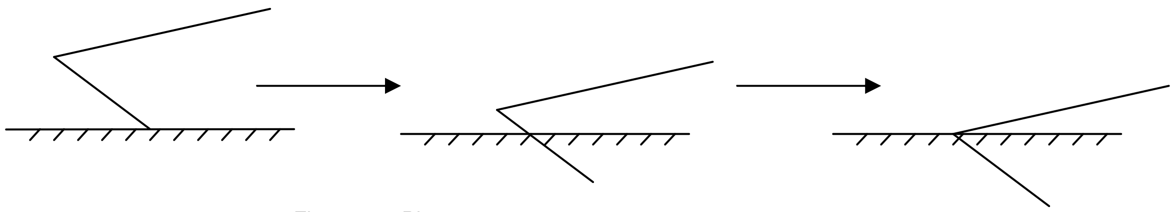


Figure 3.2: Phase 2

When the fluke starts to penetrate, the cutting theory of Miedema can be used. Forces that will play a role in the force balance are the fluke forces. When we look at the angles on the fluke a few assumption can be made. First of all the fluke/shank angle κ will be constant and will have its maximum value.
The internal friction and the external friction angles are also constant. These parameters are only depending on the material of the anchor and soil properties.

Phase 3

<u>Penetration causes fluke and shank forces.</u>

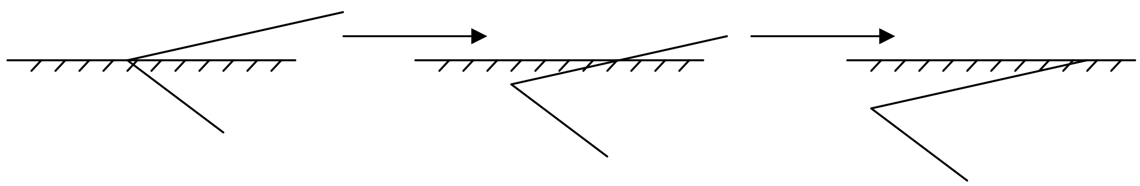


Figure 3.3: Phase 3

In this situation the fluke is completely covered by sand and the shank will become an extra factor which will cause penetration resistance. For the shank the cutting theory of Miedema can't be used. The strip footing theory as described in Verruijt will be used for determining the shank resistance. The maximum shank resistance acts when the complete shank is penetrated.

Phase 4

<u>Penetration causes fluke forces, shank forces and mooring line forces.</u>

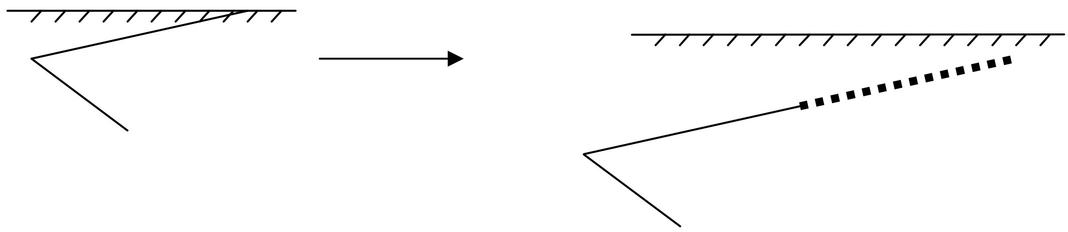


Figure 3.4: Phase 4

The fluke and the shank are completely covered by sand. When there is still no equilibrium, a part of the mooring line will enter the soil. The mooring line penetration will lead to an extra factor which will cause penetration resistance.
The anchor becomes stable when there is a balance between the vertical and horizontal forces on the anchor part, which is covered by sand.

4 Verification of the forces acting on the anchor

In this chapter you will find the modeling of the forces on the fluke. The influences of the angles will be given and verified.

4.1 *Phase 1*

As can be seen in penetration phases of the anchor in sand, there is no penetration and therefore no fluke forces will occur.

4.2 *Phase 2*

Fluke in the soil

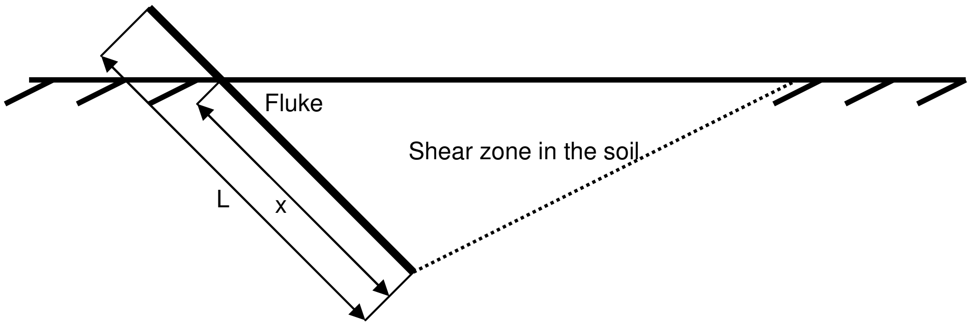


Figure 4.1: Phase 2

Forces on the soil layer

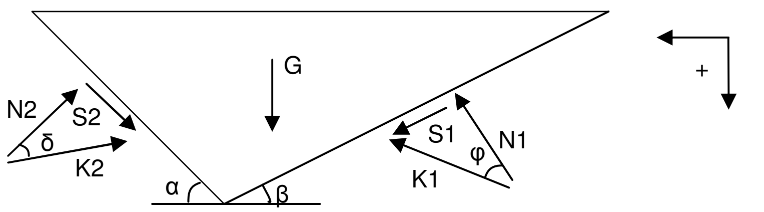


Figure 4.2: Forces on the soil layer

As discussed before the cohesion, adhesion, inertial forces and water tension can be neglected. For figure 4.1, a force balance can be calculated.

The shear force and the normal force are related according:

$S_1 = N_1 \tan\varphi$ and $S_2 = N_2 \tan\delta$

Where:

$\varphi =$ Internal friction angle of the sand
$\delta =$ External friction angle fluke/sand

The grain forces will be:

$K_1 = \sqrt{\left(S_1^{\ 2} + N_1^{\ 2}\right)}$ and $K_2 = \sqrt{\left(S_2^{\ 2} + N_2^{\ 2}\right)}$

The weight of the soil can be given as a force according:

$$G = \left(\frac{x^2 \sin^2\alpha}{2\tan\alpha} + \frac{x^2 \sin^2\alpha}{2\tan\beta}\right)\cdot\gamma$$

Where:

$x =$ Length of the fluke in the sand

$\gamma =$ Density of the in situ sand
$\alpha =$ Angle of the fluke in the sand
$\beta =$ Angle of the shear zone

Horizontal equilibrium of forces:

$$K_1 \cdot \sin\left(\beta+\varphi\right) - K_2 \cdot \sin\left(\alpha+\delta\right) = 0$$

Vertical equilibrium of forces:

$$-K_1 \cdot \cos\left(\beta+\varphi\right) + G - K_2 \cdot \cos\left(\alpha+\delta\right) = 0$$

Forces on the Fluke

The force K2 on the fluke is important to determine the horizontal and vertical acting forces on the fluke.

$$K_2 = \frac{G \cdot \sin\left(\beta+\varphi\right)}{\sin\left(\alpha+\beta+\varphi+\delta\right)}$$

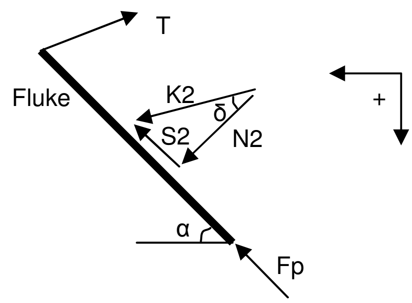


Figure 4.3: Forces on the fluke

The following forces are acting on the fluke blade:

- The Horizontal Force
  $$F_h = K_2 \cdot \sin\left(\alpha+\delta\right)$$
- The Vertical Force
  $$F_v = K_2 \cdot \cos\left(\alpha+\delta\right)$$

The force Fp can be neglected as discussed before.

Example 2.1; Influences of the angles α and β on the fluke blade forces

<u>The vertical fluke Force:</u>

When investigating the vertical fluke force (Fv) as a function of the angles α, β, a 3D plot has been made.

In this plot it's interesting to look at the influence of the angle α on Fv. As can be seen in the figures, when α reaches 1.1rad or +/- 60deg, the vertical fluke force becomes negative.

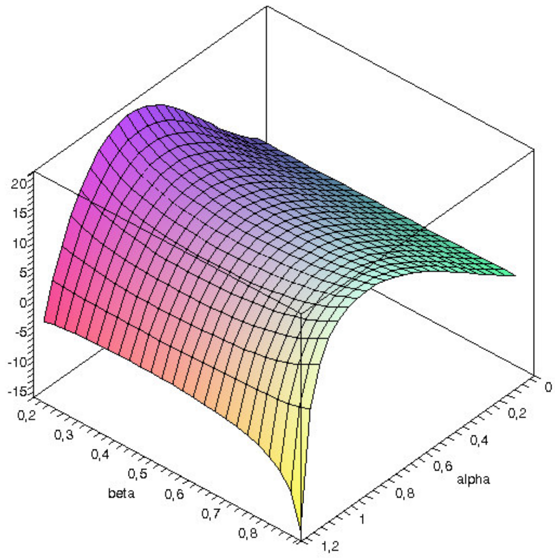


Figure 4.4: Vertical fluke force 3D

A negative Fv represents an upward force, or in other words the fluke makes an up going movement. We want the fluke to go down and this negative Fv will not give penetration.

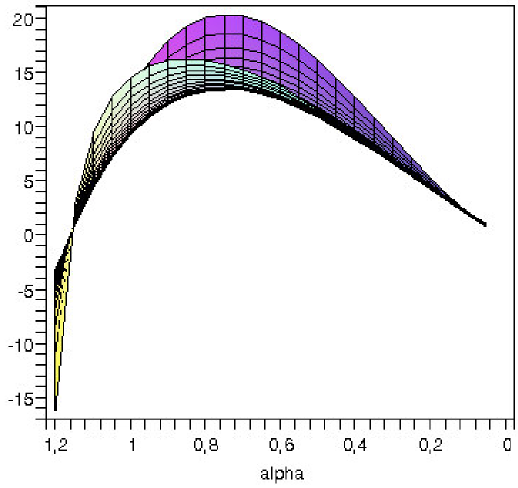


Figure 4.5: Vertical fluke force as a function of alpha

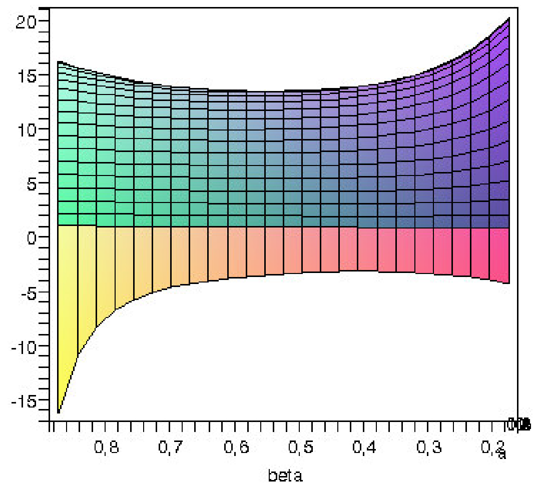


Figure 4.6: Vertical fluke force as a function of beta

Other things which can be seen, are an increasing Fv between α = 0 rad and α = 0.75 rad for a maximum angle β, and between α = 0 rad and α = 0.9 rad for a minimum angle β.

First conclusion will be that the angle α lies between 0 and 60 deg.

When investigating the vertical fluke force as a function of β, the following conclusions can be made:
β doesn't influence the Fv when looking at α = 0, this due to the horizontal orientation of the fluke in the sand. β lies between 10 and 50 deg and its influence on Fv is small.

The horizontal Fluke force:

When looking at the 3D representation of the Horizontal fluke force (Fh) as a function of the angles α and β, a relative flat figure will occur. Except, when looking at a high α and β, Fh reaches a high value then.
This result is as expected; with a high value of α and β, it becomes hard to shear the sand over the shear zone and therefore a high Fh is necessary.

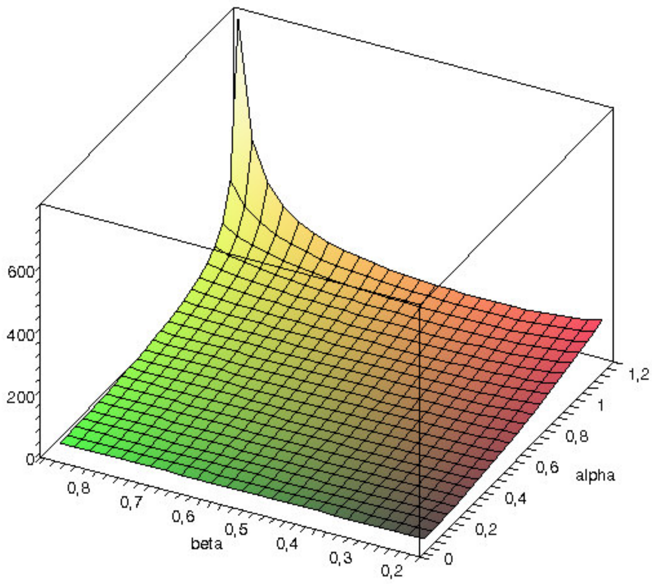


Figure 4.7: Horizontal fluke force 3D

Figure 4.8: Horizontal fluke force as a function of alpha

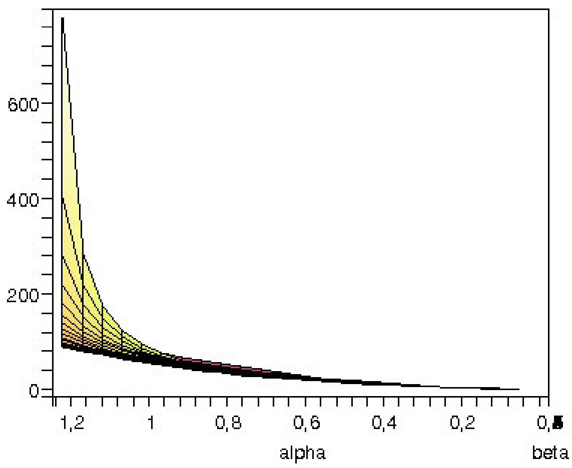
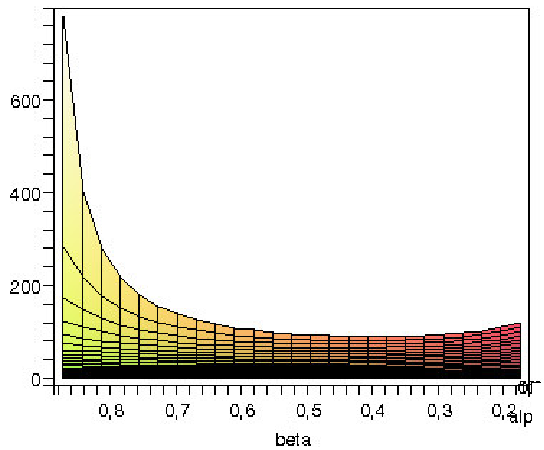


Figure 4.9: Horizontal fluke force as a function of beta



How to interpret the relationship between reality and theory

In theory we have started with the geometry in combination with the cutting theory of Miedema. So the output of this theory is the pull force. In reality it is easier to understand the inverse process.
In this way you can use the pull force as an input to determine the holding capacity and the trajectory of the anchor.

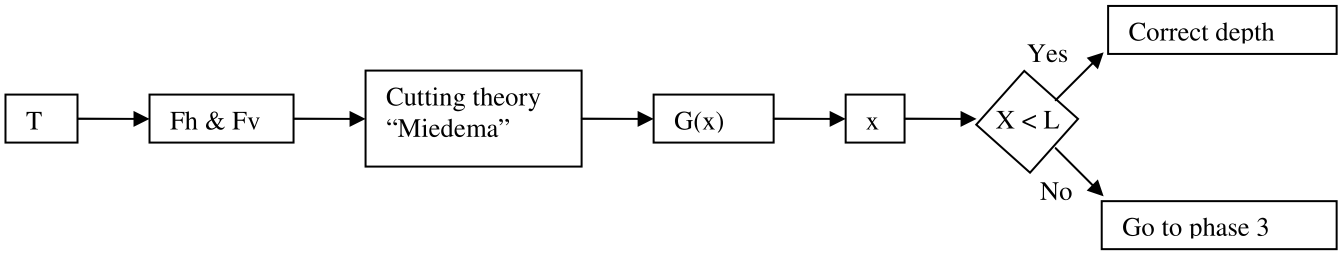


Figure 4.10: Block scheme interpretation

In practice you know the needed pull force and when using this model, you get inside information of the trajectory and the holding capacity. Because of that it's more practical to have the pull force as an input.

4.3 Phase 3

In this chapter you will find the modeling of the forces on the fluke and the shank. The influences of the angles will be given.
Cutting theory of Miedema is still valid for the fluke part of the anchor forces. For determining the forces on the shank, the strip footing theory will be used.

Fluke and shank in the soil

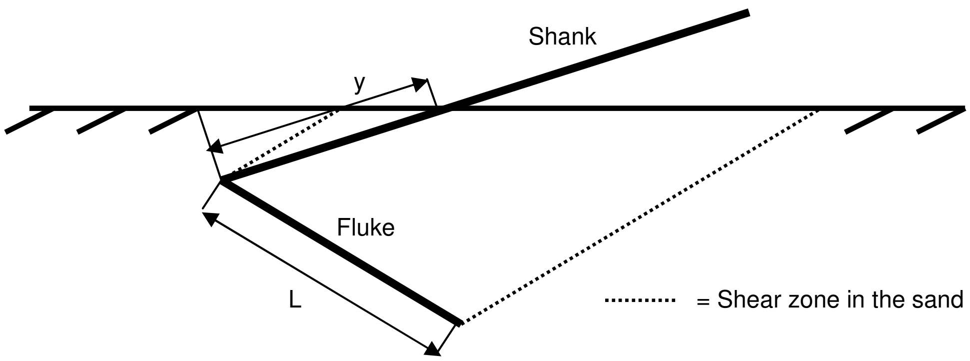


Figure 4.11: Phase 3

Forces on the soil layer

For phase 3, two shear zones are taken into account. This will lead to a geometry as discussed below.

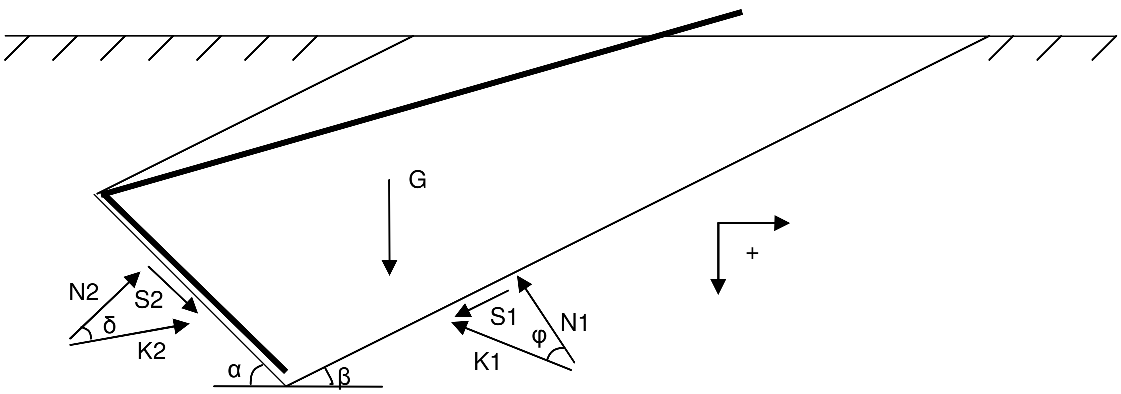


Figure 4.12: Forces on the soil layer

As discussed before; the cohesion, adhesion, inertial forces and water tension can be neglected. For the above figure a force balance can be calculated.

The shear force and the normal force are related according:

$S_1 = N_1 \tan\varphi$ and $S_2 = N_2 \tan\delta$

Where:

$\varphi =$ Internal friction angle of the sand
$\delta =$ External friction angle fluke/sand

The grain forces will be:

$K_1 = \sqrt{\left(S_1^{\,2} + N_1^{\,2}\right)}$ and $K_2 = \sqrt{\left(S_2^{\,2} + N_2^{\,2}\right)}$

The weight of the soil can be determined by using the following geometry:

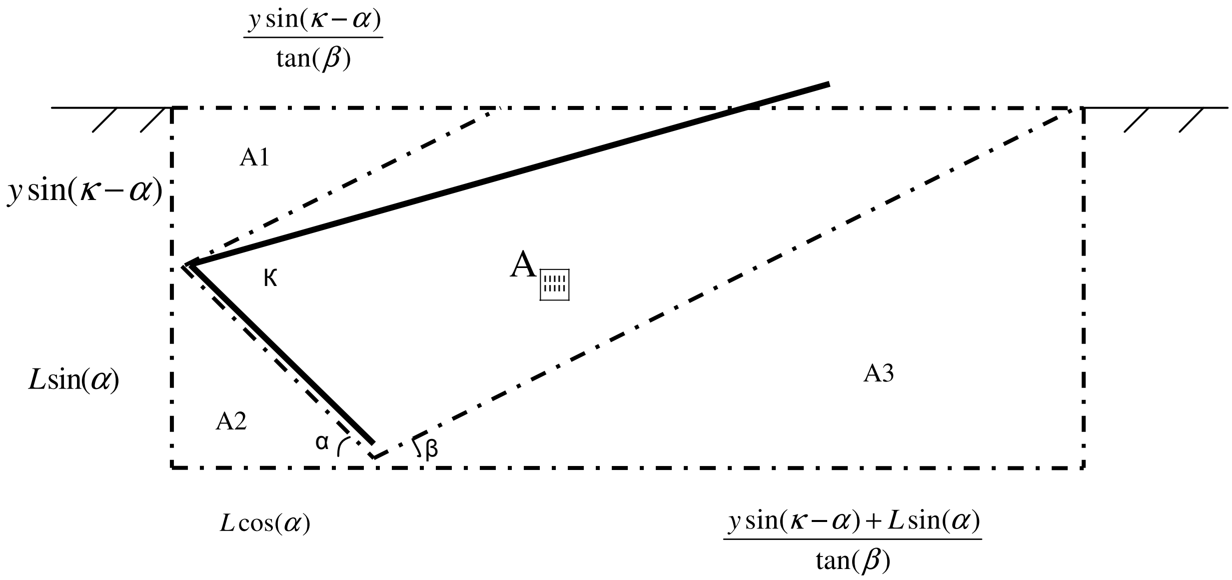


Figure 4.13: Dimensions of the soil layer

The Weight of the soil will be:

$$G = A_{\square} \cdot \gamma$$

with:

$$A_{\square} := (y*\sin(\kappa\text{-}\alpha)+L*\sin(\alpha))*\left(L*\cos(\alpha)+\frac{y*\sin(\kappa\text{-}\alpha)+L*\sin(\alpha)}{\tan(\beta)}\right)-\left(\frac{y^2*\sin(\kappa\text{-}\alpha)^2}{2*\tan(\beta)}\right)$$
$$-\frac{1}{2}*L^2*\sin(\alpha)*\cos(\alpha)-\left(\frac{\left(y*\sin(\kappa\text{-}\alpha)+L*\sin(\alpha)\right)^2}{2*\tan(\beta)}\right)$$

Where:

$\alpha =$ Angle of the fluke in the sand
$\beta =$ Angle of the shear zone
$\kappa =$ Angle between fluke and shank
$L =$ Total fluke length
$y =$ Length of the shank in the sand
$\gamma =$ Density of the in situ sand

Forces on the fluke and the shank

For the determination of the forces on the fluke and the shank, two different theories will be used. For the fluke the cutting theory of Miedema is valid, therefore forces on the soil layer and on the fluke are the same as discussed in situation 2.
For the forces on the shank the strip footing theory can be used. This theory is based on the fundamentals of Brinch Hansen and is a generalization of the Prantl theory.

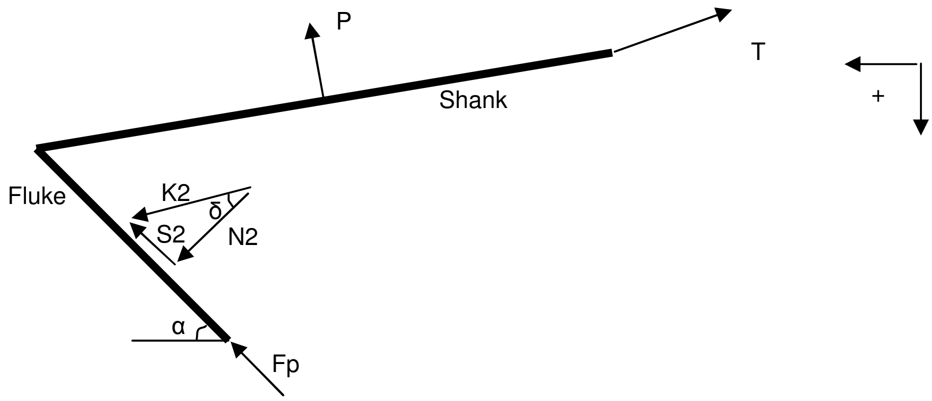


Figure 4.14: Forces on the anchor

To determine the friction Brinch Hansen force P on the shank we can make use of:

$$P = i_c s_c c N_c + i_q s_q q N_q + i_\gamma s_\gamma \frac{1}{2} \gamma B N_\gamma$$

Where c is cohesion and q is the external load on the soil

Because c and q are zero in this case (no cohesion and no external force on the soil), P will only be a function of the soil weight part of the function, so:

$$P = i_\gamma s_\gamma \frac{1}{2} \gamma B N_\gamma$$

Hereby i_γ is a correction factor for inclination factors of the load. The factor s_γ is a shape factor for the shape of the load.

In this case only a load perpendicular to the soil will be considered, so i_γ will be removed from the formula.

Inserting N_γ:

$$P = \left(1 - 0{,}3\frac{B}{y}\right) B^2 y \gamma \left(\frac{1+\sin\phi}{1-\sin\phi} e^{\pi \tan\phi} - 1\right) \tan\phi$$

where:

y = the length of the shank in the sand (at a moment in time).
B= the width of the shank

Now the friction part of the shank has to be determined.
For the friction of the shank, the next formula is valid:

$F_{friction} = \sigma_n \tan(\delta)\ y\ h$

Where:

- σ_n = Normal stress on the area of the shank
- δ = External friction angle
- y = the length of the shank in the sand
- h = the height of the shank

It is possible now to plot the results for P and $F_{friction}$ (see example 3.1) then it is possible to find out if the downward force of the fluke is big enough to pull the shank through the seabed and further.

It is also possible now to make a total force and moment balance, to predict the trajectory of the anchor.

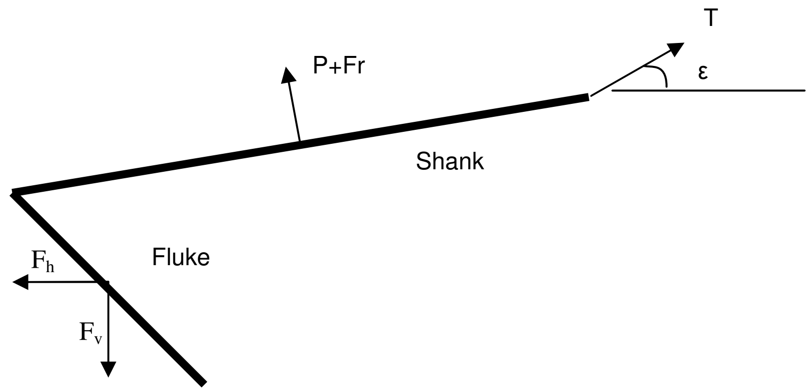


Figure 4.15: Forces on the anchor

Example 3.1; Forces calculations for a real anchor

In this example the earlier discussed theory will be used, for a real anchor at a certain depth, to check the different forces on this anchor.

- *Fluke forces:*

First of all the vertical and horizontal forces 'created' by the fluke will be determined, the results of these different forces are shown in figure:0.89

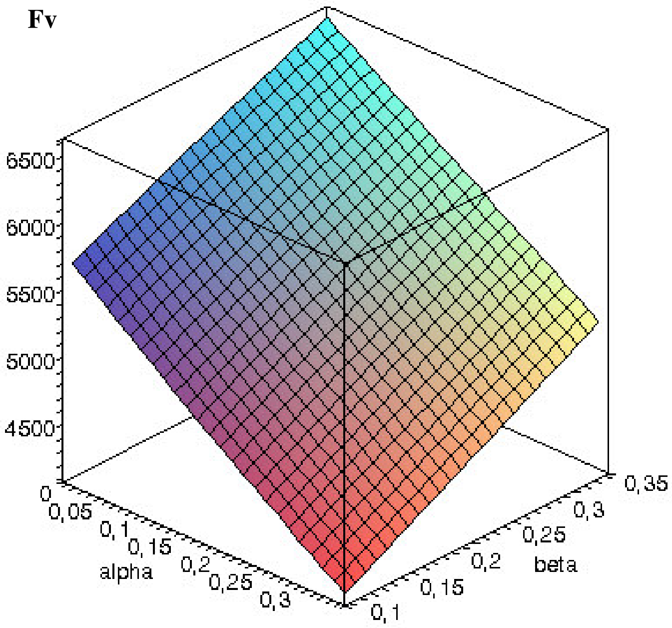


Figure 4.16: Vertical fluke force 3D

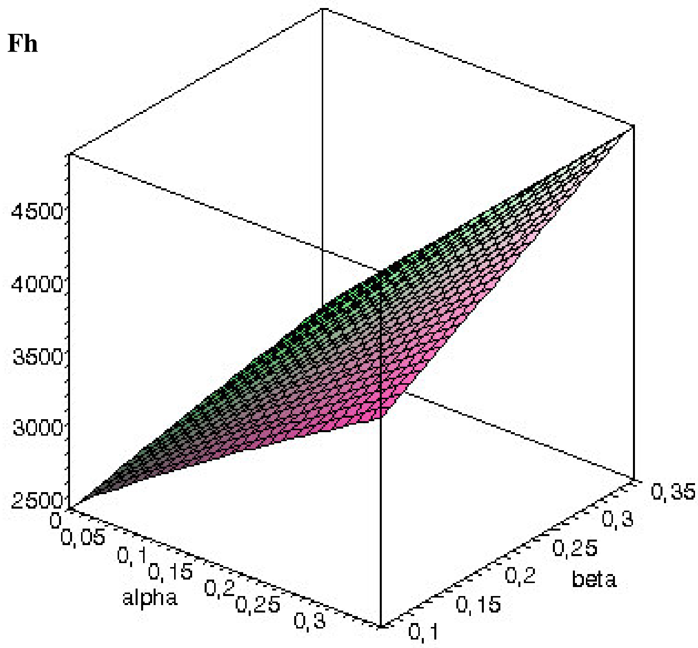


Figure 4.17: Horizontal fluke force 3D

For this situation the α = 0,15 (for example) β = 0,3.
So:

F_v = 5570N
F_h = 3470N

- *Shank forces:*

Now the forces perpendicular to the shank will be calculated for the case that:

B= 0,1m
y= 3,0 m
γ= 1900 kg/m^3
Φ= 0,611 rad

$$P = \left(1 - 0,3\frac{B}{y}\right)B^2 y\gamma\left(\frac{1+\sin\phi}{1-\sin\phi}e^{\pi\tan\phi} - 1\right)\tan\phi \qquad = 1280\text{N}$$

After this, the friction force of the shank will be determined to be:

$$F_r = \tan(\delta)\cdot y\cdot\sigma_n\cdot h \quad = 3100N$$

with:

$\sigma_n =$ 4000Pa

$h =$ 0.6m

- *Equilibrium of a stable anchor*

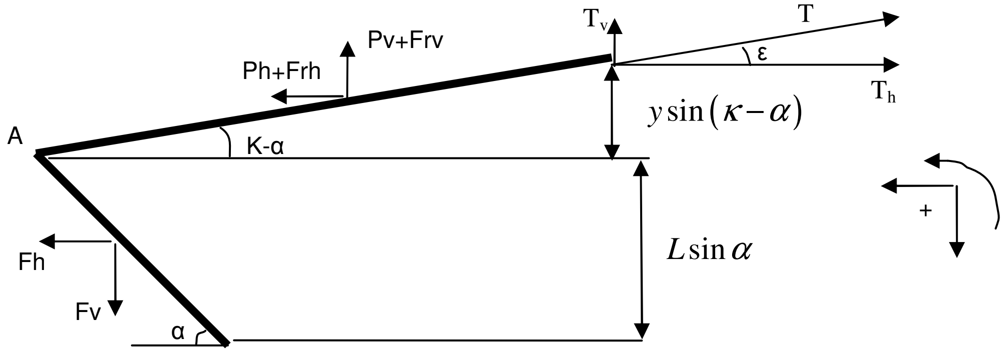


Figure 4.18: Forces on the anchor

When looking at the point where the anchor becomes stable a force and moment balance can be made out of all the forces on the anchor and mooring line. In fact this is the moment were the anchor reaches his maximum holding capacity.

Vertical equilibrium of forces:

$$F_v - P_v - F_{rv} - T_v = 0$$

Now one can calculate the maximum vertical component of the pull force T_v :

$$T_v = F_v - P_v - F_{rv} = 5570 - 1280\cos(0{,}34 - 0{,}15) - 3100\cos(0{,}34 - 0{,}15) = 1269N$$

The maximum horizontal component of the pull force (T_h) can be calculated in the same way:

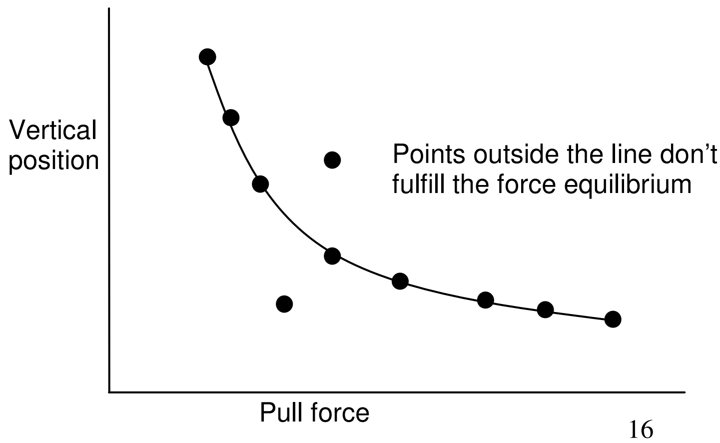
$$T_h = F_h + P_h + F_{rh} = 3470 + 1280\sin(0{,}34 - 0{,}15) + 3100\sin(0{,}34 - 0{,}15) = 4297N$$

Moment balance to point A:

$$-F_v \cdot \frac{1}{2}L\cos\alpha - F_h \cdot \frac{1}{2}L\sin\alpha + (P_v + F_{rv}) \cdot \frac{1}{2}y\cos(\kappa-\alpha) + (P_h + F_{rh}) \cdot \frac{1}{2}y\sin(\kappa-\alpha)$$
$$+T_v y\cos(\kappa-\alpha) - T_h y\sin(\kappa-\alpha) = 2449Nm$$

In practice:
When positioning an anchor, at each anchor position the maximum pull force can be determined.
For the next point this process will be repeated and the maximum pull force at that specific position can be calculated as well. As a result of these positions, a curve can be made of the holding capacity of the anchor at a certain depth.

Figure 4.19: Maximum pull force vs. depth



How to interpret the relationship between reality and theory

In theory we have started with the geometry in combination with the cutting theory of Miedema and the strip footing theory as discussed in Verruijt. So the output of this theory is the pull force. In reality it is easier to understand the inverse process.
In this way you can use the pull force as an input to determine the holding capacity and the trajectory of the anchor.

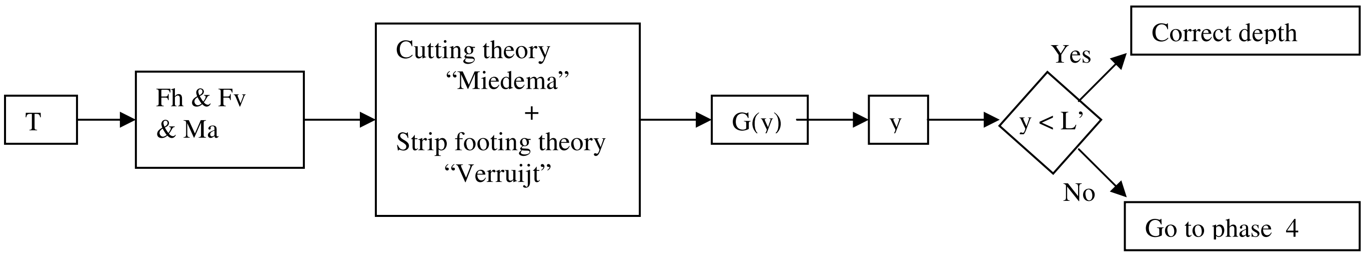


Figure 4.20: Block scheme interpretation

In practice you know the needed pull force and when using this model, you get inside information of the trajectory and the holding capacity. Because of that it's more practical to have the pull force as an input.

4.4 Phase 4

In this chapter you will find the modeling of the forces on the fluke, the shank and the mooring line. Cutting theory of Miedema is still valid for the fluke part of the anchor forces. For determining the forces on the shank and the mooring line we will use the strip footing theory as discussed in Verruijt.

Fluke, shank and mooring line in the soil

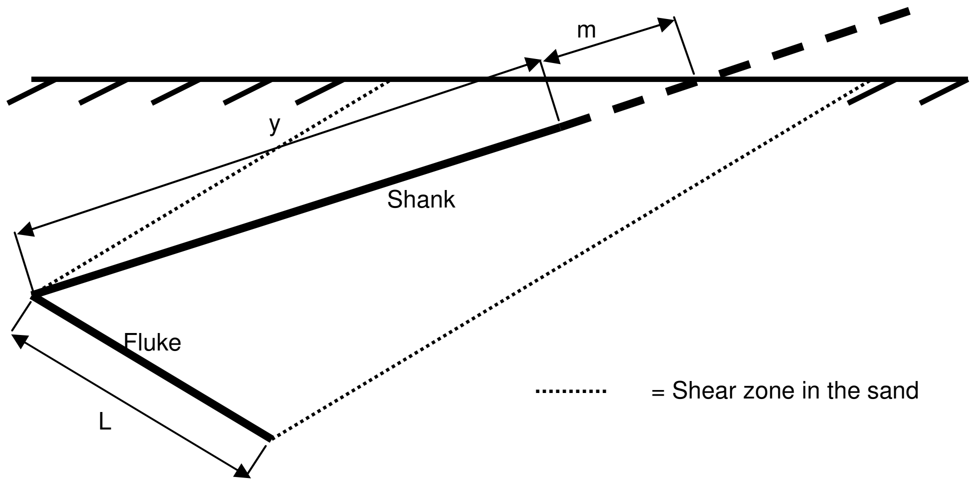


Figure 4.21: Phase 4

Forces on the soil layer

The soil layer properties can be interpreted in a same way as described in phase 3. So this way the function for G is still valid.

Forces on the fluke, shank and mooring line

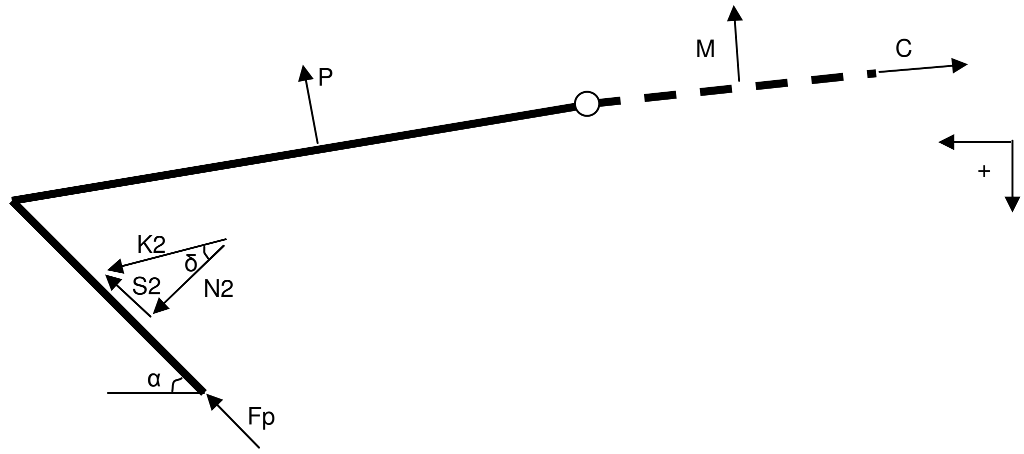


Figure 4.22: Forces on the anchor and mooring line

To determine the forces on the anchor for this situation the theory as discussed in phase 3 is valid. For the mooring line forces we will also use the Brinch Hansen theory.

- *Anchor forces:*

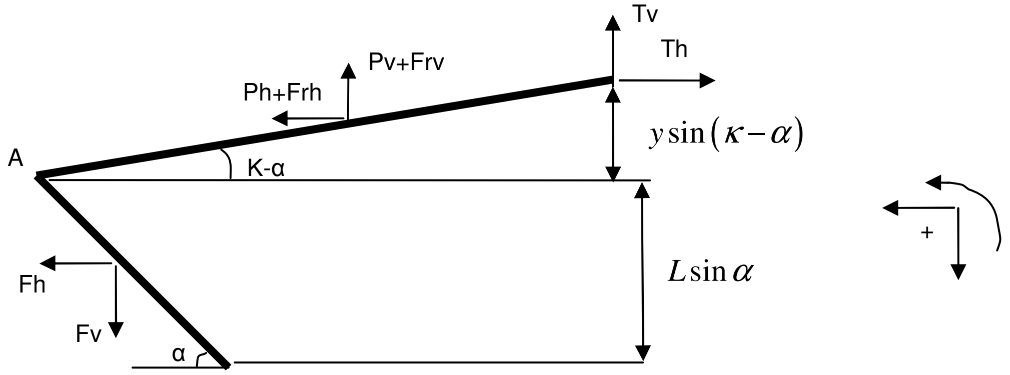
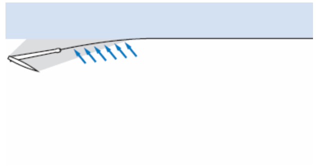
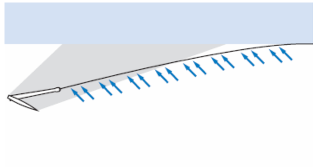


Figure 4.23: Forces on the anchor

- *Mooring line forces:*



The penetration of the mooring line causes resistance perpendicular to this line (penetration resistance).
This effect is noticeable in all soil conditions. The type of mooring line will determine the value of this resistance. Think of a wire rope mooring line which penetrates deeper (less resistance) than a chain mooring line.



During the penetration process of the anchor, the resistance increases when depth increases, which is related to the position of the anchor.

The mooring line penetration can be described by the following geometry:

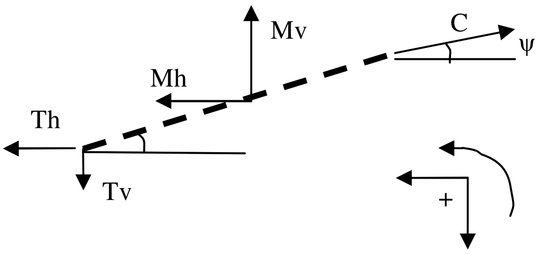


Figure 4.24: Forces on the mooring line

With:

M=	resistance on mooring line
C=	Catenary force
T=	Anchor pull force

When looking at the point where the anchor becomes stable, a force and moment balance can be made out of all the forces on the anchor and mooring line. In fact this is the moment were the anchor reaches his maximum holding capacity.

Vertical equilibrium of forces:

$$F_v - P_v - F_{rv} - M_v - T_v = 0$$

Horizontal equilibrium of forces:

$$F_h + P_h + F_{rh} + M_h - T_h = 0$$

Moment balance to point A:

$$-F_v \cdot \frac{1}{2} L\cos\alpha - F_h \cdot \frac{1}{2} L\sin\alpha + \left(P_v + F_{rv}\right) \cdot \frac{1}{2} y\cos\left(\kappa - \alpha\right) + \left(P_h + F_{rh}\right) \cdot \frac{1}{2} y\sin\left(\kappa - \alpha\right)$$
$$+T_v \cdot y\cos\left(\kappa - \alpha\right) + T_h \cdot \sin\left(\kappa - \alpha\right) = 0$$

How to interpret the relationship between reality and theory

In theory we have started with the geometry in combination with the cutting theory of Miedema and the strip footing theory as discussed in Verruijt. So the output of this theory is the pull force. In reality it is easier to understand the inverse process.
In this way you can use the pull force as an input to determine the holding capacity and the trajectory of the anchor.

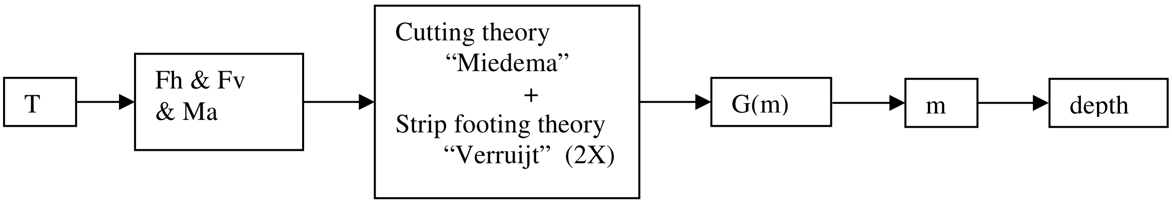


Figure 4.25: Block scheme interpretation

In practice you know the needed pull force and when using this model, you get inside information of the trajectory and the holding capacity. Because of that it's more practical to have the pull force as an input.

5 Conclusions and recommendations

In the penetration behavior the different forces and moments in all four phases can be described with the theory dealt with, in this document. These forces and moments are a function of the anchor geometry.
The holding capacity of the anchor is described as well as a function of the depth and the geometry.
To predict the trajectory of the anchor during the penetration, one has to find a relationship between the different forces and moments on the anchor and the trajectory of the anchor. The anchor trajectory will stop when the different forces are in equilibrium, or when the pull force will be too high for that particular anchor, at a certain depth. In the last case, the pull force necessary to penetrate deeper in the soil, is higher then the maximum holding capacity of that particular anchor at a certain depth. At that point the maximum holding capacity is reached (see figure:5.1).
When pulling further, the anchor will be pulled out and looses his function.

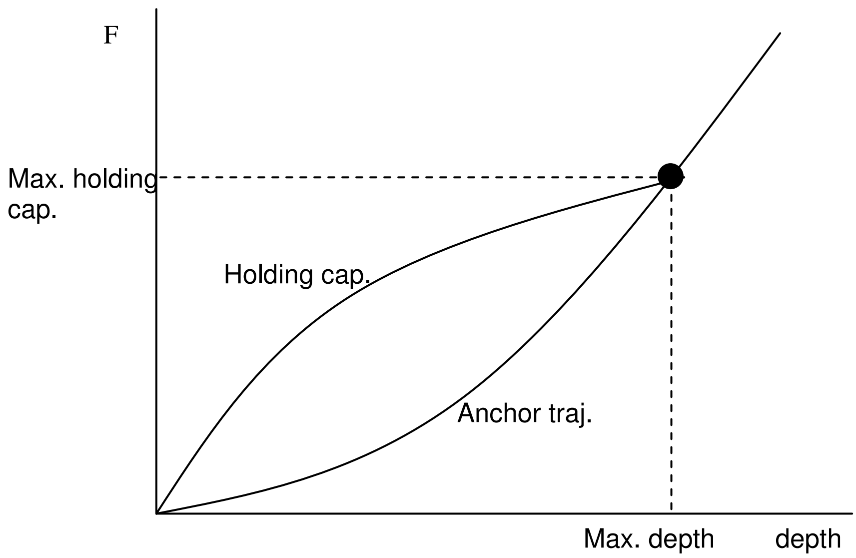


Figure 5.1: Holding capacity vs. anchor trajectory

6 References

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- Miedema, S.A., "Calculation of the Cutting Forces when Cutting Water Saturated Sand." September 15th 1987
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