# CT4450 IWRM Week 2 – Scenarios

***Excercise for on-line participants:***
***Acquiring intuition for high-dimensional spaces***

The system that you will model and analyse is very complex. Looking into the futures, there are many dimensions that need to be taken into account for scenario development. The decision and modeling spaces are high dimensional. We do not have a natural intuition for such spaes but it can be acquired quickly.
Try to answer the following questions, start with a square with sides of unit length, moving to a cube, and hypercubes of 4, 10, 100, 1000, and 10,000 dimensions:

1. What is the distance between two opposing corners? (Or: what is the longest distance within the square/cube?)
2. If you sample along 80% of each side, what percentage of total volume do you sample?
3. If you take ten samples that each represent 10% of the length along each feature dimension, what percentage of the total volume do you sample? Note: You do not sample 100% along each axis, each sample represents 10% of each axis.
4. People sometimes try to overcome the dimensionality sampling problem by using a "Latin hypercube" construction, a multi-dimensional variation of the Latin square:

| | 1 | 1 | 1 | 1 |
|---|---|---|---|---|
| 1 | **X** | | | |
| 1 | | | **X** | |
| 1 | | | | **X** |
| 1 | | **X** | | |

   The idea is that because you sample each part of each feature, your sample is representative. Do you agree?

5. As a function of the number of dimensions *n*:
   a) What is the modal distance from a random point to the **center** of the hypercube?
   b) What is the average distance from a random point to the **outside** of the hypercube?
   c) What is the average distance between two points that are (uniformly) randomly distributed within the cubes?
   d) What is the "surface" area?
   e) If 90 samples are needed to "cover" 90% of a square (2D), how many samples are needed to cover 90% of nD hypercubes?