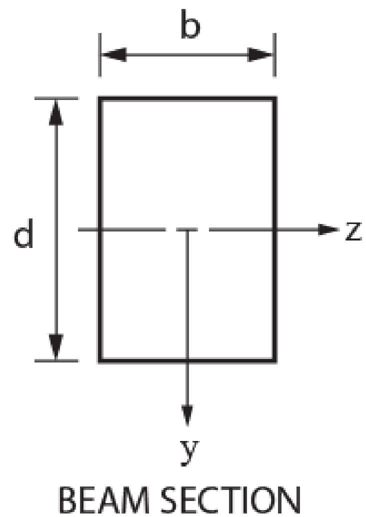
Example 7.1: Shear Stress Distribution in a Rectangular Beam

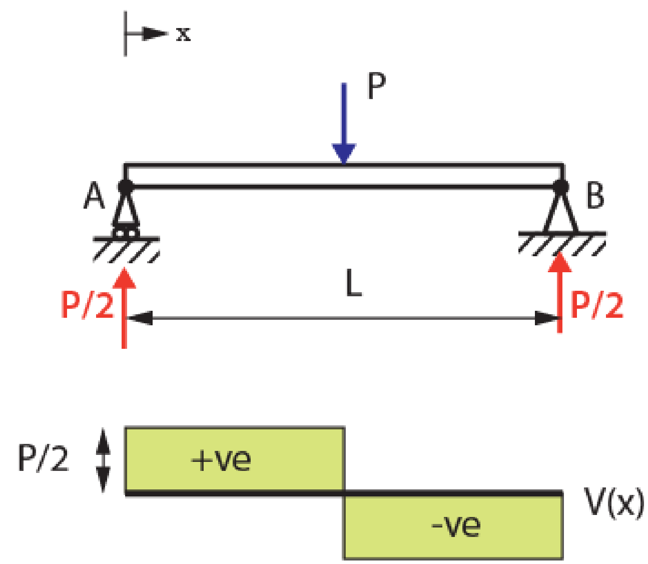
To be Attempted In Class

For the beam given below (shear force diagram provided) determine and plot the shear stress distribution for the cross-section:

Take 5 minutes and attempt to solve

$$\tau = \frac{VQ}{It}$$





Solution

Example 7.1

$$\tau = \frac{VQ}{It}$$

Determine V : $|V| = \frac{P}{2}$ (from shear force diagram)

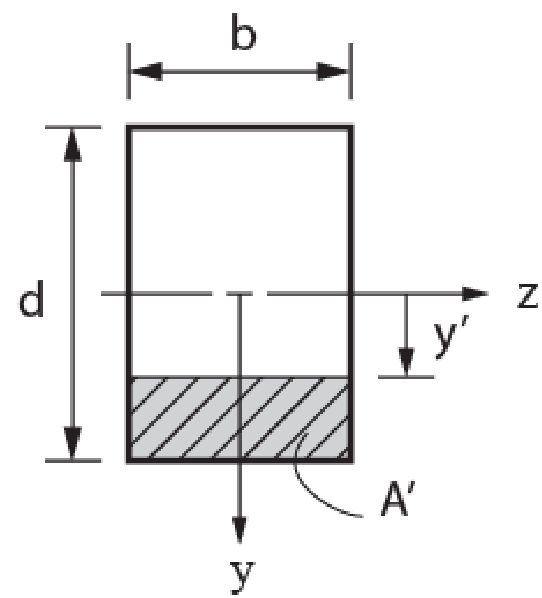
Determine I : $I = \frac{b \cdot d^3}{12}$ (standard solution for rectangle)

Determine Q :

<u>Method 1: by integration</u>

$$Q(y') = \int_{A'} y \cdot dA = \int_{y'}^{d/2} y \cdot b \cdot dy = \frac{b}{2} \cdot y^2 \Big|_{y'}^{d/2}$$

$$\therefore Q(y') = \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - y'^2\right]$$

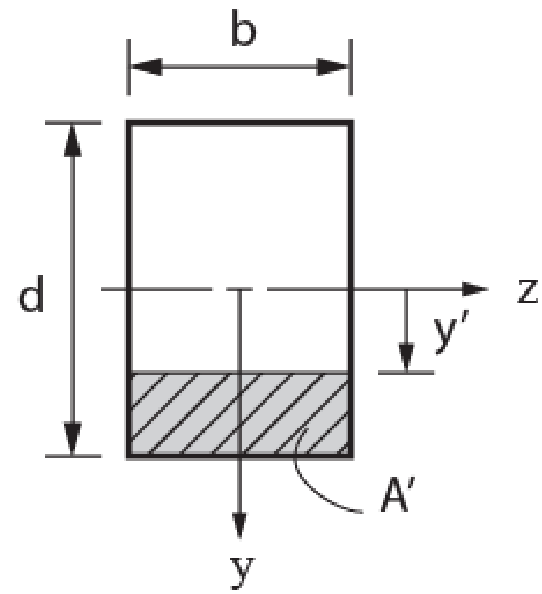


Solution

Example 7.1

Determine Q :

<u>Method 2: by centroid location</u>



$$Q(y') = \overline{y'} \cdot A'$$

$$A' = b \cdot \left(\frac{d}{2} - y'\right)$$

$$\overline{y'} = y' + \frac{1}{2}\left(\frac{d}{2} - y'\right) = \frac{1}{2}\left(y' + \frac{d}{2}\right)$$

$$\therefore Q(y') = \left[\frac{1}{2}\left(y' + \frac{d}{2}\right)\right] \cdot \left[b\left(\frac{d}{2} - y'\right)\right] = \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - y'^2\right]$$

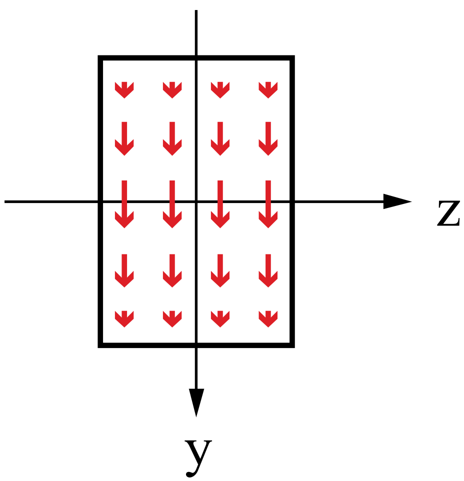
Solution

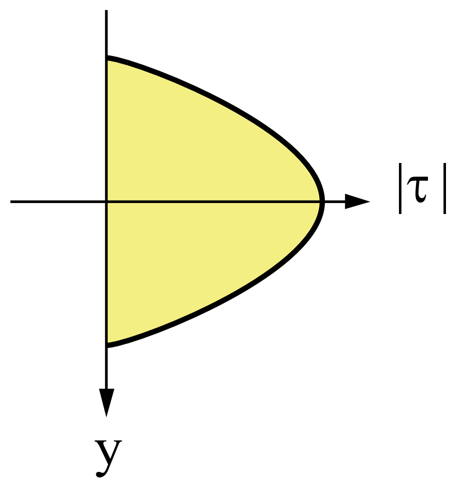
Example 7.1

$$\tau = \frac{VQ}{It} = \frac{(P/2)}{\left(\frac{1}{12}bd^3\right)b} \cdot \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - y'^2\right] = \frac{3P}{bd^3}\left[\left(\frac{d}{2}\right)^2 - y'^2\right]$$

τ_{max} at y′ = 0 (@ N.A.): $$\left|\tau_{max}\right| = \frac{3P}{4bd}$$

parabolic
- max at N.A.
- 0 at y′ = d/2





What if we did use uniform stress approximation?

$$\tau_{average} = \frac{V}{bd} = \frac{P}{2bd}$$

$$\tau_{average} = \frac{2}{3}\tau_{max}!!!!$$