

Electronic Instrumentation

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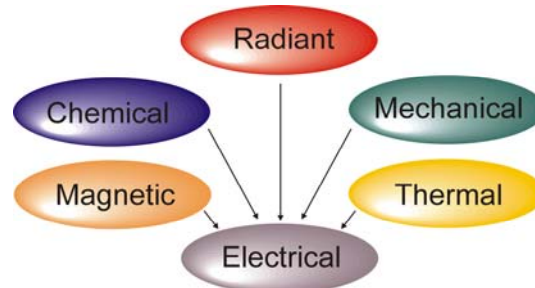
Content and Schedule

Monday Sept. 16: Transduction of information (Chapter 2)
- Sensitivity and cross-sensitivities
- Resistive transducers
- Capacitive transducers

Wednesday Sept. 18: Submit **Assignment 1**
Detection limit due to offset

What Is A Sensor?

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- A device that converts information from one energy domain into the **electrical** domain
- Where it can be easily (digitally) processed and stored

The World Is Analog

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- So sensors output **analog** information
- Requires **analog-to-digital** conversion
- Note: transducer = sensor OR actuator



Signal domains:

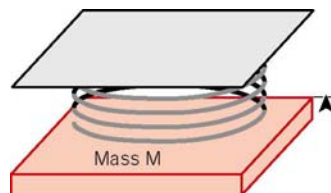
- Magnetic (Ma),
- Mechanical (Me),
- Thermal (Th),
- Optical (Op),
- Chemical (Ch) and
- **Electrical (EI).**

Sources of uncertainty (error) :

- Source loading by the measurement
- **Sensitivity to unintended el. and non-el. quantities**
- Electro-magnetic interference
- **Thermal noise**
- Many more.....

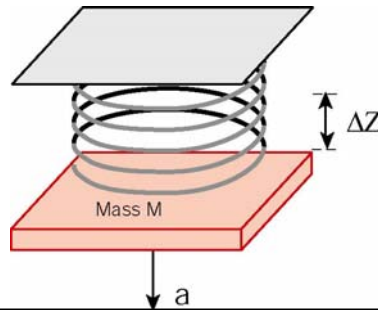
Transduction matrix:

$$\begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ EI \end{pmatrix} = \begin{pmatrix} t_{ma,ma} & t_{me,ma} & t_{th,ma} & t_{opt,ma} & t_{ch,ma} & t_{el,ma} \\ t_{ma,me} & t_{me,me} & t_{th,me} & t_{opt,me} & t_{ch,me} & t_{el,me} \\ t_{ma,th} & t_{me,th} & t_{th,th} & t_{opt,th} & t_{ch,th} & t_{el,th} \\ t_{ma,opt} & t_{me,opt} & t_{th,opt} & t_{opt,opt} & t_{ch,opt} & t_{el,opt} \\ t_{ma,ch} & t_{me,ch} & t_{th,ch} & t_{opt,ch} & t_{ch,ch} & t_{el,ch} \\ t_{ma,el} & t_{me,el} & t_{th,el} & t_{opt,el} & t_{ch,el} & t_{el,el} \end{pmatrix} \begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ EI \end{pmatrix} + \begin{pmatrix} Ma_{os} \\ Me_{os} \\ Th_{os} \\ Opt_{os} \\ Ch_{os} \\ EI_{os} \end{pmatrix}$$



Transduction matrix:

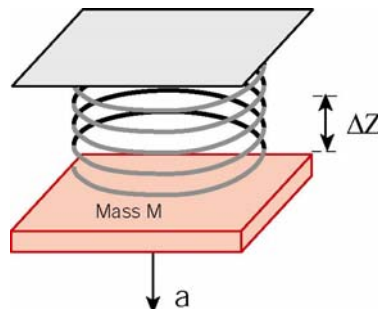
$$\begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} = \begin{pmatrix} t_{ma,ma} & t_{me,ma} & t_{th,ma} & t_{opt,ma} & t_{ch,ma} & t_{el,ma} \\ t_{ma,me} & t_{me,me} & t_{th,me} & t_{opt,me} & t_{ch,me} & t_{el,me} \\ t_{ma,th} & t_{me,th} & t_{th,th} & t_{opt,th} & t_{ch,th} & t_{el,th} \\ t_{ma,opt} & t_{me,opt} & t_{th,opt} & t_{opt,opt} & t_{ch,opt} & t_{el,opt} \\ t_{ma,ch} & t_{me,ch} & t_{th,ch} & t_{opt,ch} & t_{ch,ch} & t_{el,ch} \\ t_{ma,el} & t_{me,el} & t_{th,el} & t_{opt,el} & t_{ch,el} & t_{el,el} \end{pmatrix} \begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} + \begin{pmatrix} Ma_{os} \\ Me_{os} \\ Th_{os} \\ Opt_{os} \\ Ch_{os} \\ El_{os} \end{pmatrix}$$



Acceleration to displ. =
Transduction from
mechanical to mechanical

Transduction matrix (red text ⇒ non-zero coeffs):

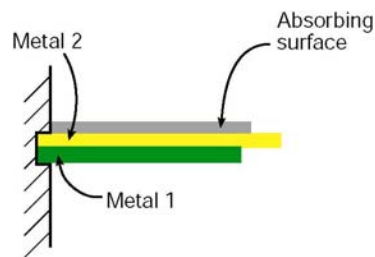
$$\begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} = \begin{pmatrix} t_{ma,ma} & t_{me,ma} & t_{th,ma} & t_{opt,ma} & t_{ch,ma} & t_{el,ma} \\ t_{ma,me} & t_{me,me} & t_{th,me} & t_{opt,me} & t_{ch,me} & t_{el,me} \\ t_{ma,th} & t_{me,th} & t_{th,th} & t_{opt,th} & t_{ch,th} & t_{el,th} \\ t_{ma,opt} & t_{me,opt} & t_{th,opt} & t_{opt,opt} & t_{ch,opt} & t_{el,opt} \\ t_{ma,ch} & t_{me,ch} & t_{th,ch} & t_{opt,ch} & t_{ch,ch} & t_{el,ch} \\ t_{ma,el} & t_{me,el} & t_{th,el} & t_{opt,el} & t_{ch,el} & t_{el,el} \end{pmatrix} \begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} + \begin{pmatrix} Ma_{os} \\ Me_{os} \\ Th_{os} \\ Opt_{os} \\ Ch_{os} \\ El_{os} \end{pmatrix}$$



Acceleration to displ. =
Transduction from
mechanical to mechanical

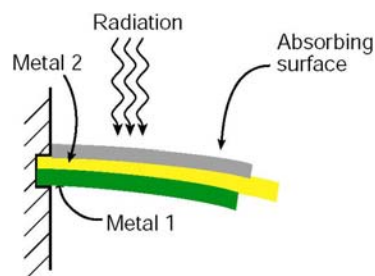
Transduction matrix:

$$\begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} = \begin{pmatrix} t_{ma,ma} & t_{me,ma} & t_{th,ma} & t_{opt,ma} & t_{ch,ma} & t_{el,ma} \\ t_{ma,me} & t_{me,me} & t_{th,me} & t_{opt,me} & t_{ch,me} & t_{el,me} \\ t_{ma,th} & t_{me,th} & t_{th,th} & t_{opt,th} & t_{ch,th} & t_{el,th} \\ t_{ma,opt} & t_{me,opt} & t_{th,opt} & t_{opt,opt} & t_{ch,opt} & t_{el,opt} \\ t_{ma,ch} & t_{me,ch} & t_{th,ch} & t_{opt,ch} & t_{ch,ch} & t_{el,ch} \\ t_{ma,el} & t_{me,el} & t_{th,el} & t_{opt,el} & t_{ch,el} & t_{el,el} \end{pmatrix} \begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} + \begin{pmatrix} Ma_{os} \\ Me_{os} \\ Th_{os} \\ Opt_{os} \\ Ch_{os} \\ El_{os} \end{pmatrix}$$



Transduction matrix:

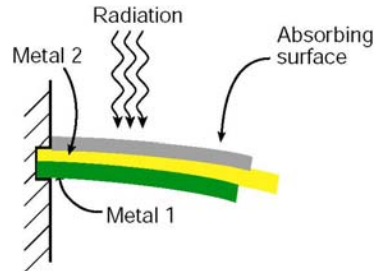
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Transduction in two steps:
1. from optical to thermal,
followed by
2. thermal to mechanical

= tandem transducer

Tandem transduction:



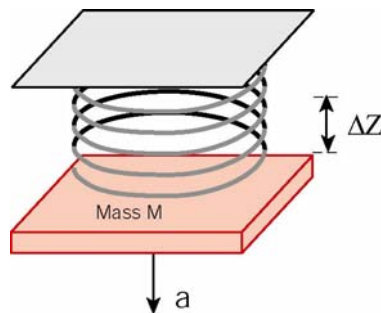
Transduction in two steps:
1. from optical to thermal,
followed by
2. thermal to mechanical

= tandem transducer

$$Me = (S_{ma,me} \ S_{me,me} \ S_{th,me} \ S_{opt,me} \ S_{ch,me} \ S_{el,me}) \bullet$$

$$\begin{pmatrix} t_{ma,ma} & t_{me,ma} & t_{th,ma} & t_{opt,ma} & t_{ch,ma} & t_{el,ma} \\ t_{ma,me} & t_{me,me} & t_{th,me} & t_{opt,me} & t_{ch,me} & t_{el,me} \\ t_{ma,th} & t_{me,th} & t_{th,th} & t_{opt,th} & t_{ch,th} & t_{el,th} \\ t_{ma,opt} & t_{me,opt} & t_{th,opt} & t_{opt,opt} & t_{ch,opt} & t_{el,opt} \\ t_{ma,ch} & t_{me,ch} & t_{th,ch} & t_{opt,ch} & t_{ch,ch} & t_{el,ch} \\ t_{ma,el} & t_{me,el} & t_{th,el} & t_{opt,el} & t_{ch,el} & t_{el,el} \end{pmatrix} \begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} + \begin{pmatrix} Ma_{os} \\ Me_{os} \\ Th_{os} \\ Opt_{os} \\ Ch_{os} \\ El_{os1} \end{pmatrix} + Me_{os2}$$

Tandem transduction requires an intermediate non-electrical domain:



So:

1. Acceleration to displacement (within mechanical domain),
- followed by
2. displacement to capacitance (to the electrical domain)

⇒ not a tandem transducer

Transduction to the electrical domain:

$$\begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} = \begin{pmatrix} t_{ma,ma} & t_{me,ma} & t_{th,ma} & t_{opt,ma} & t_{ch,ma} & t_{el,ma} \\ t_{ma,me} & t_{me,me} & t_{th,me} & t_{opt,me} & t_{ch,me} & t_{el,me} \\ t_{ma,th} & t_{me,th} & t_{th,th} & t_{opt,th} & t_{ch,th} & t_{el,th} \\ t_{ma,opt} & t_{me,opt} & t_{th,opt} & t_{opt,opt} & t_{ch,opt} & t_{el,opt} \\ t_{ma,ch} & t_{me,ch} & t_{th,ch} & t_{opt,ch} & t_{ch,ch} & t_{el,ch} \\ t_{ma,el} & t_{me,el} & t_{th,el} & t_{opt,el} & t_{ch,el} & t_{el,el} \end{pmatrix} \begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} + \begin{pmatrix} Ma_{os} \\ Me_{os} \\ Th_{os} \\ Opt_{os} \\ Ch_{os} \\ El_{os} \end{pmatrix}$$

One sensitivity plus up to 5 cross-sensitivities

Example: A photodiode

Sensitivity: $t_{opt,el}$

Cross-sensitivities: $t_{el,el}$ (?) $t_{th,el}$ (?)

Offset: due to dark current

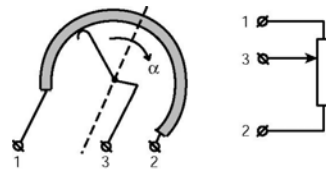
Transduction to the electrical domain:

$$\begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} = \begin{pmatrix} t_{ma,ma} & t_{me,ma} & t_{th,ma} & t_{opt,ma} & t_{ch,ma} & t_{el,ma} \\ t_{ma,me} & t_{me,me} & t_{th,me} & t_{opt,me} & t_{ch,me} & t_{el,me} \\ t_{ma,th} & t_{me,th} & t_{th,th} & t_{opt,th} & t_{ch,th} & t_{el,th} \\ t_{ma,opt} & t_{me,opt} & t_{th,opt} & t_{opt,opt} & t_{ch,opt} & t_{el,opt} \\ t_{ma,ch} & t_{me,ch} & t_{th,ch} & t_{opt,ch} & t_{ch,ch} & t_{el,ch} \\ t_{ma,el} & t_{me,el} & t_{th,el} & t_{opt,el} & t_{ch,el} & t_{el,el} \end{pmatrix} \begin{pmatrix} Ma \\ Me \\ Th \\ Opt \\ Ch \\ El \end{pmatrix} + \begin{pmatrix} Ma_{os} \\ Me_{os} \\ Th_{os} \\ Opt_{os} \\ Ch_{os} \\ El_{os} \end{pmatrix}$$

Two types of sensors:

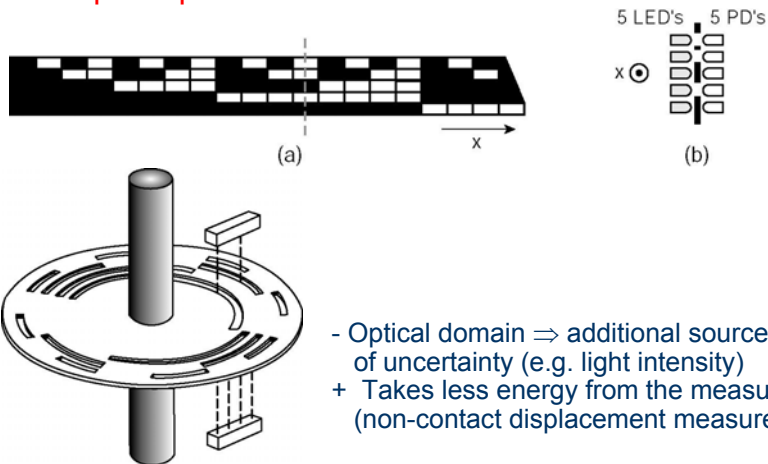
- Self generating sensors
- Modulating sensors

Self-generating sensors:
Example: Potentiometer



- + No additional sources of uncertainty
- + No external power supply
- Draws all its energy from the measurand (mechanical source loading)

Modulating sensors:
Example: Optical encoder

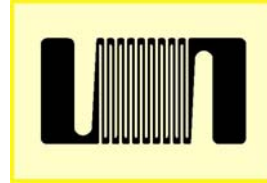


- Optical domain \Rightarrow additional sources of uncertainty (e.g. light intensity)
- + Takes less energy from the measurand (non-contact displacement measurement)

Force sensors using strain gauges

Resistance value is defined by specific resistivity and dimensions:

$$R = \rho \frac{L}{A}$$



In case of strain (deformation) due to mechanical stress:

$$\frac{\partial R}{R} = \frac{\partial \rho}{\rho} + \frac{\partial L}{L} - \frac{\partial A}{A}$$

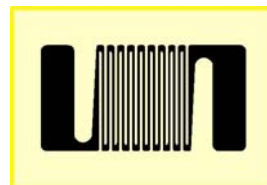
When assuming constant volume:

$$\frac{\partial V}{V} = 0 \rightarrow \frac{\partial L}{L} = -\frac{\partial A}{A}$$

Force sensors using strain gauges

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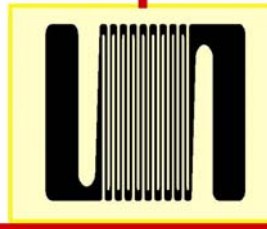
$$\frac{\partial R}{R} = \frac{\partial \rho}{\rho} + \frac{\partial L}{L} - \frac{\partial A}{A}$$

When assuming constant volume:

$$\left. \begin{array}{l} \frac{\partial V}{V} = 0 \rightarrow \frac{\partial L}{L} = -\frac{\partial A}{A} \\ \text{Metal film: } \frac{\partial \rho}{\rho} \propto \frac{\partial L}{L} \end{array} \right\} \rightarrow \frac{\partial R}{R} = 2 \frac{\partial L}{L} = k \frac{\partial L}{L}$$

k= gauge factor

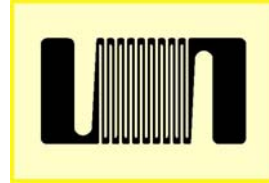
TENSILE STRESS in sensitive direction: F $\left(\frac{\Delta R}{R}\right)_{strain} > 0$



Force sensors using strain gauges

Resistance value is defined by specific resistivity and dimensions:

$$R = \rho \frac{L}{A}$$



In case of strain (deformation) due to mechanical stress:

$$\frac{\partial R}{R} = \frac{\partial \rho}{\rho} + \frac{\partial L}{L} - \frac{\partial A}{A}$$

When assuming constant volume:

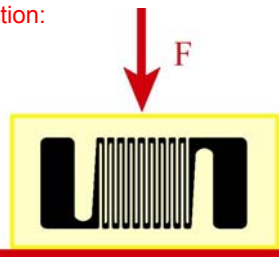
$$\frac{\partial V}{V} = 0 \rightarrow \frac{\partial L}{L} = -\frac{\partial A}{A}$$

$$\text{Metal film: } \frac{\partial \rho}{\rho} \approx \frac{\partial L}{L} \rightarrow \frac{\partial R}{R} = 2 \frac{\partial L}{L} = k \frac{\partial L}{L}$$

k = gauge factor

COMPRESSIVE STRESS in sensitive direction:

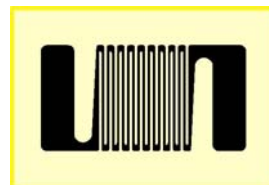
$$\left(\frac{\Delta R}{R}\right)_{comp} = -\left(\frac{\Delta R}{R}\right)_{tens}$$



Force sensors using strain gauges

Resistance value is defined by specific resistivity and dimensions:

$$R = \rho \frac{L}{A}$$



In case of strain (deformation) due to mechanical stress:

$$\frac{\partial R}{R} = \frac{\partial \rho}{\rho} + \frac{\partial L}{L} - \frac{\partial A}{A}$$

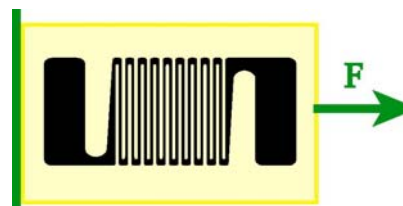
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$$\text{Metal film: } \frac{\partial \rho}{\rho} \approx \frac{\partial L}{L} \rightarrow \frac{\partial R}{R} = 2 \frac{\partial L}{L} = k \frac{\partial L}{L}$$

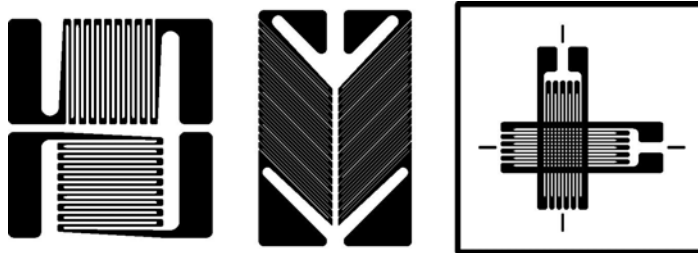
k = gauge factor

Applying load in insensitive direction:

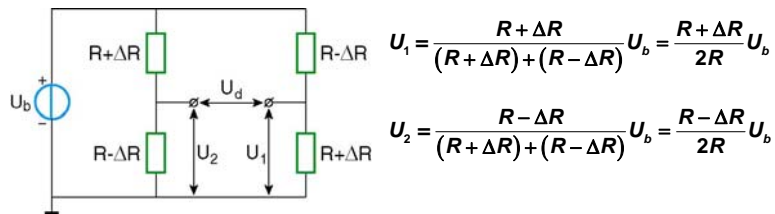


$$\left(\frac{\Delta R}{R}\right)_{perp.} \approx 0$$

Measuring force in two dimensions



Wheatstone bridge:

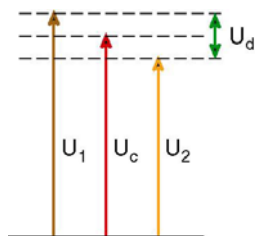


Information is contained in the differential signal:

$$U_d = U_1 - U_2 = \frac{\Delta R}{R} U_b < 10mV \text{ at } U_b = 10V$$

Which, however, is superimposed on the common-mode signal:

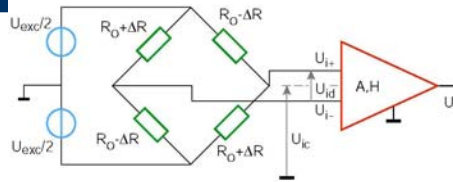
$$U_c = \frac{U_1 + U_2}{2} = \frac{U_b}{2} = 5V \text{ at } U_b = 10V$$



Bridge configurations

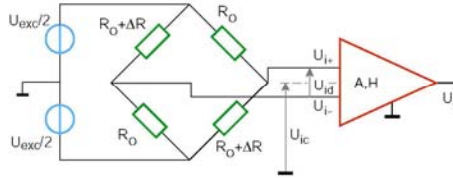
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Full bridge



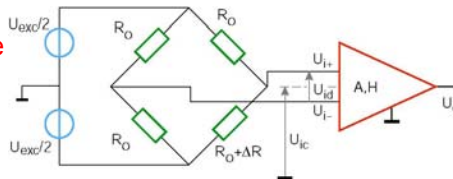
- Linear
- All elements on structure
- Increasing AND decreasing with measurand

1/2 bridge



- Non-linear
- 1/2 of elements on structure
- Increasing OR decreasing with measurand

1/4 bridge



- Non-linear
- 1/4 of elements on structure - minimum dimensions.

Capacitive sensor readout

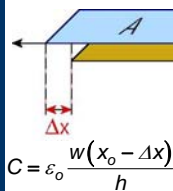
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El. Instr.

Very suitable for displacement measurements

From the parallel-plate approximation:

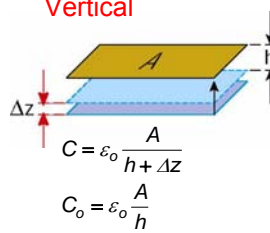
$$C = \epsilon_0 \epsilon_r \frac{A}{h} = \epsilon_0 \frac{A}{h}$$

Lateral



Linear

Vertical



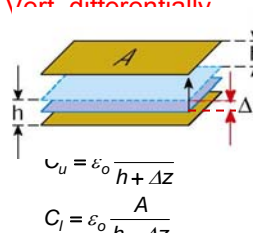
$$C_0 = \epsilon_0 \frac{A}{h}$$

$$C = \epsilon_0 \frac{A}{h + \Delta z}$$

$$\frac{\epsilon_0 A}{h^2} \frac{\Delta z}{1 + \Delta z/h} \approx \frac{\epsilon_0 A}{h^2} \Delta z$$

Non-linear

Vert. differentially



$$C_u = \epsilon_0 \frac{A}{h + \Delta z}$$

$$C_l = \epsilon_0 \frac{A}{h - \Delta z}$$

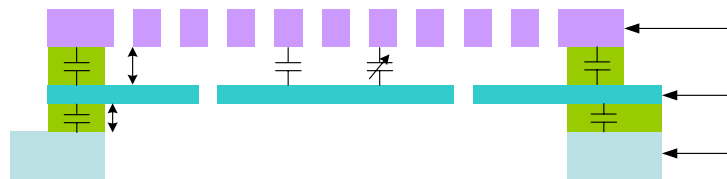
$$\Delta C = C_0 - C = \epsilon_0 A \left(\frac{1}{h} - \frac{1}{h + \Delta z} \right) = C_l - C_u = \epsilon_0 \frac{A}{h} \left(\frac{1}{1 - \Delta z/h} - \frac{1}{1 + \Delta z/h} \right)$$

$$\Delta C = \epsilon_0 \frac{A}{h} \left(\frac{2 \Delta z/h}{1 + (\Delta z/h)^2} \right) \approx \frac{2 \epsilon_0 A}{h^2} \Delta z$$

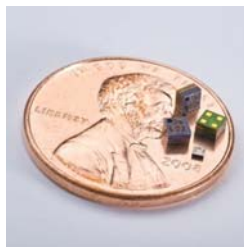
Non-linearity reduced by differential measurement

MEMS Microphone

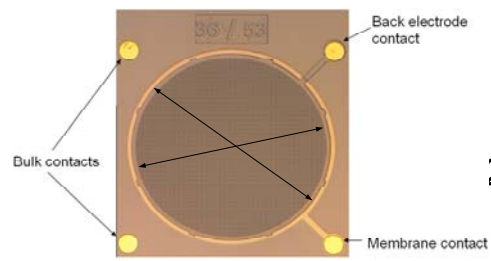
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- Sound waves move membrane \Rightarrow capacitance between the membrane and the fixed back plate varies
- Found in many cell phones



Courtesy: Akustica (website)



Courtesy: R. van Veldhoven, NXP

2um

Cmic

h_{box}=500nm

Capacitive sensor readout

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The charge amplifier (1)

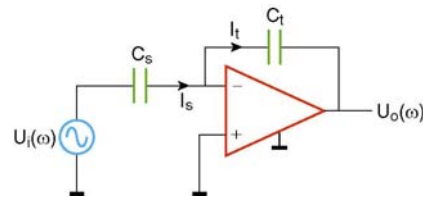
For an "ideal" opamp:

- (a) $U_- = U_+ = 0$
 (b) $I_s(\omega) = j\omega C_s \cdot U_i(\omega)$
 (c2) $I_t(\omega) = -j\omega C_t \cdot U_o(\omega)$, hence:

$$U_o(\omega) / U_i(\omega) = -C_s / C_t$$

(can also be derived from the expression of an inverting amplifier with: $Z_s = 1 / j\omega C_s$ and $Z_t = 1 / j\omega C_t$)

Use C_s for lateral displacement sensing ($U_o \propto A$) and C_t for vertical displacement sensing with linear output ($U_o \propto 1/(1/d)=d$).

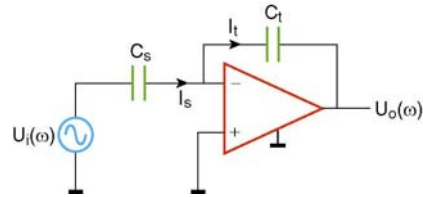


$$C = \epsilon_0 \epsilon_r \frac{A}{h} = \epsilon_0 \frac{A}{h}$$

The charge amplifier (2)

For an "ideal" opamp:

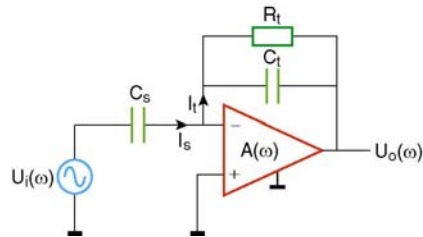
$$U_o(\omega) / U_i(\omega) = -C_s / C_t$$



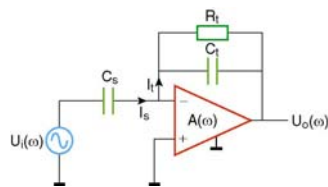
In practice:

1. R_t required to avoid saturation of the output by I_{bias} .

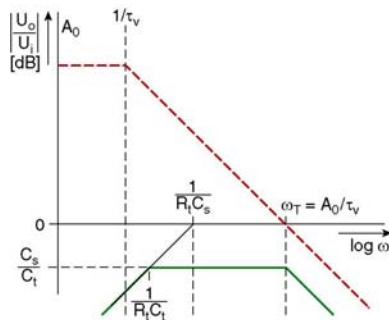
2. Open-loop gain is finite:
 $A(\omega) = A_o / (1 + j\omega\tau_v)$.



Modulus transfer of the charge amplifier



$$\frac{U_o}{U_i} = \frac{-j\omega R_t C_s}{\left(1 + j\omega \frac{\tau_v}{A_o}\right) (1 + j\omega R_t C_t)}$$

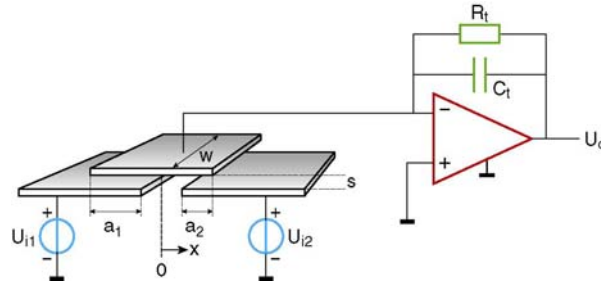


For sensor readout the charge amplifier typically attenuates the excitation voltage U_i ($C_s \ll C_t$).

Features:

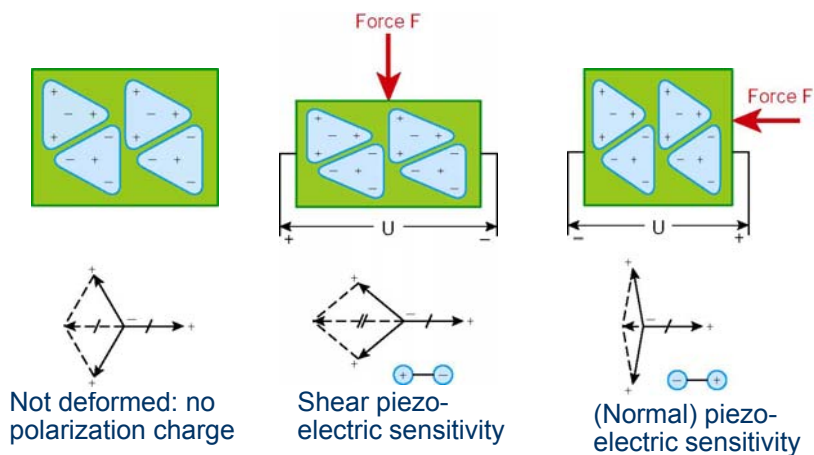
1. Can be used for excitation voltage frequencies up to ω_T .
2. U_i is virtual ground and can be used for adding currents in a differential sensor structure.

Application in differential sensor readout



Topology is often used for the readout of MEMS accelerometers and gyroscopes

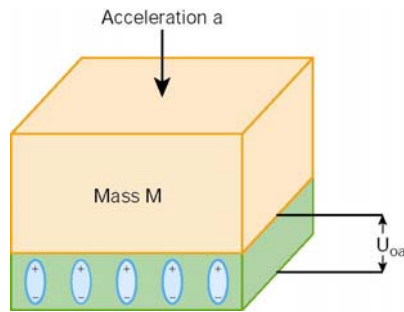
Piezo-electricity



Piezo-electricity is reversible: connecting voltage source results in a deformation and, hence, in force.

Piezo-electricity in all crystal directions: described by a matrix.

Application in acceleration sensing

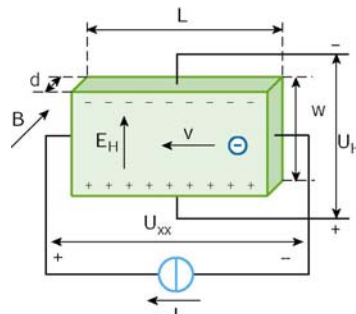


Charge sensitivity: $S_q = 5 \text{ pC/N}$.
Capacitance: $C_s = 20 \text{ pF}$
Mass: $M = 100 \text{ g}$.

U_{0a} @ 1 m/s^2 ?

Force due to load: $F = M \cdot a = 0.1 \text{ N}$
Voltage sensitivity: $S_u = S_q / C_s = 250 \text{ mV/N}$.
Hence: $U_0 = 25 \text{ mV}$

The Hall effect is based on the Lorentz force acting on electrons moving through a (semi)conductor.

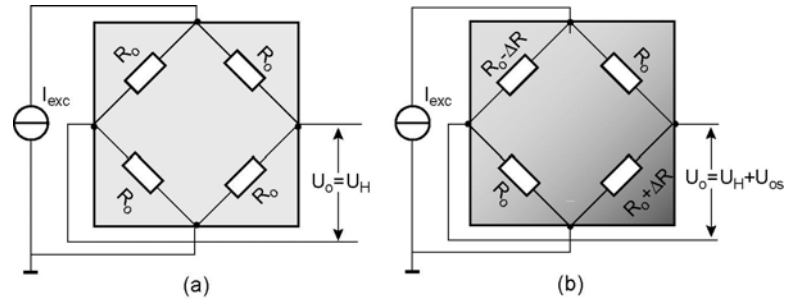


The deflection results in charge accumulation. The electrostatic field due to the charge counteracts the Lorentz force and equilibrium is reached at the Hall field, $E_H = F_L / e = vB$.

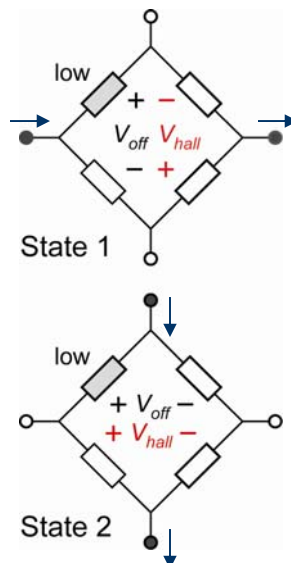
The Hall voltage $U_H = E_H \cdot w$ is the output signal and is proportional to both I and B .

Note that $U_H / I = f(B)$ is like a resistance measured in a 4-terminal resistance measurement.

Stress in the conductive layer results in offset.



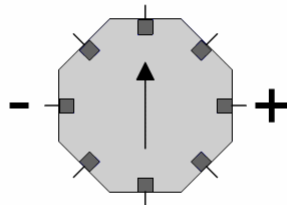
Wheatstone bridge not balanced at $B = 0$ T.



- Changing the **direction** of the bias current changes the **relative** polarity of V_{off} & V_{hall}
- So **averaging** the bridge output cancels V_{off} !

In practice

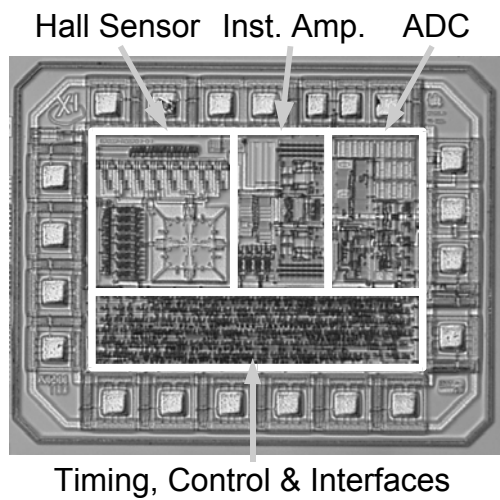
- n-well resistance depends on current direction (anisotropic)
- ⇒ 3 current directions needed for optimal offset cancellation!



- 3 current directions
⇒ Octagonal Hall plate
- Bias current rotated, while Hall voltages are summed
- Cancels offset due to **static** bridge mismatch
⇒ 10 - 100 μ T offset

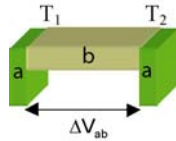
Earth's magnetic field \sim 50 μ T

So electronic compass applications are possible ...



- Standard 0.5 μ m CMOS
- Spinning current technique + low-offset amplifier
⇒ 4 μ T offset
- State-of-the-art!
- J. van der Meer et al., ISSCC '05

The Seebeck effect



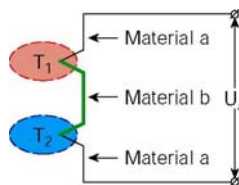
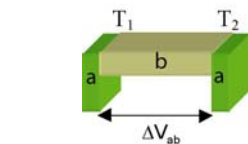
The **Seebeck effect** is due to the excess kinetic energy of free carriers at the hot side of a (semi)-conducting material, which results in a net diffusion of carriers towards the cold side. The resulting charge build-up creates an internal electric field that opposes further diffusion and is externally measurable as an open-circuit potential, ΔV (conventional).

$$\alpha = \left. \frac{\partial V}{\partial T} \right|_T, \text{ where } \alpha \text{ is the temperature coef of the Seebeck voltage at temperature } T \Rightarrow \text{the Seebeck coefficient}$$

Equivalent definition: The Seebeck effect is due to the temperature-dependence of the Fermi level in a material (quantum mechanical).

$$\alpha = \frac{1}{q} \left. \frac{\partial E_F}{\partial T} \right|_T$$

Practical use of the Seebeck effect: a thermocouple with two different materials and a temperature difference



$$U_{ab} = [\alpha_{a,T1} \cdot T_1 - \alpha_{b,T1} \cdot T_1] + [\alpha_{b,T2} \cdot T_2 - \alpha_{a,T2} \cdot T_2] \square$$

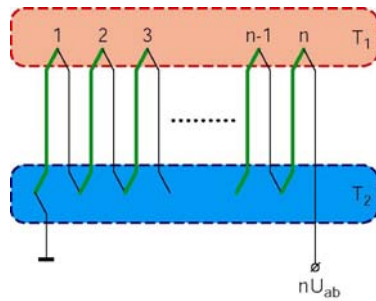
$$(\alpha_a - \alpha_b)(T_1 - T_2) = \alpha_{ab}(T_1 - T_2)$$

MATERIALCOMB.	SENSITIVITY (AT 0°C) [μV/K]	RANGE [°C]
iron/constantan	45	0..760
copper/constantan	35	-100..370
chromel/alumel	40	0..1260
platinum/Pt+ Rd	5	0..1500

No offset, however very small DC voltages (typ. < 1mV) are generated.

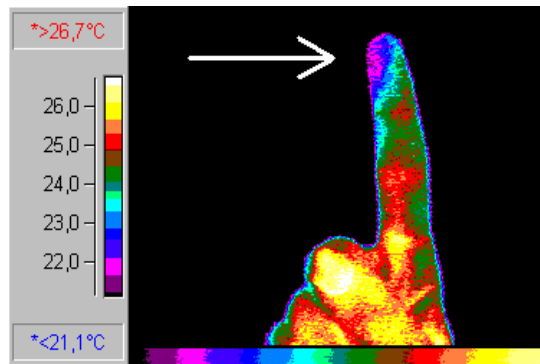
Problem: readout circuit has offset

Thermopile for increased output signal

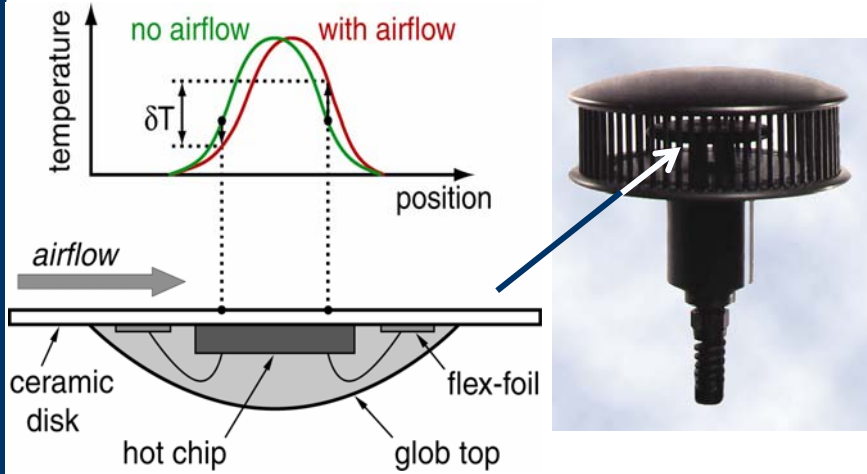


N-times the thermocouple voltage

- Problems:**
- increased **electrical** resistance (noise)
 - increased **thermal** conduction between cold and hot parts \Rightarrow
 1. Increased source loading or
 2. More heat flux required to build up a temperature difference.



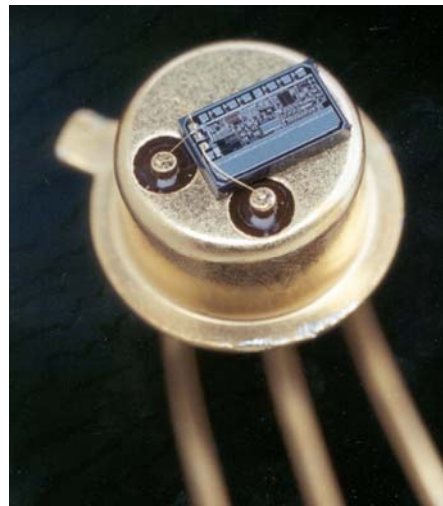
Convective cooling \Rightarrow temperature gradient
 \Rightarrow wind speed and direction



On-chip thermopiles measure temperature gradient

K. Makinwa et al., ISSCC '02

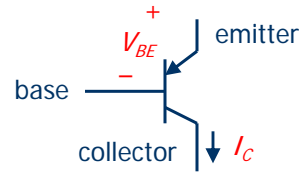
- Transistors are natural temperature sensors
- But manufacturing tolerances cause errors of up to 3°C



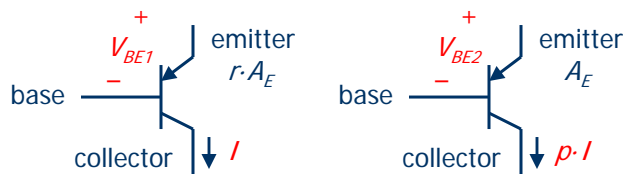
- For $I_C \gg I_S$

$$I_C \approx I_S \exp\left(\frac{qV_{BE}}{kT}\right)$$

$$\Rightarrow V_{BE} = \frac{kT}{q} \ln \frac{I_C}{I_S}$$



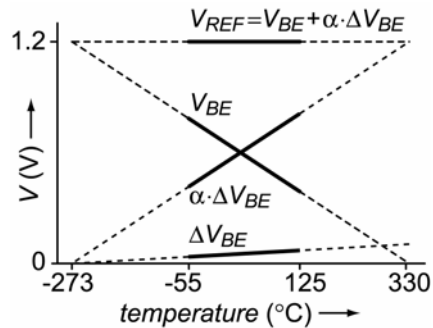
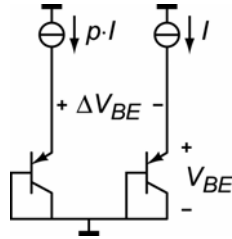
- V_{BE} is a near-linear function of temperature
- With a **negative** temperature coefficient: $\sim -2\text{mV}/^\circ\text{C}$
- But it is **process dependent** (via I_S and I_C)



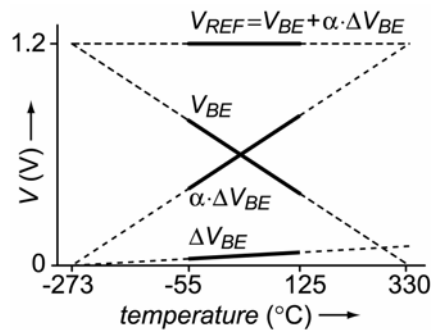
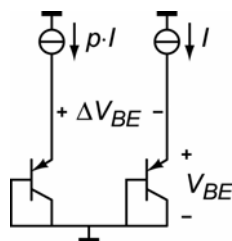
- By biasing two transistors at a fixed collector current ratio p , we can eliminate the process dependence:

$$\Delta V_{BE} = V_{BE2} - V_{BE1} = \frac{kT}{q} \ln(p \cdot r)$$

- ΔV_{BE} is a linear function of temperature
- With a **positive** tempco: $\sim 180\mu\text{V}/^\circ\text{C}$ (for $p \cdot r = 10$)



- Two bipolar transistors can generate:
 - ΔV_{BE} proportional to absolute temperature (PTAT)
 - V_{BE} complementary to absolute temperature (CTAT)

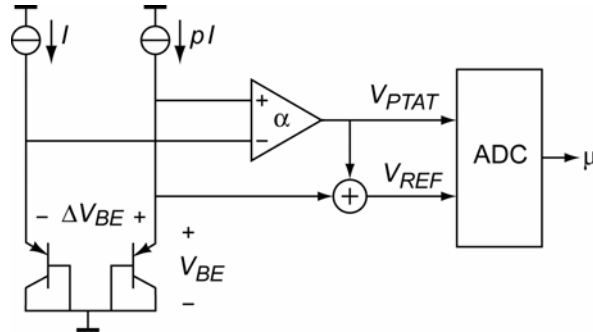


- These voltages can be combined to give:
 - V_{REF} temperature independent (band-gap) voltage
 - $\alpha \cdot \Delta V_{BE}$ temperature dependent voltage
- Their ratio is a measure of temp:

$$\mu = \frac{\alpha \cdot \Delta V_{BE}}{V_{BE} + \alpha \cdot \Delta V_{BE}}$$

A practical Bandgap temperature sensor

ET8.017
El. Instr.



Current mirror ratio p , emitter area ratio r and amplifier gain α generate

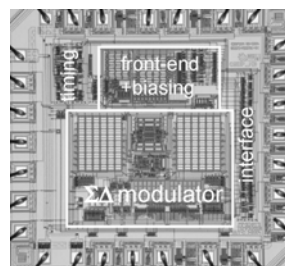
$$V_{PTAT} = \alpha \frac{kT}{q} \ln(p \cdot r)$$

Current source I_{bias} generates V_{BE}
ADC then computes the ratio:

$$\mu = \frac{\alpha \cdot \Delta V_{BE}}{V_{BE} + \alpha \cdot \Delta V_{BE}} = \frac{V_{PTAT}}{V_{REF}}$$

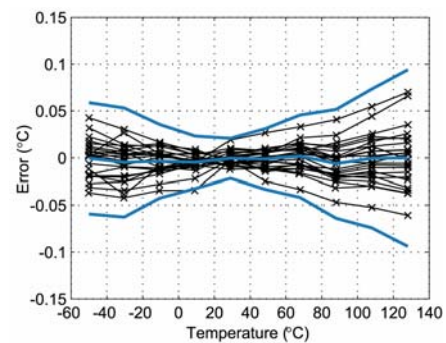
State-of-the-Art

ET8.017
El. Instr.



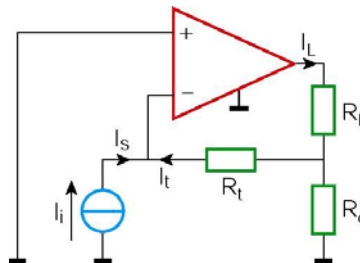
Pertijs et al., JSSC, Dec. '05

- CMOS \Rightarrow substrate PNPs
- I_S spread? \Rightarrow one room temperature trim
- Inaccuracy $< \pm 0.1^\circ\text{C}$ (3σ) from -55 to 125°C



$$\mu = \frac{\alpha \cdot \Delta V_{BE}}{V_{BE} + \alpha \cdot \Delta V_{BE}}$$

- The accuracy of a bandgap temp sensor will be limited by the offset and gain error of the “ α ” amplifier
- Assuming $p=1$, $r=16$ and $\alpha = 8$, what offset corresponds to 0.1°C error?
- Similarly what gain error corresponds to an error of less than 0.1°C over the military range -55°C to 125°C ?
- Assume that the tempco of $V_{BE} = -2\text{mV}/^\circ\text{C}$ and that ideally, $V_{REF} = 1.2\text{V}$
- Please submit this before the class on Wed. 19 Sept.



- Derive an expression for the transfer function I_L/I_i of the circuit shown above.
- Assuming the opamp suffers from offset and bias current, calculate the corresponding detection limit
- Please submit this before the class on Wed. 19 Sept.

