

Electronic Instrumentation

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Outline

Monday Sept. 24: Deterministic and Random Errors
 Common-mode Rejection Ratio
 Power supply Rejection Ratio
 Gain error

Two kinds of errors (uncertainty):

- a. **Deterministic errors**
e.g. source loading, offset, gain error
- b. Random (stochastic) errors
e.g. thermal noise

Assuming that the output of a system $z = f(a, b, c, \dots)$

Then the total error due to deterministic parameter uncertainty is:

$$\Delta z = \left| \frac{\partial f(a, b, c, \dots)}{\partial a} \right| \Delta a + \left| \frac{\partial f(a, b, c, \dots)}{\partial b} \right| \Delta b + \left| \frac{\partial f(a, b, c, \dots)}{\partial c} \right| \Delta c + \dots$$

where Δz denotes the uncertainty in z due to parameter a , which is specified with uncertainty Δa etc..

The derivatives above are called sensitivities.

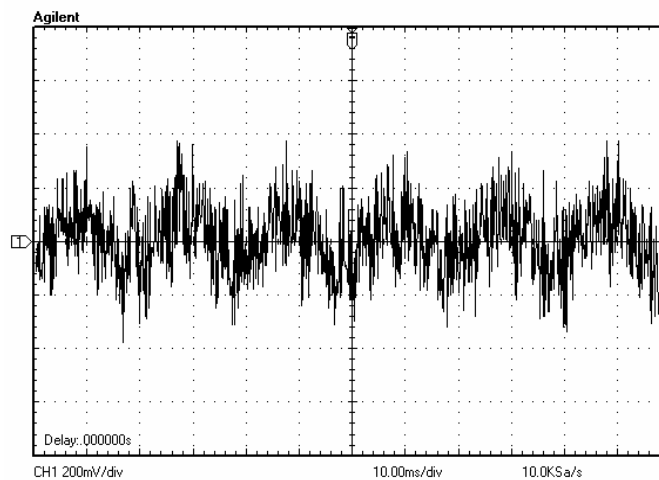
Example: Combining additive errors:

For $z = a + b$ or $z = a - b$ with Δa the uncertainty in a and Δb the uncertainty in b , the overall uncertainty follows as:

$$\Delta z = \left| \frac{\partial(a \pm b)}{\partial a} \right| \Delta a + \left| \frac{\partial(a \pm b)}{\partial b} \right| \Delta b = \Delta a + \Delta b$$

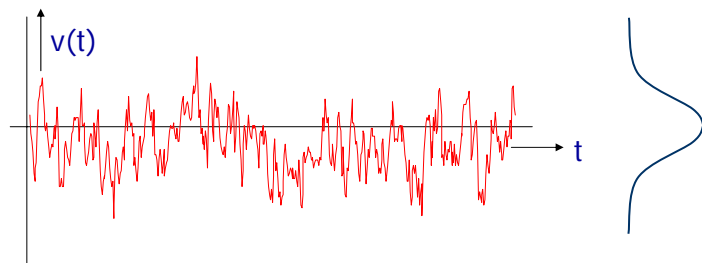
By extension, the effect of deterministic errors (e.g. offset, gain error etc.) in a linear time-invariant (LTI) system can be determined by superposition.

- *Interference* is caused by *external* signals e.g. 50 Hz hum
- *Noise* is caused by stochastic processes in the measurement system itself e.g. thermal noise.
- Noise and interference are usually additive error signals
⇒ they can both be modeled by extra error sources to the system
- Note that interference and noise should not be confused with signal *distortion*, which is caused non-linearities in either the amplitude or the frequency domain.



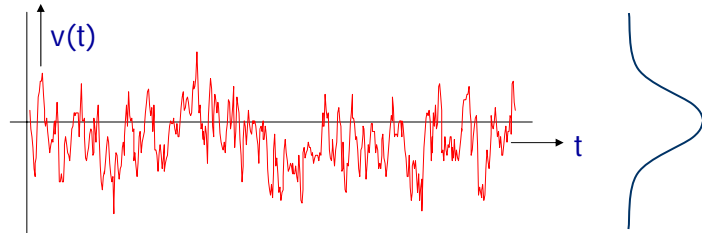
- ALL resistors exhibit thermal noise
- Thermal noise has a uniform (white) frequency spectrum
- noise power density: $P_n = 4kT$ (W/Hz)
 - where k is Boltzmann's constant and T is absolute temperature
- At room temperature (293 K): $P_n = 4kT = 1.62 \times 10^{-20}$ (W/Hz)
- For a resistor: $P = V_{\text{rms}}^2/R = I_{\text{rms}}^2 R$
- => noise voltage with spectral density: $v_n = \sqrt{4kTR}$ (V/Hz^{1/2})
- => noise current with spectral density: $i_n = \sqrt{4kT/R}$ (A/Hz^{1/2})

- (Thermal) noise is stochastic, i.e. its amplitude at a given time t cannot be predicted
- Thermal noise has a Gaussian amplitude distribution



$$p(V)dV = \frac{1}{V_n \sqrt{2\pi}} e^{-\left(\frac{V^2}{2V_n^2}\right)} dV$$

- In the presence of thermal noise, the detection limit is usually specified as the standard deviation (or sigma) V_n
- A good estimate of its peak value (e.g. as seen on a wide-band scope) is 5-sigma



$$p(V)dV = \frac{1}{V_n \sqrt{2\pi}} e^{-\left(\frac{V^2}{2V_n^2}\right)} dV$$

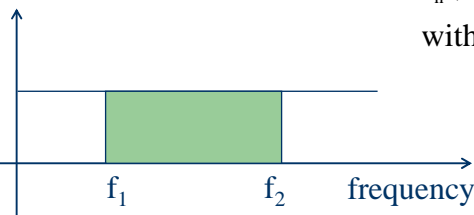
- Shot noise is the result of the fact that an electrical current is the result of the motion of discrete charge carriers.
- This can be compared with hail on a roof: even if the average current (mass flow) is constant, the amount of charge carriers (ice lumps), measured in different intervals with the same length Δt , will, in general, be different. The relative fluctuation becomes greater as Δt becomes smaller and smaller.
- Like thermal noise, shot noise has a uniform frequency spectrum and so is also a form of white noise.
- If a current I consists of charge carriers with charge q that move independently of each other, then the spectral noise current is given by $i_n = \sqrt{2qI}$ (A/Hz^{1/2}).
- This equation applies to p-n junctions (diodes, transistors, etc.) but not to metallic conductors where there is more long-range correlation between the movement of the charge carriers. In such conductors, the shot noise will thus be smaller!

i_n is proportional to the square-root of current!

$$i_n = \sqrt{2qI} \quad (\text{A/Hz}^{1/2})$$

$$I_n(\text{rms}) = i_n \sqrt{B} = \sqrt{2qI_{\text{DC}} B} \quad (\text{A})$$

with bandwidth $B = f_2 - f_1$



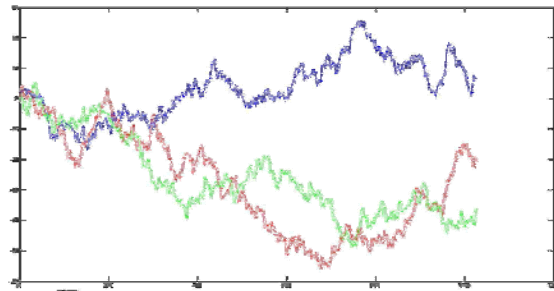
Example (for $B = 10 \text{ Hz}$):

- $I = 1 \text{ A} \Rightarrow$ noise current $I_n(\text{rms}) = 57 \text{ nA}$ (0.000006%)
- $I = 1 \text{ pA} \Rightarrow$ noise current $I_n(\text{rms}) = 56 \text{ fA}$ (5.6%)
- Shot noise is important at low current levels

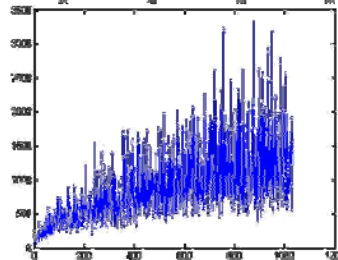
- Both thermal and shot noise are the result of fundamental physical processes and so form a fundamental detection limit.
- Apart from these types of noise, “real” electronic components also suffer from several sources of “excess noise.”
- Resistors exhibit current-dependent 1/f noise: $V_n(\text{rms}) = c \cdot I$ where the constant c is determined by resistor quality (price).
- Transistors (especially MOSFETs) exhibit area-dependent 1/f noise: where $V_n(\text{rms}) \propto 1/\sqrt{\text{Area}}$
- 1/f noise has a 1/f POWER spectrum, i.e. the spectral noise power per decade (or per octave) is constant.
- 1/f noise is sometimes referred to as “pink” or “flicker” noise

1/f noise example

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3 instances
of 1/f noise



Peak amplitude increases for as
measurement time increases?!

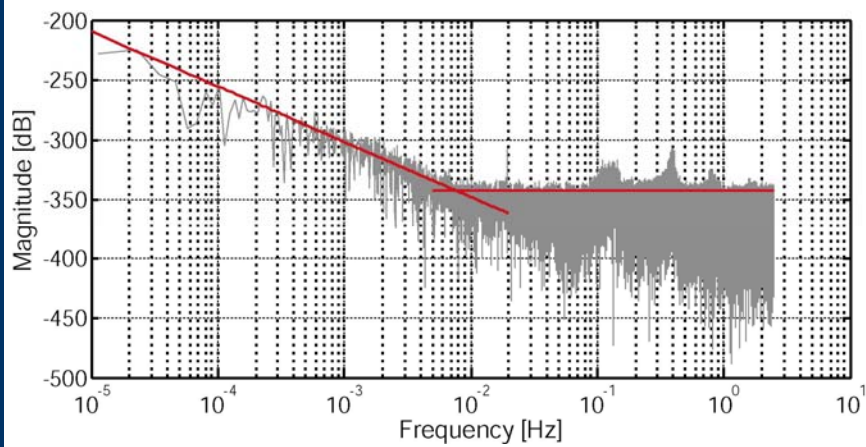
$$x \sim t$$

More and more important for long
(low-frequency) measurements!

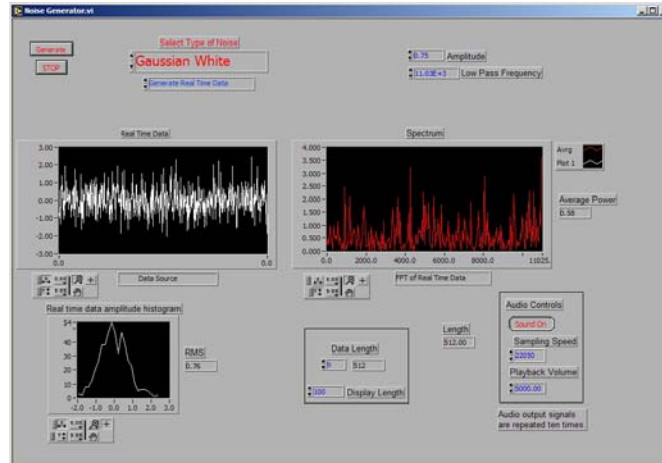
Real noise spectrum

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El. Instr.

Spectrum of a critically damped micro-mechanical structure



Two spectral regimes: a. 1/f noise and
b. frequency independent white noise.



<http://socrates.berkeley.edu/~phylabs/bsc/Supplementary/NoiseGenerator.html>

- Noise is also present in non-electrical domains
- Thermal noise \Rightarrow some kind of energy dissipation

- In the mechanical domain this is due to damping
- In very small structures (MEMS), Brownian noise is also important
- In the thermal domain this is due to thermal resistance

- Shot noise \Rightarrow small amount of energy-carrying particles
- In the optical domain, photon shot noise is significant

Errors (uncertainties) due to:

- a. Noise,
- b. Electro-Magnetic Interference (EMI)

are referred to as **Stochastic (random) sources of error (uncertainty)**.

The effect of stochastic error(s) on overall system specifications can be evaluated using Gauss law of error propagation:

$$\underline{z} = f(a, b, c, \dots)$$

$$\sigma_z^2 = \left(\frac{\partial f(a, b, c, \dots)}{\partial a} \right)_{a, b, c, \dots}^2 \cdot \sigma_a^2 + \left(\frac{\partial f(a, b, c, \dots)}{\partial b} \right)_{a, b, c, \dots}^2 \cdot \sigma_b^2 + \left(\frac{\partial f(a, b, c, \dots)}{\partial c} \right)_{a, b, c, \dots}^2 \cdot \sigma_c^2 + \dots$$

where σ_z^2 denotes the variance (uncertainty) in parameter z with average value \underline{z} due to parameter a , which is specified with uncertainty σ_a etc..

Combining additive errors:

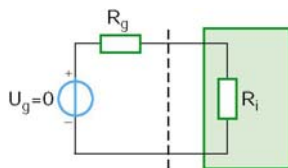
For $z = a + b$ or $z = a - b$ the overall uncertainty follows as:

$$\underline{z} = a \pm b$$

$$\sigma_z^2 = \left(\frac{\partial(a \pm b)}{\partial a} \right)_{a, b, c, \dots}^2 \cdot \sigma_a^2 + \left(\frac{\partial(a \pm b)}{\partial b} \right)_{a, b, c, \dots}^2 \cdot \sigma_b^2 = \sigma_a^2 + \sigma_b^2$$

Hence, the **variances** of stochastic errors in a linear system can be linearly added.

In case of distributed noise voltages or currents:



Combining additive errors:

For $z = a + b$ or $z = a - b$, the overall uncertainty follows as:

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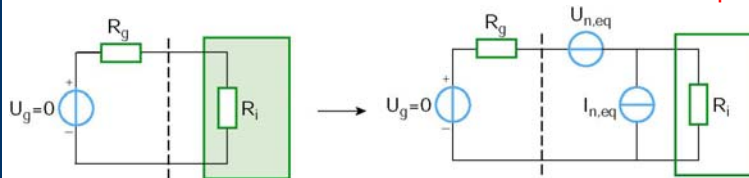
Hence, the **variances** of stochastic errors in a linear system can be linearly added.

In case of distributed noise voltages or currents:

$$u_{n,eq} = \sqrt{u_{n1}^2 + u_{n2}^2}$$

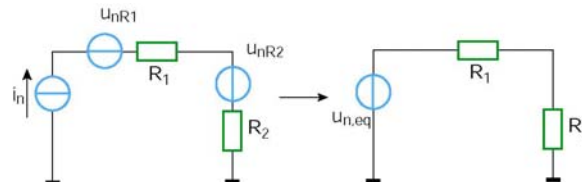
$$i_{n,eq} = \sqrt{i_{n1}^2 + i_{n2}^2}$$

Provided these are independent !



Noise paradox:

What is the equivalent noise voltage in a circuit with two resistors connected to a noise current source ?



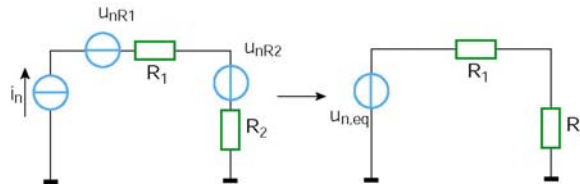
Adding the thermal noise powers of R_1 and R_2 to the noise power due to i_n flowing through R_1 and i_n flowing through R_2 :

$$(a) u_{n1,eq}^2 = u_{n,R1}^2 + u_{n,R2}^2 + i_n^2 R_1^2 + i_n^2 R_2^2 =$$

$$u_{n,R1}^2 + u_{n,R2}^2 + i_n^2 (R_1^2 + R_2^2)$$

Noise paradox:

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Adding the thermal noise powers of R_1 and R_2 to the noise power due to i_n flowing through R_1 and i_n flowing through R_2 :

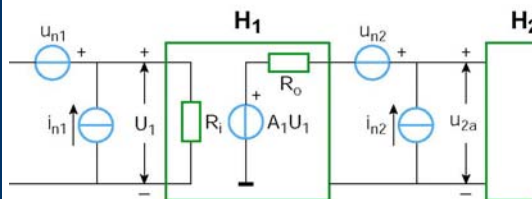
$$(a) \overline{u_{n1,eq}^2} = \overline{u_{n,R1}^2} + \overline{u_{n,R2}^2} + \overline{i_n^2 R_1^2} + \overline{i_n^2 R_2^2} =$$

$$\times \quad \overline{u_{n,R1}^2} + \overline{u_{n,R2}^2} + \overline{i_n^2 (R_1^2 + R_2^2)}$$

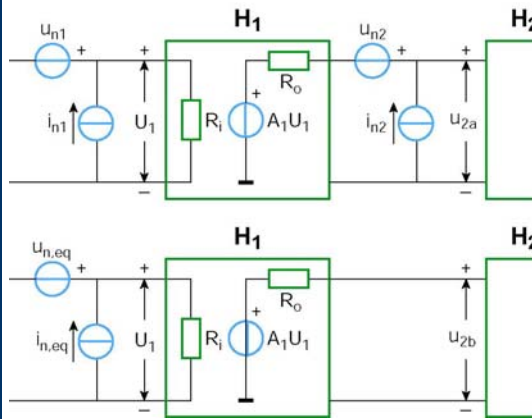
Adding the thermal noise powers of R_1 and R_2 to the noise power due to i_n flowing through R_1 plus R_2 :

$$\checkmark \quad (b) \overline{u_{n2,eq}^2} = \overline{u_{n,R1}^2} + \overline{u_{n,R2}^2} + \overline{i_n^2 (R_1 + R_2)^2}$$

General approach for finding equivalent input noise sources:



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Note: equivalent noise is independent of source impedance!

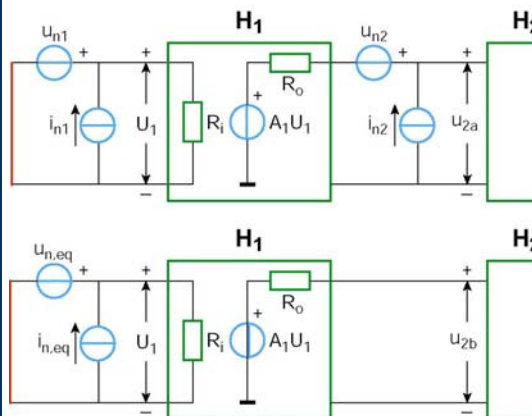
Extreme case 1:
open input.

$$\left. \begin{aligned} u_{2a}^2 &= (i_{n1} R_i A_1)^2 + (i_{n2} R_o)^2 + u_{n2}^2 \\ u_{2b}^2 &= (i_{n,eq} R_i A_1)^2 \\ u_{2a}^2 &= u_{2b}^2 \end{aligned} \right\}$$

Hence:

$$i_{n,eq}^2 = i_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2 R_i^2}$$

General approach for finding equivalent input noise sources:



Note: equivalent noise is independent of source impedance!

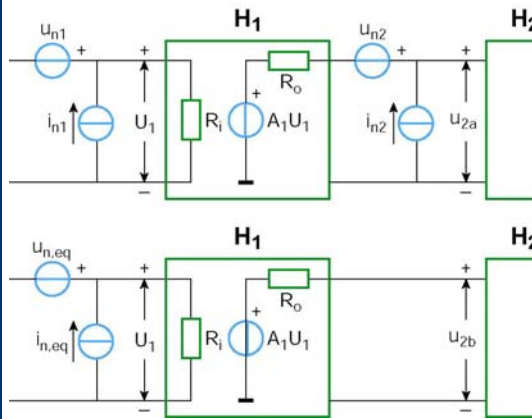
Extreme case 2:
short-circuited input.

$$\left. \begin{aligned} u_{2a}^2 &= (u_{n1} A_1)^2 + (i_{n2} R_o)^2 + u_{n2}^2 \\ u_{2b}^2 &= (u_{n,eq} A_1)^2 \\ u_{2a}^2 &= u_{2b}^2 \end{aligned} \right\}$$

Hence:

$$u_{n,eq}^2 = u_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2}$$

General approach for finding equivalent input noise sources:

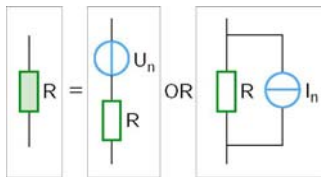


$$u_{n,eq}^2 = u_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2}$$

$$i_{n,eq}^2 = i_{n1}^2 + \frac{u_{n2}^2 + i_{n2}^2 R_o^2}{A_1^2 R_i^2}$$

Recommendations for low-noise performance:
Focus design effort on first stage: low noise and high gain, A_1 .

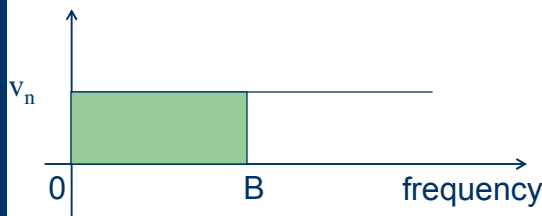
Noise in components is usually specified in terms of the **noise spectral power, s_n** . In case of a resistor:



$$s_{nu,Z} = 4k_B T |Z| = 4k_B T R \text{ [V}^2/\text{Hz]}$$

$$s_{ni,Z} = \frac{4k_B T}{|Z|} = \frac{4k_B T}{R} \text{ [A}^2/\text{Hz]}$$

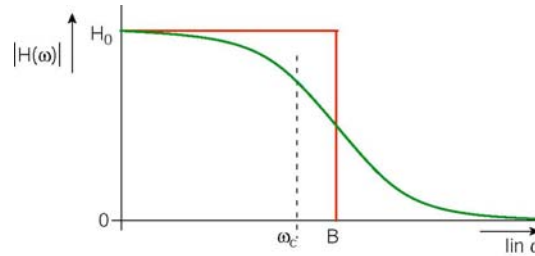
Noise bandwidth is the bandwidth of a theoretical low-pass filter with an abrupt cut-off frequency at B . A practical low-pass filter will have a more gradual transition from pass-band to stop-band, $H(\omega)$.



At 20°C and into a 10kHz bandwidth, a 10kΩ resistor generates 1.3 μVrms.

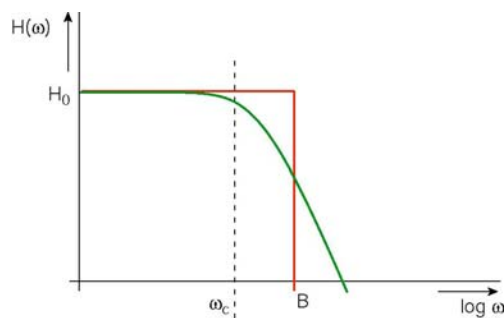
is defined as the bandwidth of a “brick wall” filter with an abrupt cut-off frequency at B. A practical system is specified by a low-pass filter with a more gradual transition from pass-band to stop-band, $H(\omega)$.

The **Noise bandwidth** B of a practical low-pass filter can be obtained from its transfer function, $H(\omega)$ as follows:



$$P_n = |H_o|^2 B = \int_0^\infty |H(\omega)|^2 d\omega \rightarrow B = \frac{1}{|H_o|^2} \int_0^\infty |H(\omega)|^2 d\omega$$

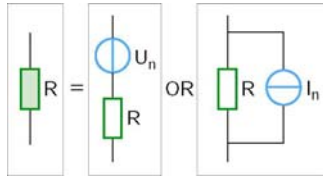
$$P_n = |H_o|^2 B = \int_0^\infty |H(\omega)|^2 d\omega \rightarrow B = \frac{1}{|H_o|^2} \int_0^\infty |H(\omega)|^2 d\omega$$



$$\text{For } H_o=1: B = \int_0^\infty \frac{d\omega}{1+(\omega\tau)^2} = \frac{1}{\tau} \text{arctg}(\omega\tau) \Big|_0^\infty = \frac{\pi}{2\tau} = \frac{\pi}{2} \omega_c \text{ [rad / s]},$$

For noise calculations, a “correction factor” $\pi/2$ is required

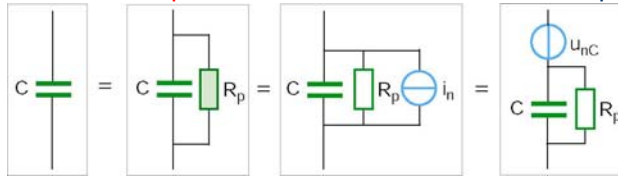
Noise in impedance, Z , is primarily due to the dissipating (resistive) part, R :



$$s_{nu,z} = 4k_B T |Z| = 4k_B TR \text{ [V}^2/\text{Hz]}$$

$$s_{ni,z} = \frac{4k_B T}{|Z|} = \frac{4k_B T}{R} \text{ [A}^2/\text{Hz]}$$

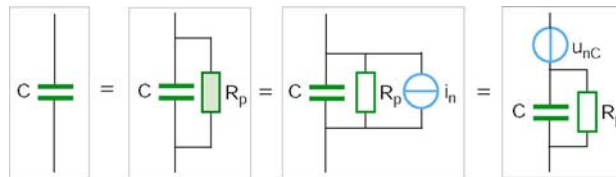
Reactive components are not free of noise due to parasitics:



$$\overline{u_{nC}^2} = \int_0^\infty i_n^2 |Z(\omega)|^2 d\omega = i_n^2 \int_0^\infty R_p^2 \frac{d\omega}{1 + (\omega R_p C)^2} = i_n^2 R_p^2 \frac{2\pi}{4R_p C} = \frac{4k_B T}{2\pi R_p} R_p^2 \frac{\pi}{2R_p C} = \frac{k_B T}{C}$$

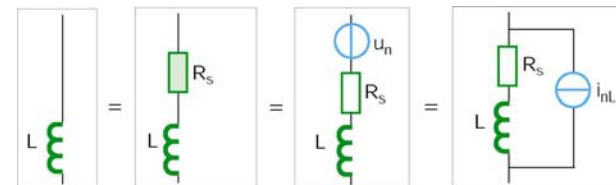
Reactive components are not free of noise due to parasitics:

In a capacitor:



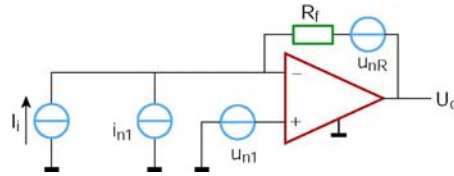
$$\overline{u_{nC}^2} = \int_0^\infty i_n^2 |Z(\omega)|^2 d\omega = i_n^2 \int_0^\infty R_p^2 \frac{d\omega}{1 + (\omega R_p C)^2} = i_n^2 R_p^2 \frac{2\pi}{4R_p C} = \frac{4k_B T}{2\pi R_p} R_p^2 \frac{\pi}{2R_p C} = \frac{k_B T}{C}$$

Similarly,
in an inductor:



$$\overline{i_{nL}^2} = \int_0^\infty \frac{u_n^2}{|Z(\omega)|^2} d\omega = u_n^2 \int_0^\infty \frac{d\omega}{R_s^2 + (\omega L)^2} = \frac{4k_B TR_s}{2\pi} \times \frac{1}{R_s^2} \int_0^\infty \frac{d\omega}{1 + \left(\frac{\omega L}{R_s}\right)^2} = \frac{k_B T}{L}$$

Trans-impedance amplifier:



$$U_o/I_i = (-)R_f$$

Ideal current source, thus $i_{n,eq}$ only (open-input)

$$\overline{u_{o1}^2} = \overline{u_{n1}^2} + \overline{i_{n1}^2 R_f^2} + \overline{u_{nR}^2} = \overline{u_{o2}^2} = \overline{i_{n,eq}^2 R_f^2} \quad \text{Hence,} \quad \overline{i_{n,eq}^2} = \frac{\overline{u_{n1}^2}}{R_f^2} + \overline{i_{n1}^2} + \frac{\overline{u_{nR}^2}}{R_f^2}$$

For:
 $s_{ni,1} = 1 \text{ pA}/\sqrt{\text{Hz}}$,
 $R_f = 10 \text{ k}\Omega$ and
 $4k_B T = 1.65 \cdot 10^{-20} \text{ J}$
 @300 K

$$\overline{i_{n,eq}^2} = 10^{-24} + \left(\frac{4 \cdot 10^{-9}}{10^4}\right)^2 + \frac{1.65 \cdot 10^{-20}}{10^4} = 2.8 \cdot 10^{-24} \text{ [A}^2/\text{Hz]}$$

$$\sqrt{\overline{i_{n,eq}^2}} = 1.67 \text{ pA}/\sqrt{\text{Hz}}$$

SNR is defined as the ratio between signal power and noise power expressed in dB:

$$\frac{S}{N} = 10 \log \left(\frac{P_S}{P_N} \right) = 10 \log \left(\frac{V_S(\text{rms})}{V_N(\text{rms})} \right)^2 = 20 \log \left(\frac{V_S(\text{rms})}{V_N(\text{rms})} \right)$$

Notes:

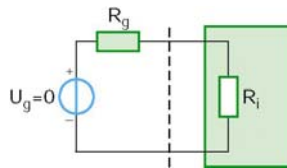
- Since the form of the signal and that of the noise may differ significantly, SNR should not be calculated from signal amplitudes but from rms values.
- A given SNR is always associated with a certain bandwidth.
- For maximum SNR, the bandwidth of a measurement system should be about the same as the expected signal bandwidth

Best noise performance implies that readout adds no noise to the noise power already included in the source.

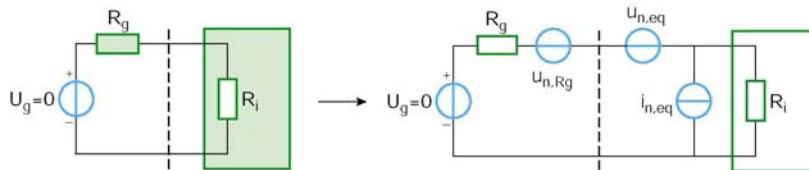
The noise added by the readout is specified in the **Noise factor**, F:

$$F = \frac{SNR_{input}}{SNR_{output}} = \frac{\frac{P_s(input)}{P_n(source)}}{\frac{P_s(output)}{H(\omega)^2 [P_n(source + readout)]}} = \frac{P_n(source) + P_n(readout)}{P_n(source)} = 1 + \frac{P_n(readout)}{P_n(source)}$$

How can we optimize the NF of a practical system?



Noise matching in a practical system:



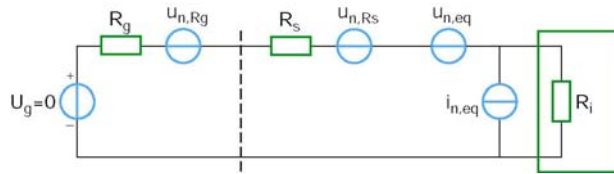
$$NF = \frac{SNR_{input}}{SNR_{output}} = 1 + \frac{P_n(readout)}{P_n(source)} = 1 + \frac{\overline{u_n^2} + \overline{i_n^2} R_g^2}{4k_B T R_g}$$

$$\text{Minimum at: } \frac{\partial NF}{\partial R_g} = \frac{4k_B T R_g \times 2\overline{i_n^2} R_g - 4k_B T (\overline{u_n^2} + \overline{i_n^2} R_g^2)}{(4k_B T R_g)^2} = 0 \rightarrow R_{g,opt}^2 = \frac{\overline{u_n^2}}{\overline{i_n^2}}$$

$$\text{Resulting in a minimum noise factor: } NF_{min} = 1 + \frac{2\overline{u_n^2}}{4k_B T R_g}$$

Minimum NF indicates best **redistribution of noise** over $u_{n,eq}$ and $i_{n,eq}$ at the minimum noise power, $P_n(readout)$, derived before.

Noise matching using a series resistance?

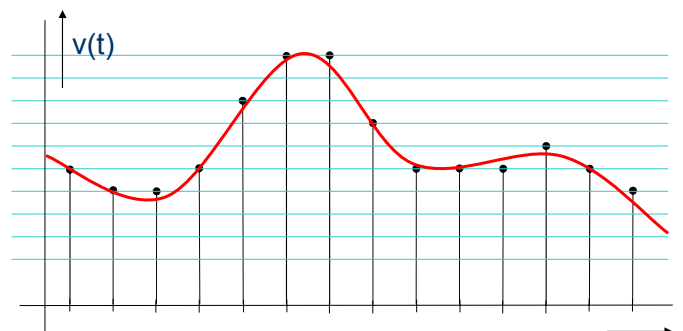


$$P_n(\text{source} + \text{readout}) = 4k_b T R_g + 4k_b T R_s + \overline{u_n^2} + \overline{i_n^2} (R_g + R_s)^2 \rightarrow$$

$$NF = \frac{4k_b T (R_g + R_s) + \overline{u_n^2} + \overline{i_n^2} (R_g + R_s)^2}{4k_b T R_g} = 1 + \frac{R_s}{R_g} + \frac{\overline{u_n^2} + \overline{i_n^2} (R_g + R_s)^2}{4k_b T R_g}$$

- Noise matching at higher noise level.
- Attenuation of U_g

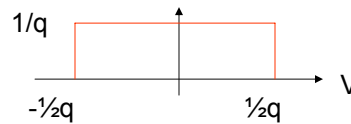
Noise matching using series resistance is counter productive



Signal is quantized in both amplitude and in time

- Range: $0 - V_{ref}$
- Number of bits: n
- Number of levels: 2^n
- “step-size”: $q = V_{ref} \cdot 2^{-n}$
- error: $-\frac{1}{2}q < V < \frac{1}{2}q$
- For a busy signal (one that jumps between many quantization levels), this error can be modeled as random noise with a uniform pdf
- The associated standard deviation e is then given by:

Probability distr. function $p(V)$



$$e = \sqrt{\frac{1}{q} \int_{-q/2}^{q/2} V^2 dV} = \frac{1}{\sqrt{12}} q$$

Interference source

Interference sensitive system

Coupling mechanism

electrostatic discharge

conduction

integrated circuit

transformer (50 Hz)

magnetic field

audio system

Switched-mode PSU

electric field

high-impedance sensor

HF digital circuit

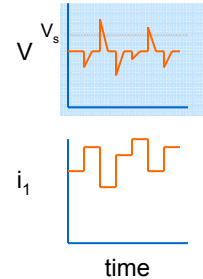
electromagnetic field

FM radio

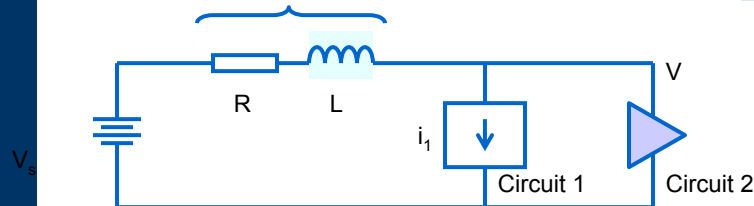
- Coupling via a common impedance
- E.g. via a common supply line or a ground connection

Circuit 1 : HF digital circuit

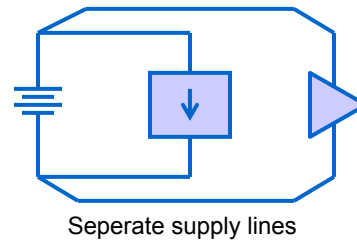
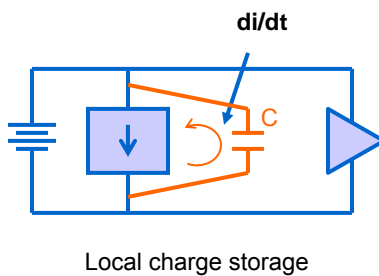
⇒ i_1 is a rapidly varying current (high di/dt)



$$\Delta V = R i_1 + L di_1/dt$$

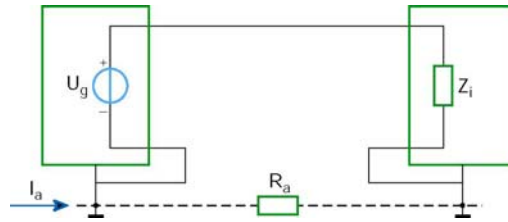


- Quick fix: use a bypass capacitor near a HF circuit to locally deliver its rapidly changing supply currents (this can be seen as a form of filtering)
- Good design: avoid shared impedances



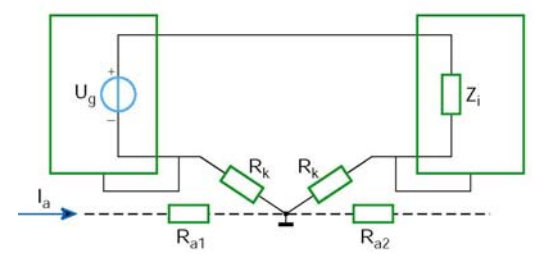
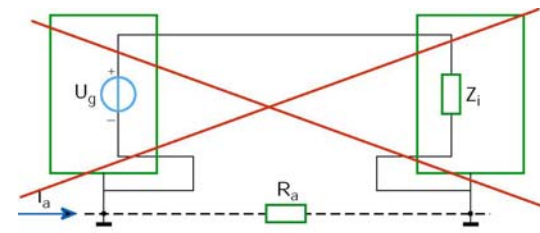
Ground loops

Additive error due to $I_a R_a$ in series with u_g .

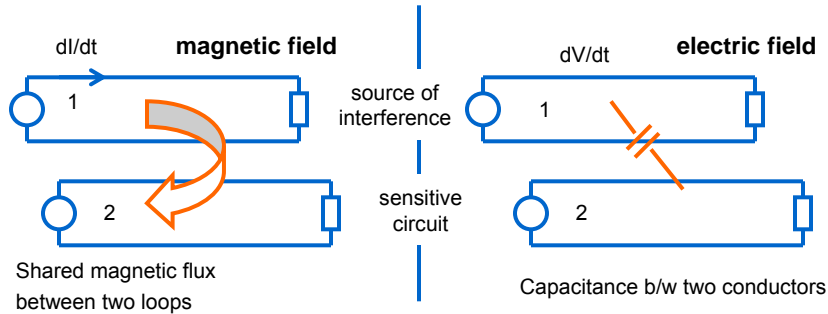


Ground loops

“star” connection to limit error due to $I_a R_a$.

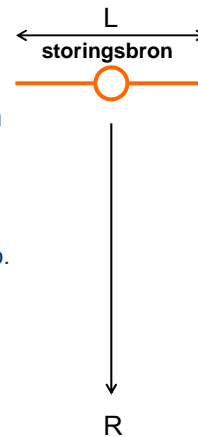


Electric (capacitive) coupling is due to changing voltages, i.e. dV/dt
Magnetic (inductive) coupling is due to changing currents, i.e. dI/dt



- In the same circuit there can be multiple coupling mechanisms
 - Depends on source and load-impedances, size and geometry

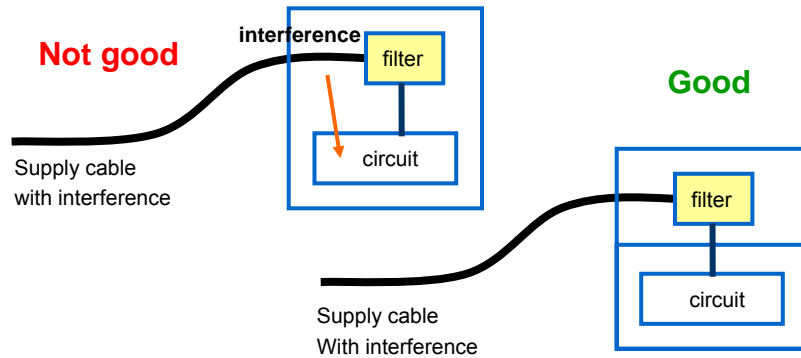
- When an “antenna” is driven by a source, it creates
 - an electromagnetic field
 - an electric field
 - a magnetic field
- In a given situation, the dominant field depends on the shape and size of the antenna, the frequency and the distance to the antenna
- Field strengths vary with:
 - $1/R^2$ and $1/R^3$ voor electric and magnetic fields, resp. $1/R$ for EM-waves (R = distance to antenna)
 - $R < \lambda$: E and M fields can dominate (near field)
 - $R > \lambda$: EM waves can dominate (far field)
- The following applies to EM waves:
 - $L = \lambda/2$: good antenna (lots of EM interference)
 - $L < \lambda/20$: bad antenna (less problems)



Interference reduction: Filtering

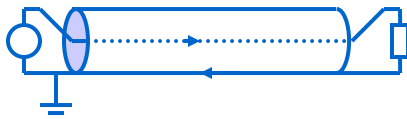
ET8.017
El. Instr.

- Filtering can help suppress interference that is coupled into a circuit via its supply lines
- Filtering can also be used on signal lines if the interference does not occur at the same frequencies as the wanted signal

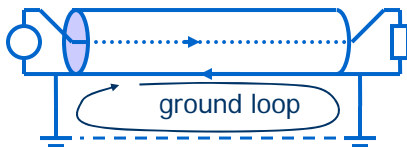


Shielding: Coax cable

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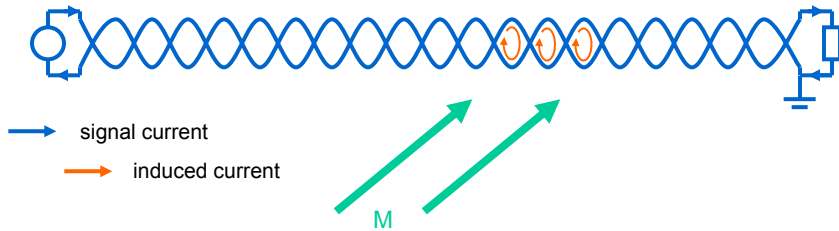
If the outer conductor of a coax cable (on 1 side) is grounded, electric field cannot reach the inner conductor. The outer conductor also forms the return path *and* the signal's reference.



If the outer is grounded at both sides (often the case) a ground loop will be created. This can lead to interference because of the voltage differences between the two grounds and/or magnetic coupling via the loop.

Twisted pair

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El. Instr.



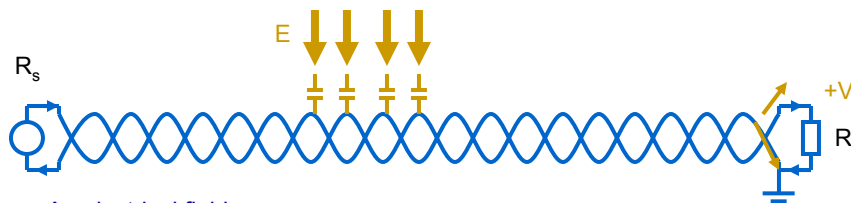
- Magnetic field induces current in coils
 - coils are “twisted”
 - currents in neighboring loops have opposite signs
 - currents cancel each other
 - *net induced current is zero*

	Attn [dB]
untwisted pair (ref)	0
twisted pair	43
untwisted pair + alu foil shield	3
untwisted pair + steel shield	32

50 Hz **Magnetic** shielding

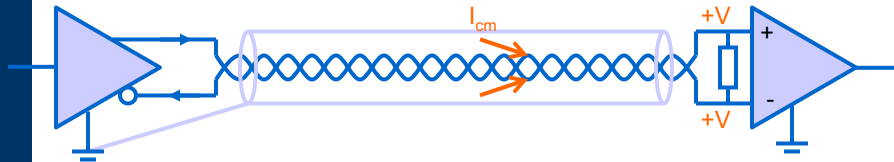
Twisted pair (2)

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- An electrical field
 - Couples capacitively to both conductors
 - Generates equal currents in both signal and return lines
 - Signal line: current flows through R_s/R_L generates +V
 - Return line: current flows to ground (low R)
- Result: electric fields are not shielded!
- Solutions:
 - Add capacitive shielding (e.g. a conducting outer layer)
 - Balance the circuit

$$(+V) - (+V) = 0$$



Balancing:

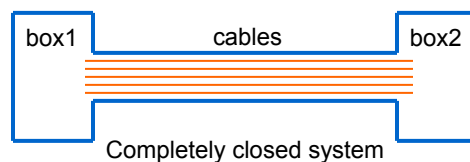
- Use differential inputs and outputs
- Ensure that capacitively coupled currents in both lines are the same (i.e. that they are common-mode signals) and that they “see” the same impedance
- The current-induced voltages will then cancel each other out
- Adding an outer shield will then result in excellent suppression of both magnetic and electric interference

- EM waves will not penetrate a closed conducting enclosure if it is at least 10 “skin depths” thick
- The enclosure does not need to be grounded
- EM waves can leak into the enclosure via:
 - Cables that enter the enclosure
 - Use filters and/or shield the cables
 - Openings in the enclosure such as:
 - plastic knobs
 - displays
 - Ventilation holes
 - Poorly designed connectors

$$\delta = \frac{1}{\sqrt{\pi \cdot \sigma \cdot \mu \cdot f}}$$

δ = skin depth
 σ = conductivity
 μ = permeability
 f = frequency

$$1\delta = 8.7 \text{ dB attn.} \\ (= 2.7 \times)$$



Types of cable

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untwisted pair



coaxial



twin-axial



twisted-pair



twisted-pair shielded



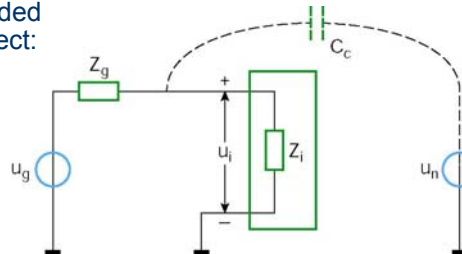
Most types of cable can be obtained with an extra outer shield for even better suppression of EM interference

Shielding

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El. Instr.

Shielding to reduce capacitive coupling

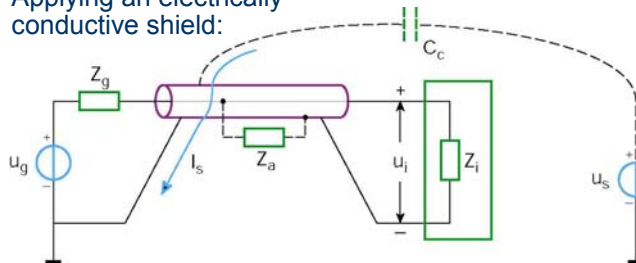
Non-shielded interconnect:



$$u_{i,g} = \frac{Z_i}{Z_g + Z_i} u_g$$

$$u_{i,n} = \frac{Z_g // Z_i}{Z_g // Z_i + \frac{1}{j\omega C_c}} u_n$$

Applying an electrically conductive shield:



$$C_c \downarrow \rightarrow u_{i,n} / u_n \downarrow$$

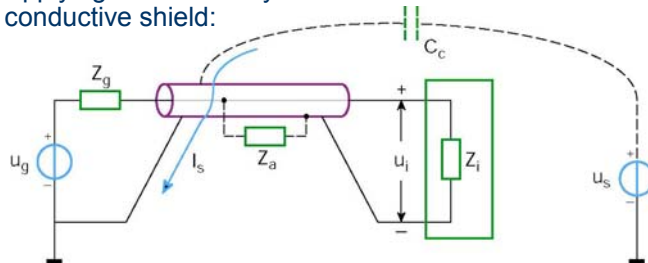
$$u_{i,g} = \frac{Z_i // Z_a}{Z_g + Z_i // Z_a} u_g$$

$$Z_i // Z_a \square Z_i$$

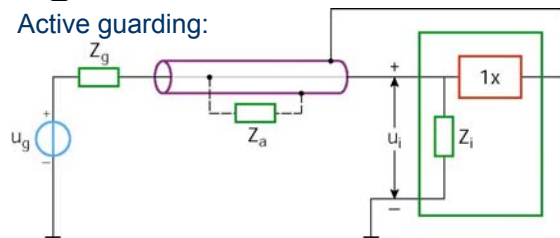
$$\rightarrow BW(u_{i,g} / u_g) \downarrow$$

Active guarding reduces capacitive coupling, and maintains BW

Applying an electrically
conductive shield:



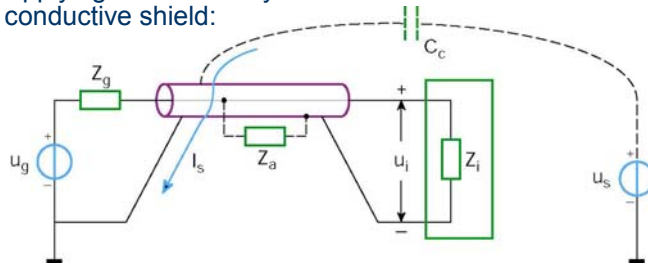
Active guarding:



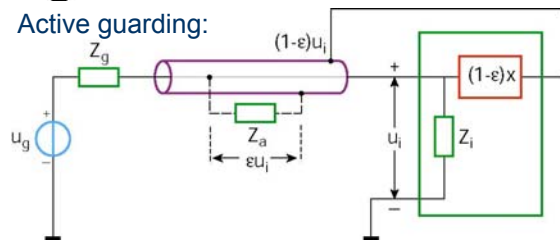
No current flowing
through Z_a . Hence:
 $Z'_a = U_i / I_a \gg Z_a$

Active guarding reduces capacitive coupling, and maintains BW

Applying an electrically
conductive shield:



Active guarding:

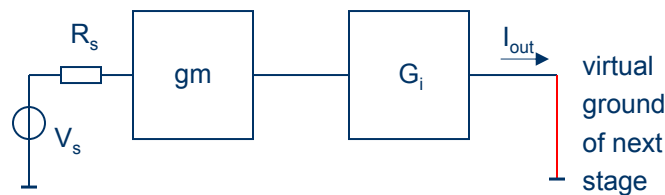


$I_a = u_i - (1-\epsilon)u_i / Z_a$.
Hence: $Z'_a = U_i / I_a =$
 $u_i Z_a / \epsilon u_i = Z_a / \epsilon$

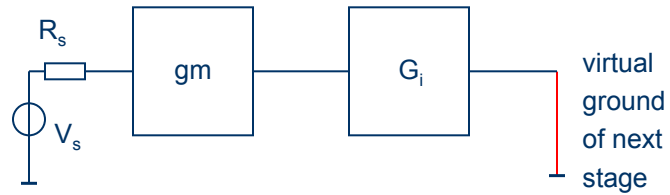
Stability: ϵ positive !

Knowing how electric (capacitive) coupling arises, it is “easy” to avoid this type of interference:

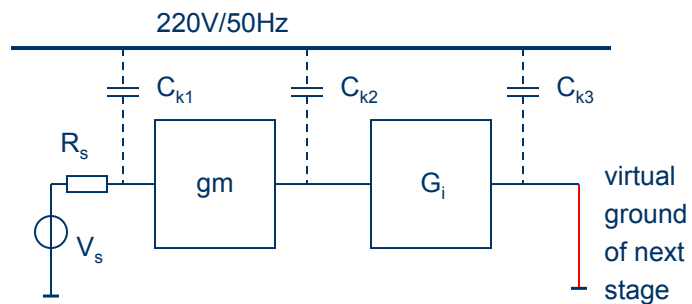
- Reduce the signal impedances
 - The coupling cap and the signal impedance form a voltage divider
- Drop the frequency and amplitude of the interfering signal
 - Usually not possible
- Decrease the coupling capacitance
 - Increase the distance between interference and circuit
 - Ground floating conductors between interference and circuit
- Break any return path for interference
 - These usually form ground loops
- Capacitive shielding:
 - Shielded cables (e.g. coax)
 - Conducting box around sensitive circuits
 - Ground the shielding!



- A voltage-to-current converter with trans-conductance $G_m = 1 \text{ A/V}$ is used to readout a sensor with source resistance $R_s = 4 \text{ k}\Omega$.
- The voltage-to-current converter consists of a transconductor with $g_m = 10 \text{ mA/V}$ followed by a current amplifier with $G_i = 100$
- The transconductor's equivalent noise voltage and current are: $u_n = 1 \text{ nV}/\sqrt{\text{Hz}}$ and $i_n = 0.5 \text{ pA}/\sqrt{\text{Hz}}$. Its input resistance is $100 \text{ k}\Omega$.
- The current amplifier's equivalent noise voltage and current are: $u_n = 100 \text{ nV}/\sqrt{\text{Hz}}$ and $i_n = 10 \text{ pA}/\sqrt{\text{Hz}}$. Its input resistance is 10Ω .
- Calculate the converter's equivalent noise voltage and current



- Calculate the detection limit due to noise of the readout, if the bandwidth of interest lies between 20Hz and 20kHz.
- The source also produces both $1/f$ and thermal noise. Assuming that the latter is due to the 4k resistor and that the $1/f$ corner frequency is 1kHz, calculate the new detection limit



- The circuit is mounted close to a power cable and so suffers from capacitively-coupled 50Hz interference.
- Calculate the input-referred voltage if $C_{k1} = C_{k2} = C_{k3} = 1\text{pF}$
- What can be done to reduce the power-line coupling?