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# Electronic Instrumentation

Lecturer: Kofi Makinwa

K.A.A.Makinwa@TUDelft.NL

015-27 86466
Room. HB13.270 EWI building

Also involved:
Saleh Heidary
(S.H.Shalmany@TUDelft.NL)

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## Introduction

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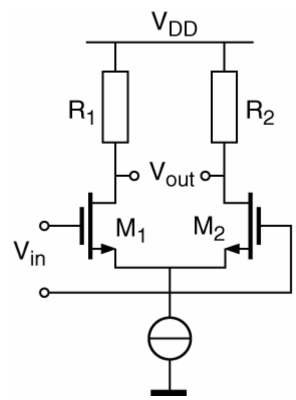
- **Differential (as opposed to single-ended) measurements have many advantages:**
  - **inherently reject common-mode interference and noise**
  - **allow a wider swing for a fixed power-supply voltage**
  - **minimize offset and even order distortion**
  - **(in some cases) lead to improved sensor linearity**
- **BUT most practical amplifiers also respond to some degree to common-mode signals**
  - **The appropriate figure of merit is the common-mode rejection ratio (CMRR).**
  - **Good amplifiers can achieve CMRRs > 120dB**

# Differential Amplifiers



- Widely used to amplify DC signals
- Balanced structure is
- Nominally offset free
- Rejects common-mode and power supply interference
- Easily realized in both CMOS and bipolar technologies

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# Practical Diff. Amps

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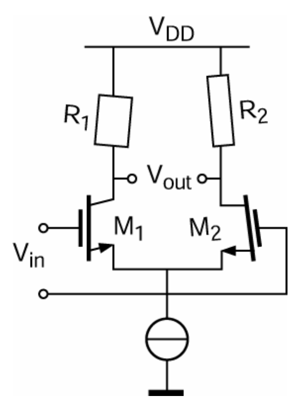
- Component mismatch e.g. $R_1 \neq R_2$, $M_1 \neq M_2$
  $\Rightarrow$ offset, finite CMRR & PSRR
- Mismatch is mainly due to
- Process variation
- Lithographic errors
- All things being equal:
- CMOS amplifiers are 10x worse than Bipolar amplifiers



## Wheatstone bridge

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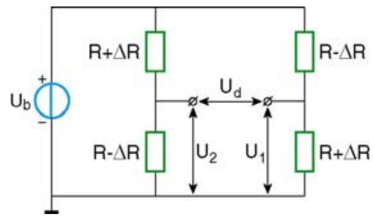
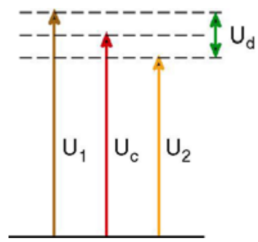



$$U_1 = \frac{R+\Delta R}{(R+\Delta R)+(R-\Delta R)}U_b = \frac{R+\Delta R}{2R}U_b$$

$$U_2 = \frac{R-\Delta R}{(R+\Delta R)+(R-\Delta R)}U_b = \frac{R-\Delta R}{2R}U_b$$




Information is contained in the small differential signal:

$$U_d = U_1 - U_2 = \frac{\Delta R}{R}U_b = 0.1V \;\; for \;\; \frac{\Delta R}{R} = 1\%,\;\; U_b = 10V$$

Which, however, is superimposed on a much larger common-mode signal:

$$U_c = \frac{U_1+U_2}{2} = \frac{U_b}{2} = 5V \;\; at \;\; U_b = 10V$$

## Common-mode rejection in differential measurements

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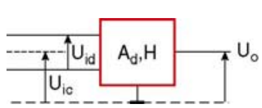
A differential measurement is characterized by a (small) differential signal of interest, which is superimposed on a (large) common-mode signal.

The differential measurement should be sensitive to the differential signal only, regardless of the magnitude of the common-mode signal. The extent to which this is achieved by the read-out is by the common-mode rejection (*CMRR* or *H*).




$$H = \frac{A_d}{A_C} = \frac{\frac{U_o}{U_{id}}}{\frac{U_o}{U_{ic}}} = \left(\frac{U_{ic}}{U_{id}}\right)_{Uo=const}$$

Ideally: $U_o = A_d U_{id}$, but practically: $U_o = A_d(U_{id} + U_{ic}/H)$.

The term $U_{ic}/H$ is an error signal, so $H$ should be maximized.

Note: Reader uses "$G_d$" rather than "$A_d$"

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## Reducing the effect of capacitive coupling

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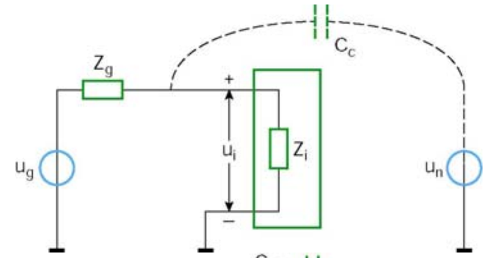
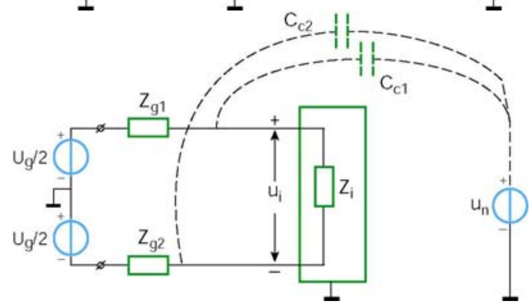
Single-ended input:




Differential readout:

In a balanced system
$U_n$ results in $U_{ic}$ only
⇒ high CMRR required

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## Causes of finite CMRR (1)

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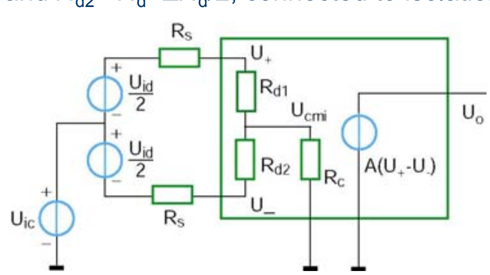
Generalized input circuit with differential input resistance $R_{d1}= R_d+\Delta R_d/2$ and $R_{d2}= R_d- \Delta R_d/2$, connected to isolation resistance $R_c$:.




$$U_{cmi} \approx \frac{2R_c U_{ic}}{2R_c + R_s + R_d}$$

$$\frac{U_+ - U_-}{U_{ic}} \approx \frac{\Delta R_d}{R_s + R_d}\left(1 - \frac{R_c}{R_c + \frac{R_s + R_d}{2}}\right)$$

$$\frac{U_+ - U_-}{U_{id}} \approx \frac{R_d}{R_s + R_d}$$

Hence,
$$CMRR = H = \frac{\frac{U_+ - U_-}{U_{id}}}{\frac{U_+ - U_-}{U_{ic}}} = \frac{R_d}{\Delta R_d} \times \frac{2R_c + R_s + R_d}{R_s + R_d}$$

## Causes of finite CMRR (2)

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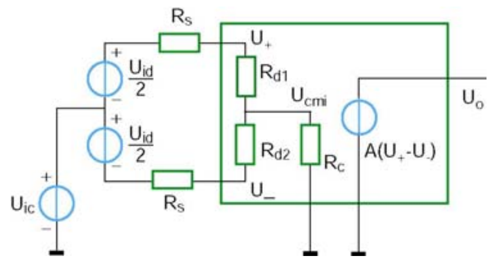
Generalized input circuit with differential input resistance $R_{d1} = R_d + \Delta R_d/2$ and $R_{d2} = R_d - \Delta R_d/2$, connected to isolation resistance $R_c$:.




$$U_{cmi} \approx \frac{2R_c U_{ic}}{2R_c + R_s + R_d}$$

$$\frac{U_+ - U_-}{U_{ic}} \approx \frac{\Delta R_d}{R_s + R_d}\left(1 - \frac{R_c}{R_c + \frac{R_s + R_d}{2}}\right)$$

$$\frac{U_+ - U_-}{U_{id}} \approx \frac{R_d}{R_s + R_d}$$

Hence, $$CMRR = H = \frac{\frac{U_+ - U_-}{U_{id}}}{\frac{U_+ - U_-}{U_{ic}}} = \frac{R_d}{\Delta R_d} \times \frac{2R_c + R_s + R_d}{R_s + R_d}$$

matching isolation

## Common-mode rejection and discrimination factor

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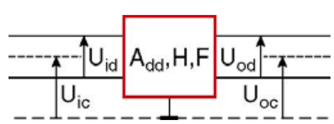
A system with differential input AND differential output has differential AND common-mode signals at input AND output. Hence it is incompletely specified by differential gain and CMRR.




$$H = \frac{A_{dd}}{A_{cd}} = \frac{\frac{U_{od}}{U_{id}}}{\frac{U_{od}}{U_{ic}}} = \left(\frac{U_{ic}}{U_{id}}\right)_{Uod=const}$$

$U_{oc}$ is not defined, thus: $$F = \frac{A_{dd}}{A_{cc}} = \frac{\frac{U_{od}}{U_{id}}}{\frac{U_{oc}}{U_{ic}}}$$

$F$ = Discrimination factor, and it should be maximized.

Alternatively, the common-mode gain $A_{cc}$ can be specified.

## Using an opamp as differential amplifier (1)

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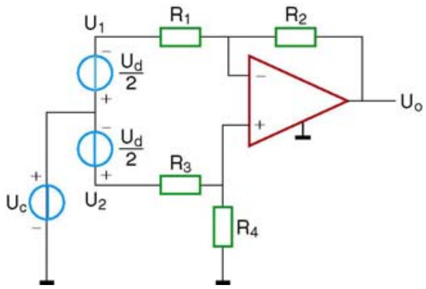


Transfer function via superposition:

## Using an opamp as a differential amplifier (2)

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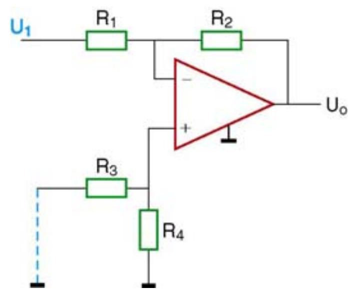


$$U_{o1} = -\frac{R_2}{R_1} U_1$$

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## Using an opamp as a differential amplifier (3)

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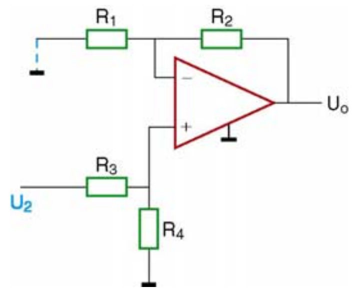


$$U_{o2} = \frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} U_2$$

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## Using an opamp as a differential amplifier (4)

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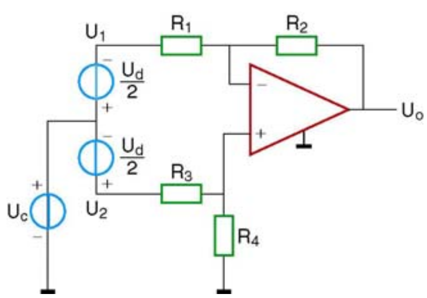


$$U_{o1} = -\frac{R_2}{R_1} U_1$$

$$U_{o2} = \frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} U_2$$

Superposition: $U_o = U_{o1} + U_{o2} = -\frac{R_2}{R_1} U_1 + \frac{R_4(R_1+R_2)}{R_1(R_3+R_4)} U_2$

For $R_1/R_2 = R_3/R_4$: $U_o = -\frac{R_4}{R_3} U_1 + \frac{R_3+R_4}{R_3} \frac{R_4}{R_3+R_4} U_2 = \frac{R_4}{R_3}(U_2 - U_1)$

## CMRR of the one-opamp differential amplifier

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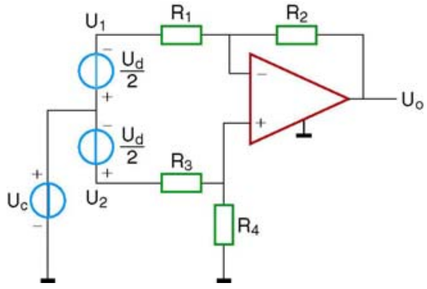


$$U_o = -\frac{R_2}{R_1}U_1 + \frac{R_4(R_1+R_2)}{R_1(R_3+R_4)}U_2$$

But practical components have tolerances! Assume

$(R_1+R_2)/R_2 = (1+\delta)(R_3+R_4)/R_4$

$$U_o = -\frac{R_2}{R_1}U_1 + \frac{R_4(R_2+R_1)}{(R_3+R_4)R_1}U_2 = -\frac{R_2}{R_1}U_1 + \frac{R_2}{R_1}(1+\delta)U_2 =$$

$$\frac{R_2}{R_1}\left(1+\frac{\delta}{2}\right)(U_2-U_1) + \frac{\delta R_2}{R_1}\frac{(U_1+U_2)}{2} = A_{dd}(U_2-U_1) + A_{cd}\frac{(U_1+U_2)}{2} \rightarrow A_{dd} = \frac{R_2}{R_1}$$

$$H = \frac{A_{dd}}{A_{cd}} = \frac{(R_2/R_1)(1+\delta/2)}{(R_2/R_1)\delta} = \frac{1+\delta/2}{\delta} \approx \frac{1}{\delta}$$

~ 1000 for resistors with 0.1% tolerance i.e. of no practical use !

## Improving the CMRR of a 1-opamp diff. amplifier

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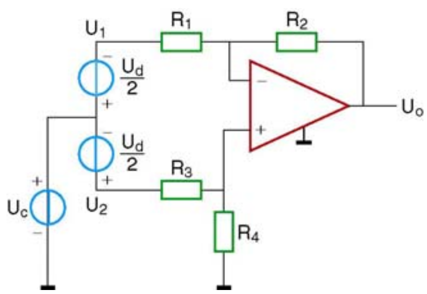


$$H \approx \frac{1}{\delta}$$

Due to component tolerances!

- CMRR is caused by resistor mismatch
  (in general, by component mismatch)
- CMRR can be improved by trimming e.g. one of the resistors
- Alternatively a better topology can be used
  – the three-opamp **instrumentation** amplifier

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## The 3-opamp instrumentation amplifier

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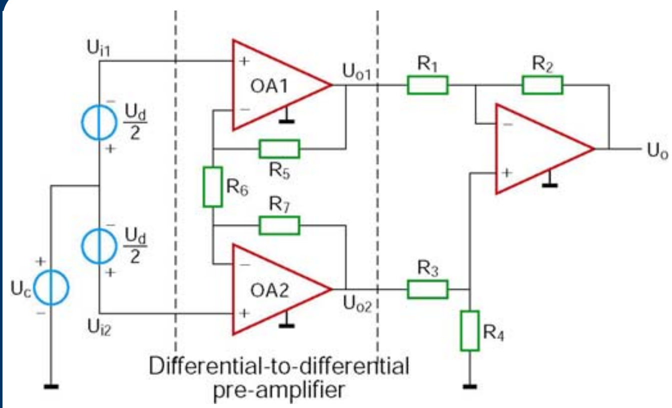


The 1-opamp differential amplifier is preceded by a differential-to-differential preamp with differential input voltage $U_{i1}$-$U_{i2}$ and differential output voltage $U_{o1}$-$U_{o2}$. Both input and output have the differential voltage superimposed on a common-mode level.

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## The 3-opamp INA – transfer function

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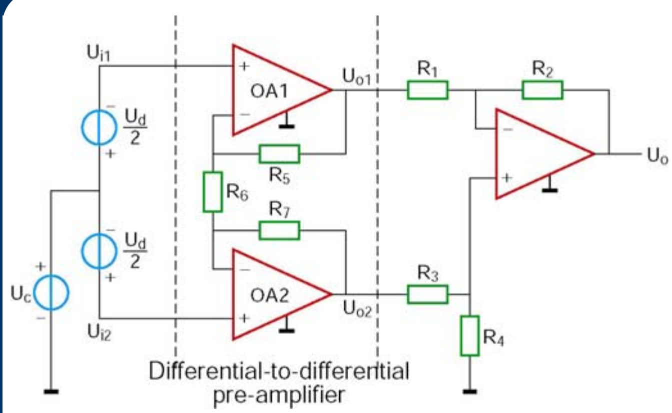


Deriving the expression for transfer function $(U_{o1}-U_{o2})/(U_{i1}-U_{i2})$.

Can you identify the topologies of the inverting and non-inverting amplifier around OA1 and OA2 ?

Use the virtual ground concept and superposition to derive $U_{o1}/U_{i1}$…..

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## Transfer function $U_{o1}/U_{i1}$

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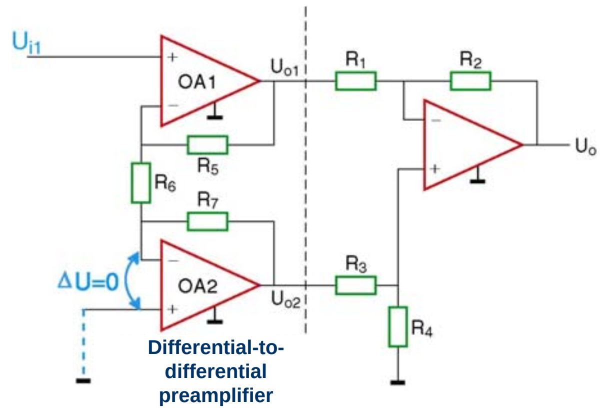


Node $R_6/R_7$ is at virtual ground potential.
OA1 is a non-inverting amplifier with: $U_{o1}/U_{i1}= (R_5+R_6)/R_6$.

Next transfer function $U_{o2}/U_{i1}$…..

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## Transfer function $U_{o2}/U_{i1}$

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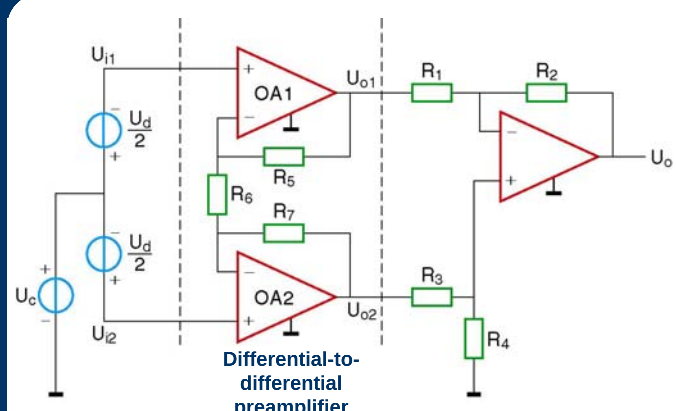

## Transfer function $U_{o2}/U_{i1}$

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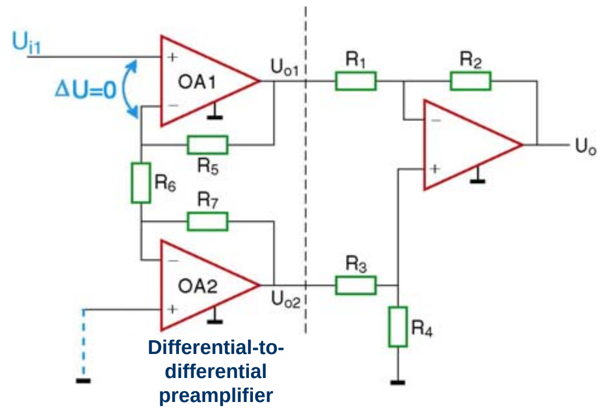


$U_{i1}$ virtually available at node $R_5/R_6$.
With respect to this node OA2 is an inverting amplifier:
$U_{o2}/U_{i1} = -R_7/R_6$.

Calculation of transfer function $U_{o2}/U_{i2}$ is similar to that of $U_{o1}/U_{i1}$..

## Transfer function $U_{o2}/U_{i2}$

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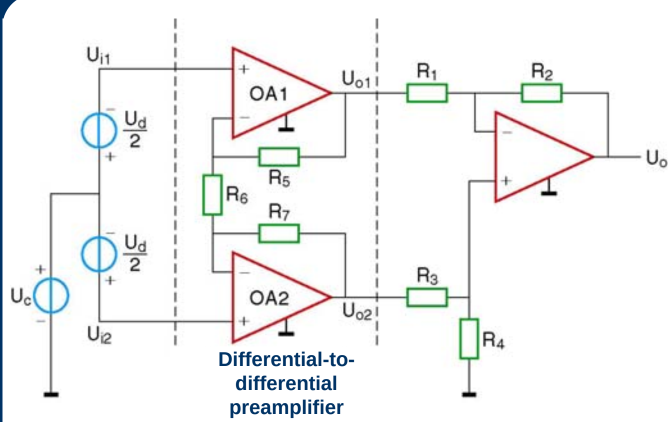

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## Transfer function $U_{o2}/U_{i2}$

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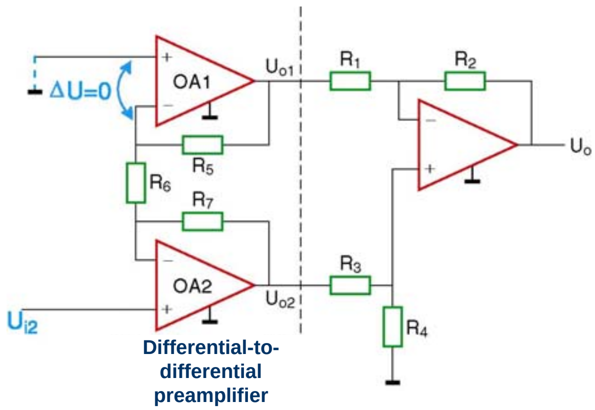


Node $R_5/R_6$ is at virtual ground potential.
OA2 is a non-inverting amplifier with: $U_{o2}/U_{i2} = (R_6+R_7)/R_6$.

Finally, the transfer function $U_{o1}/U_{i2}$ …..

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## Transfer function $U_{o1}/U_{i2}$

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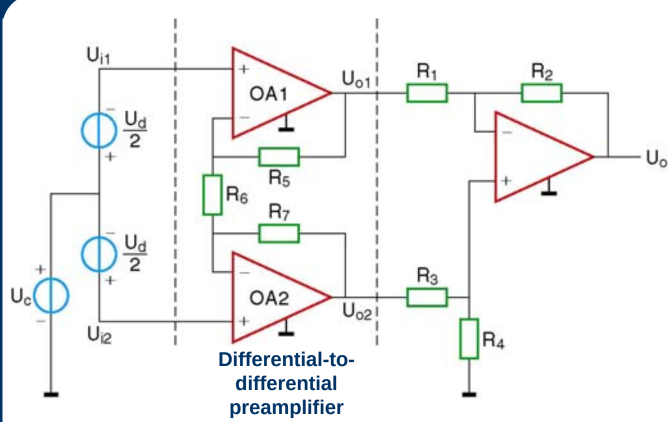

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## Transfer function $U_{o1}/U_{i2}$

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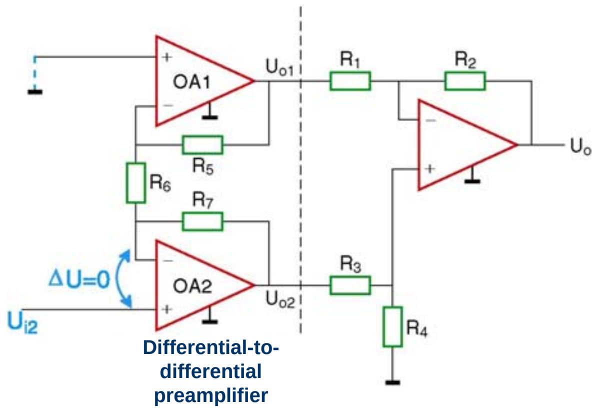


$U_{i2}$ is virtually available at node $R_7/R_6$.
With respect to this node, OA1 is an inverting amplifier:
$U_{o1}/U_{i2} = -R_5/R_6$.
Superposition: The overall transfer function $(U_{o1}-U_{o2})/(U_{i1}-U_{i2})$ follows by summation of these four components ……

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## Preamp transfer function (1)

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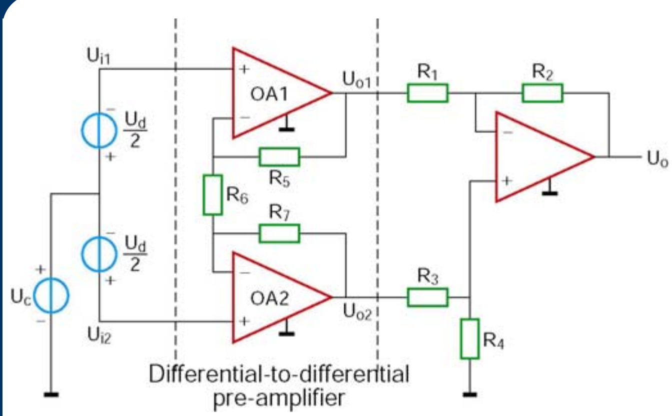


Transfer function follows as:

$$U_{o1} = \frac{R_5 + R_6}{R_6} U_{i1} - \frac{R_5}{R_6} U_{i2}$$

$$U_{o2} = -\frac{R_7}{R_6} U_{i1} + \frac{R_6 + R_7}{R_6} U_{i2}$$

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## Preamp transfer function (2)

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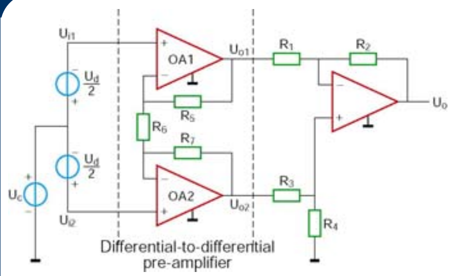


$$U_{o1} = \frac{R_5 + R_6}{R_6} U_{i1} - \frac{R_5}{R_6} U_{i2}$$

$$U_{o2} = -\frac{R_7}{R_6} U_{i1} + \frac{R_6 + R_7}{R_6} U_{i2}$$

Diff. output: $$U_{o1} - U_{o2} = \frac{R_5 + R_6 + R_7}{R_6} U_{i1} - \frac{R_5 + R_6 + R_7}{R_6} U_{i2} = \frac{R_5 + R_6 + R_7}{R_6} U_d$$

Generally: $$U_{o1} - U_{o2} = A_{dd} U_d + A_{cd} U_c \rightarrow A_{dd} = \frac{R_5 + R_6 + R_7}{R_6},\ A_{cd} = 0$$

Also: $$\frac{U_{o1} + U_{o2}}{2} = \frac{R_5 + R_6 - R_7}{2R_6} U_{i1} + \frac{R_6 + R_7 - R_5}{2R_6} U_{i2} = U_c + \frac{R_5 - R_7}{R_6} \frac{U_d}{2}$$

$$\frac{U_{o1} + U_{o2}}{2} = A_{cc} U_c + A_{dc} U_d \rightarrow A_{cc} = 1$$

So, what is the advantage ?

Preamp boosts only the differential signal ⇒ increased CMRR

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## Preamp transfer function: the easy way!

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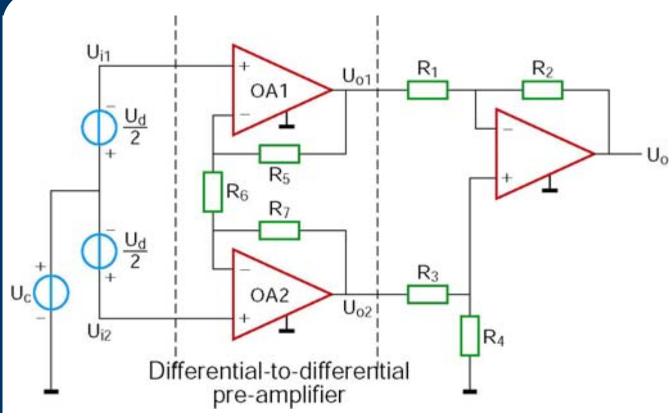


Set $U_d = 0$ then by inspection: $U_{01} + U_{o2} = 2U_c$

Set $U_c = 0$ then by inspection $U_d$ appears across $R_6$

So $U_{01} - U_{o2} = U_d + U_d(R_5+R_7)/R_6 = U_d(R_5+R_6+R_7)/R_6$

## Commercial Products

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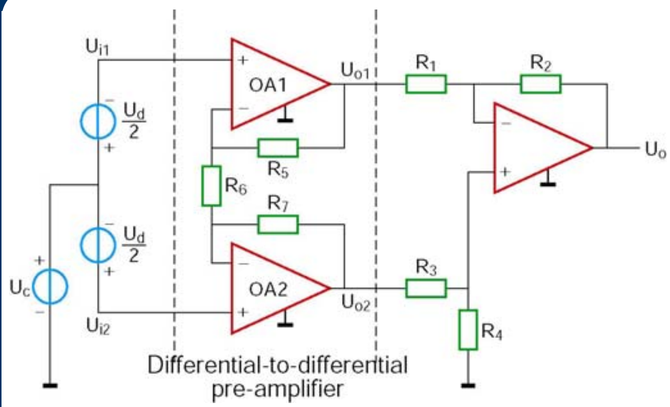


You don't have to make one yourself! Instrumentation amplifiers based on laser trimmed resistors can be purchased from several manufacturers (including Texas Instruments, Analog Devices, Linear Technology and Maxim Integrated Products)

## Common-mode rejection vs. discrimination factor

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The common-mode rejection (CMRR, $H$) specifies the extent to which a common-mode signal at the input results in a differential signal at the output ($A_{cd}$ non-zero results in signal mixing) and is normalized by the differential gain, $A_{dd}$. This signal mixing directly limits the amplifier's detectivity.

The discrimination factor ($F$) specifies to what extent a common-mode signal at the input of a fully differential amplifier results in a common-mode signal at the output ($A_{cc}$ non-zero does not remove common-mode signal) and is normalized by the differential gain, $A_{dd}$.

Attenuating the input common-mode signal reduces the required CMRR of the subsequent stage.

## Limits on Pre-amp CMRR $H_1$

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$$H_1 = \frac{A_{dd}}{A_{cd}} = \frac{R_5 + R_6 + R_7}{0.R_6} \rightarrow \infty$$

Non-realistic due to assumptions implicitly made:

1 Opamp common-mode impedance is finite

2 The CMRR of each of the opamps in the differential-to-differential pre-amplifier is finite and frequency dependent

3 The open-loop gain of the opamps in the differential-to-differential pre-amplifier is finite and frequency dependent

## Effect of finite opamp CMRR $H_o$ on $H_1$

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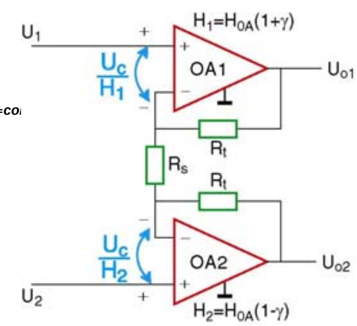
From definition: $H = \frac{A_{dd}}{A_{cd}} = \frac{\frac{U_{od}}{U_{id}}}{\frac{U_{od}}{U_{ic}}} = \left(\frac{U_{ic}}{U_{id}}\right)_{U_{od}=co}$




Hence finite CMRR can be modeled as an equivalent differential signal: $U_{id,eq} = \frac{U_{ic}}{H}$

Applying to the input stage yields: $H_{pre-amp,eq} = \frac{1}{\frac{U_1 - U_2}{U_c}} = \frac{1}{\frac{1}{H_2} - \frac{1}{H_1}} = \frac{H_{OA}\left(1-\gamma^2\right)}{2\gamma} \approx \frac{H_{OA}}{2\gamma}$

Conclusion: $H_{pre\text{-}amp}$ is limited by the mismatch $\gamma$ between the opamp CMRR's. Assume $H_o = 10^5$ and $\gamma = 1\%$ yields: $H_{pre\text{-}amp,max,} = 10^7$

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## ECG Readout

ET8.017
El. Instr.

ECG
= small differential biopotential
$U_{id}$ ~ 10 μV (0 - 100 Hz)

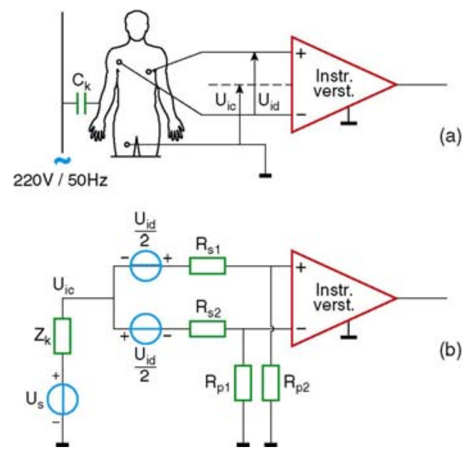
Superimposed on a large common-mode mains voltage
$U_{ic}$ ~ 1 mV, 50 Hz.

Application requires:

1. High CMRR > 80dB
(Filtering not possible)

2. High input impedance
(electrode impedances $R_s$ are not well defined)

⇒ Instrumentation Amplifier

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## Achieving higher CMRR (1)

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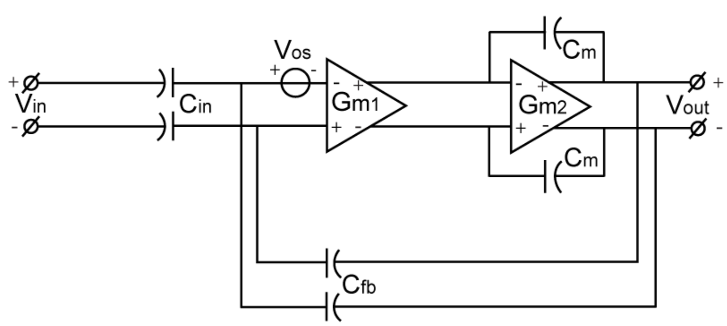


- Capacitors will block the common-mode voltage
⇒ a capacitively coupled amplifier? Closed-loop gain?
- Two stage amplifier ⇒ high open-loop DC gain (> 100dB)
- DC stability?

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## Achieving higher CMRR (2)

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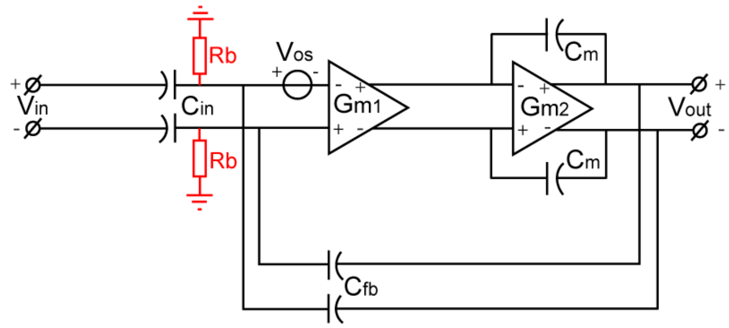


- Extra resistors are needed to establish DC voltages
- Trade-off: should be quite large (MΩ) to minimize noise, should be small to minimize CM settling time
- DC closed-loop gain?

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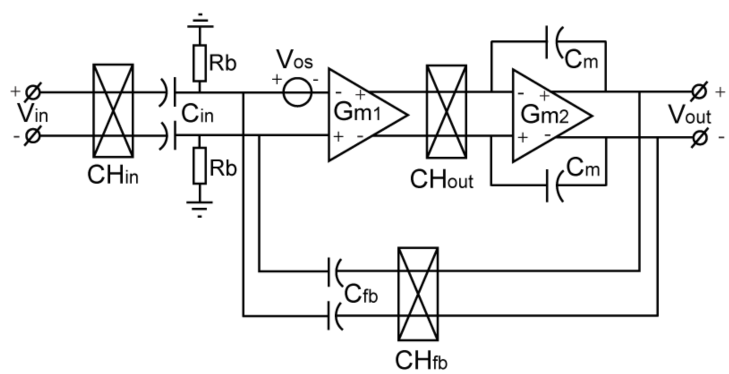
## Achieving higher CMRR (3)

*ET8.017*
*El. Instr.*




- Polarity reversing switches (known as choppers) convert input DC signals into AC signals (square-waves)
- To maintain feedback polarity, the feedback path and the amplifier also contain choppers

## Achieving higher CMRR (4)

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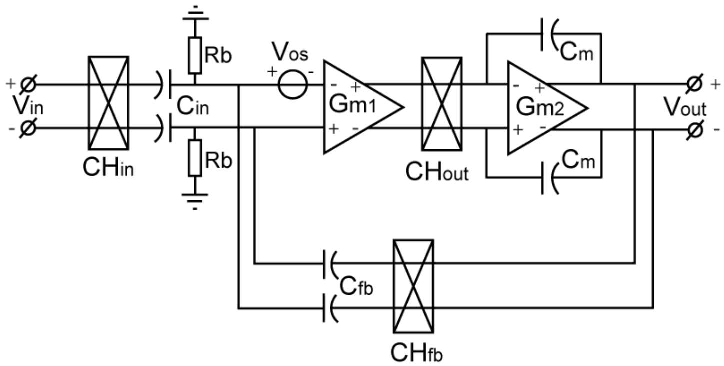


- Result: 1μV offset, 0.16% gain error, 134dB CMRR, 120dB PSRR


Q. Fan, F. Sebastiano, J.H. Huijsing and K.A.A. Makinwa, "A 1.8μW 1μV-Offset Capacitively-Coupled Chopper Instrumentation Amplifier in 65nm CMOS," *J. Solid-State Circuits,* vol. 46, is. 7, pp. 1534 - 1543, July 2011.


## PSRR

**ET8.017 El. Instr.**

- A close cousin of CMRR is the power-supply rejection ratio (PSRR), which expresses how sensitive the amplifier is to changes in its supply voltage.

$$PSRR = \frac{A_d}{A_{sy}} = \frac{\frac{U_o}{U_{id}}}{\frac{U_o}{\Delta U_{sy}}} = \left(\frac{\Delta U_{sy}}{U_{id}}\right)_{Uo=const}$$

- Here $\Delta U_{sy}$ represents the changes in the supply voltage
- A good instrumentation amplifier will have a PSRR > 100dB

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## Assignment 4a

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*El. Instr.*

- One drawback of the 3-opamp instrumentation amplifier is that its differential input range is somewhat limited
- Consider an instrumentation amplifier made from 3 identical opamps, with a preamp gain of 100 and a unity gain 1-opamp differential amplifier. The opamps are powered from a 0 to 5V supply and have rail-to-rail output and input capability

- Make a plot of the maximum differential input signal (y-axis) versus the input common-mode voltage (x-axis)
- How can the amplifier be modified to increase its differential input range? Discuss any drawbacks of the chosen approaches.

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## Assignment 4b

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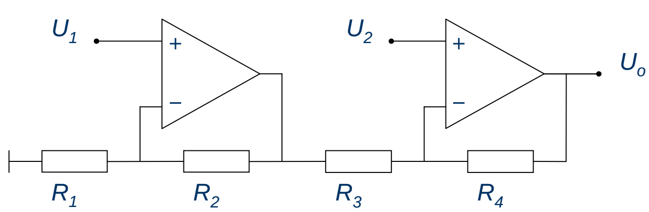


- Another way to make an instrumentation amplifier is to use the 2-opamp circuit shown above.
- Assuming that this INA also has a gain of 100, and that the opamps are the same as those used in Ass. 4a, again make a plot of the maximum differential input signal (y-axis) versus the input common-mode voltage (x-axis)
- In terms of input differential range, which is better: the two opamp or the 3-opamp INA?