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Electronic Instrumentation

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Introduction

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- Synonyms: coherent detection, synchronous demodulation, lock-in amplification, chopping??
- These are all modulation techniques that are used to improve the low frequency performance of measurement systems
- When square-wave modulation is employed, the technique is referred to as chopping

Coherent detection

Theory

Why and when to use?
Basic principle (time domain)
Amplitude and phase measurement
Signal-to-Noise Ratio (frequency domain)
Switching detector (choppers)

Summary

Application example (DEMO 1)

Strain gauges
Wheatstone bridge
Coherent detector: design considerations

Coherent detection

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When to use it?

- When measuring low-bandwidth or quasi-static signals in the presence of high noise or disturbance levels.
- If a high dynamic range is required

Therefore it is often used to readout sensors e.g.

- (MEMS) Accelerometers (fF capacitance changes)
- Optical (infrared) detectors (fA's of current)
- Magnetic sensors (mV's)
- Strain gauges (mV's)

A coherent detector behaves like a bandpass filter

Time domain analysis

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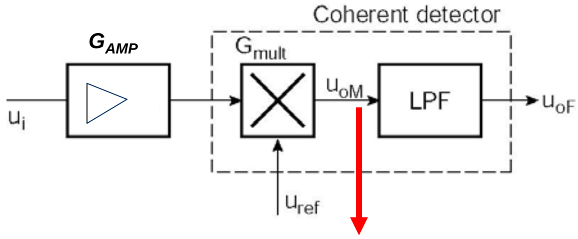

$$u_{oM} = G_{Amp} G_{mult} \left[\hat{u}_i \sin(\omega_i t) \hat{u}_r \sin(\omega_r t) \right]$$

$$u_{oM} = G_{Amp} G_{mult} \frac{\hat{u}_r \hat{u}_i}{2} \left[\cos((\omega_r - \omega_i) t) - \cos((\omega_r + \omega_i) t) \right]$$

Now if: $\omega_r = \omega_i$ (Coherent signals !) then:

DC

$$u_{oM} = G_{Amp} G_{mult} \frac{\hat{u}_r \hat{u}_i}{2} \left[1 - \cos(2\omega_i t) \right]$$

Amplitude detection

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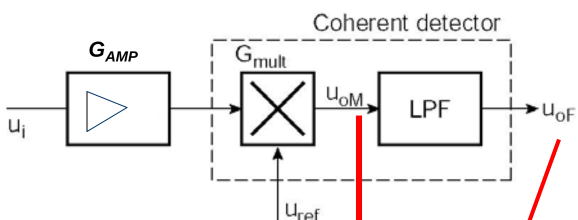

$$u_{oM} = G_{Amp} G_{mult} \frac{\hat{u}_r \hat{u}_i}{2} \left[1 - \cos(2\omega_i t) \right] \qquad \omega_r = \omega_i$$

The low-pass filter removes the second harmonic:

$$u_{oF} = G_{LPF} G_{Amp} G_{mult} \frac{\hat{u}_r \hat{u}_i}{2}$$

This coherent detector detects the **amplitude** of the input signal.

Note that in this case **U_i and U_{ref}** are in-phase (Synchronous detection).

What if they are not in-phase ????

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Phase detection

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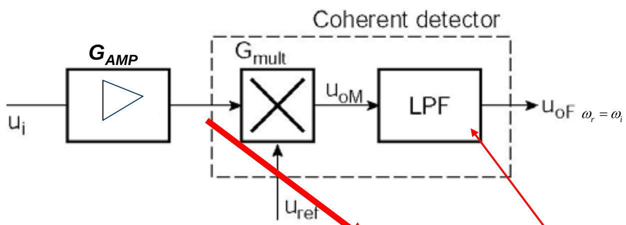
Suppose U_i and U_{ref} have a phase difference:



$$u_{oM} = G_{Amp} G_{mult} \left[\hat{u}_i \sin(\omega_i t + \varphi) \hat{u}_r \sin(\omega_r t) \right]$$

$$u_{oM} = G_{Amp} G_{mult} \frac{\hat{u}_r \hat{u}_i}{2} \left[\cos((\omega_r - \omega_i)t + \varphi) - \cancel{\cos((\omega_r + \omega_i)t + \varphi)} \right]$$

$$u_{oF} = G_{LPF} G_{Amp} G_{mult} \frac{\hat{u}_r \hat{u}_i}{2} [\cos(\varphi)]$$

=0

This coherent detector is also **phase sensitive**.

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Signal-to-noise ratio

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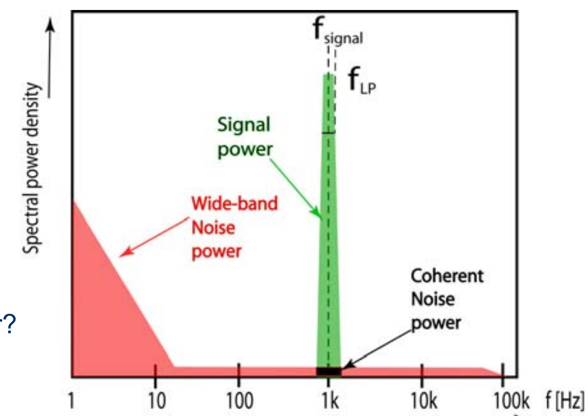
What would be
the S/N ratio of
– an oscilloscope?
– a coherent detector?



Some numbers:
What would be the detection limit of the coherent detector if the white noise spectral density = 10 nV/√Hz and f_{LP} = 1Hz ???

Coherent detection in practice

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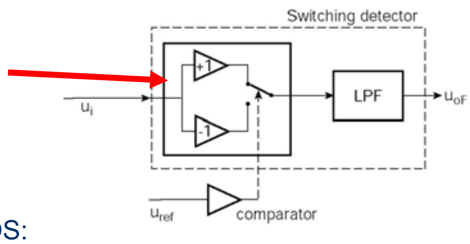
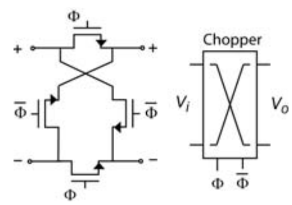
Linear operation of an analog multiplier circuit, working over a large range of input signals is difficult to realize in practice.

A much simpler realization is a switching detector a.k.a. a **chopper**



Chopper realization in CMOS:

Easy: just 4 transistors!
Accurate: does not introduce offset
But: switching spikes can cause problems (residual offset)



Summary

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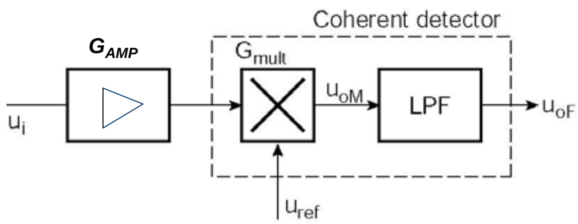
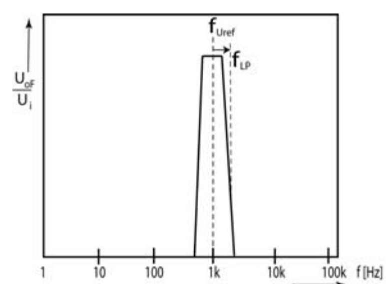


REMEMBER:

- **An ideal coherent detector is only sensitive to frequency components in a narrow band around a reference signal U_{ref}.**
- **The low-pass filter determines the signal bandwidth**



So what happens if U_{ref} is a square wave ??

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DEMO: Coherent detection for the readout of strain-gauges

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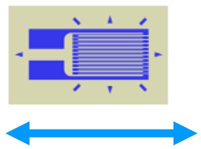

R is typically 100-2000 Ω

ΔR is typically 10^{-1}- 10^{-5} Ω

Very small changes in resistance should be measured

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DEMO: Using strain gauges for force measurement

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DEMO: Using strain gauges for force measurement

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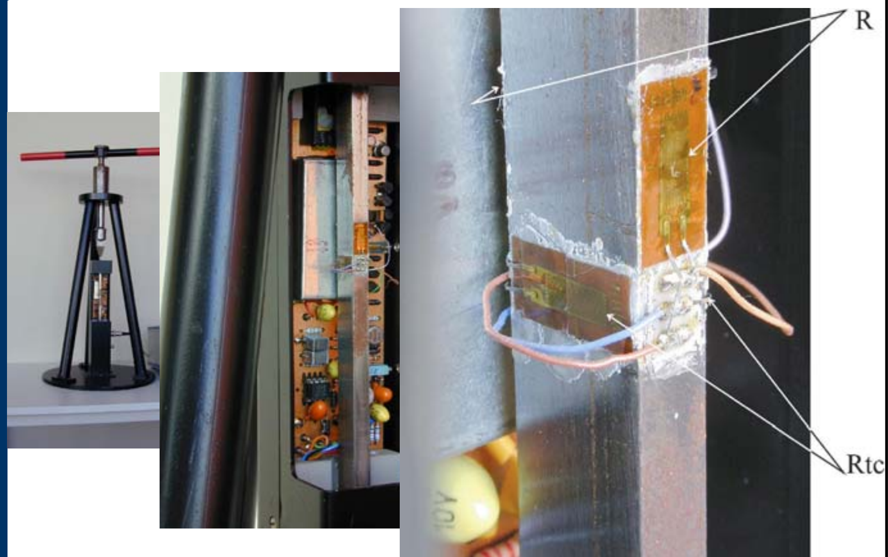

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DEMO: Wheatstone bridge with strain gauges

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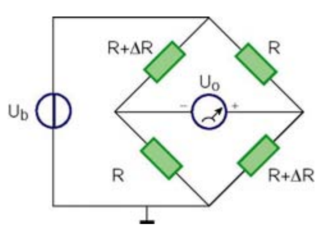
Wheatstone ½ bridge

Construction enables only stressed and non-stressed strain gauges



$$U_{o+} = \frac{R+\Delta R}{(R+\Delta R)+R}U_b = \frac{R+\Delta R}{2R+\Delta R}U_b$$

$$U_{o-} = \frac{R}{(R+\Delta R)+R}U_b = \frac{R}{2R+\Delta R}U_b$$

$$U_d = U_{o+} - U_{o-} = \frac{\Delta R}{2R+\Delta R}U_b$$

Non-linearity error is small if $\Delta R \ll 2R$

U_b can be an AC source of well-known frequency
U_o is a very small AC signal (typically a few uV)

with a high noise level !

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Strain-gauge readout design

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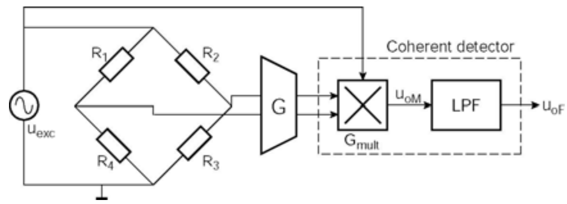

Design considerations:

What is the typical response time for this system used as a weighing scale ?

Frequency of u_{exc} ?

What are the effects of offset in amplifier G

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Design: Frequency domain

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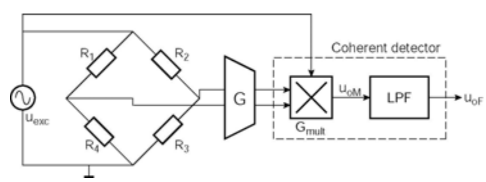
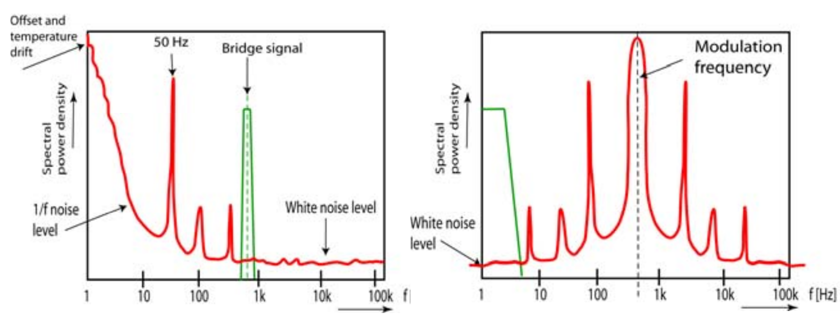


Demodulation causes a shift in the frequency domain



DEMO: Coherent detection for the readout of strain-gauges

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Some data:

Strain gauges : 120 ohm, length 6mm
Steel bar: area = 1cm^2

1 kg gives: Δl = 2,86 nm
Resistance change: ΔR = 120 $\mu\Omega$
Relative change: $\Delta R/R = 10^{-6}$

Noise level:

Force:	**10N , or 1kg**
Max force:	**15 kN, or 1500 kg (Steel bar limit)**
Dynamic range:	**1500/0.1 = 15*10^3 = 84 dB**

Coherent detection: Overload and dynamic range

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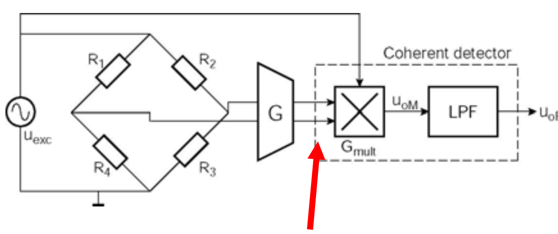
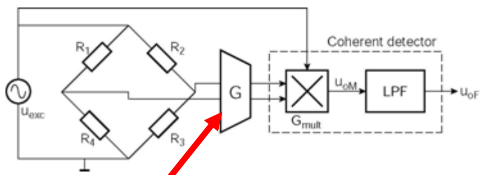


Important limitation of a coherent detector is the maximum allowable signal at the input of the multiplier.

(saturation and non-linearity will lead to unwanted effects such as harmonic sensitivity and self-detection as will be shown later)

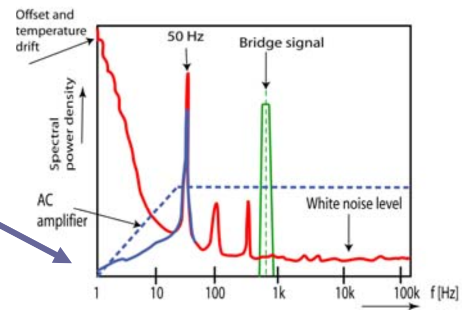
Generally a maximum signal indicator is placed at the signal input of the multiplier to indicate overload situations

Increasing the dynamic range



Use an AC amplifier

Offset suppression reduces overload on multiplier

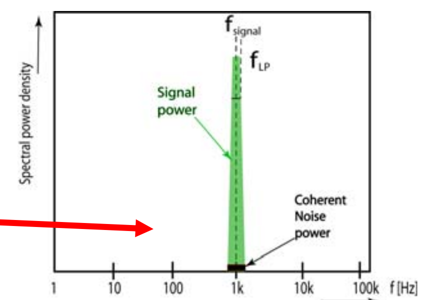


Detection limits

Signal-Noise ratio

=

$$\frac{\textbf{Green area}}{\textbf{Black area}}$$



Other detection limits are due to:

- o Offsets in both input- and output channel
- o Non-linearities in the reference channel
- o Non-linearities in the input channel
- o Non-linearities at the output of the multiplier

Coherent detection: Offset

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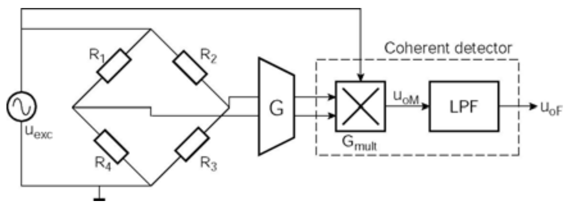

Offset in the input AND reference channels are DC signals that are simply multiplied and pass through the low-pass filter, therefore:

$$U_{oF} = G G_{mult} G_{LPF} U_{excDC} U_{bridgeDC}$$

Coherent detection: Non-linearities

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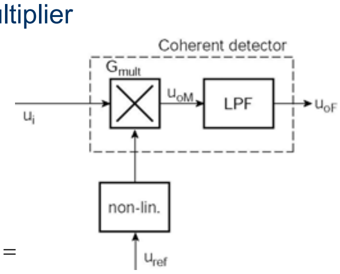
- o Non-linearities in the reference channel
- o Non-linearities in the input channel
- o Non-linearities at the output of the multiplier



In case of a second-order harmonic distortion only:

$$u_{oM} = G_{mult}\left[\hat{u}_r\left(\sin(\omega_r t) + \underline{\beta_2 \sin(2\omega_r t)}\right) \times \hat{u}_i \sin(\omega_i t)\right] =$$

$$G_{mult}\frac{\hat{u}_r \hat{u}_i}{2}\left[\cos((\omega_r - \omega_i)t) - \cos((\omega_r + \omega_i)t) + \underline{\beta_2 \cos((2\omega_r - \omega_i)t)} - \beta_2 \cos((2\omega_r + \omega_i)t)\right]$$

So input components at $\omega_i = 2\,\omega_r$ cause output!

Makes the coherent detector sensitive to SUPERHARMONICS in the input signal

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Coherent detection: Non-linearities

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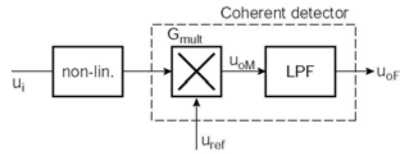
- o Non-linearities in the reference channel
- o Non-linearities in the input channel
- o Non-linearities at the output of the multiplier

In case of a third-order harmonic distortion only:



$$u_{oM} = G_{mult}\left[\hat{u}_i\left(\sin(\omega_i t)+\underline{\beta_1\sin(3\omega_i t)}\right)\times\hat{u}_r\sin(\omega_r t)\right]=$$

$$G_{mult}\frac{\hat{u}_r\hat{u}_i}{2}\left[\cos\left((\omega_r-\omega_i)t\right)-\cos\left((\omega_r+\omega_i)t\right)+\underline{\beta_1\cos\left((3\omega_i-\omega_r)t\right)}-\beta_1\cos\left((3\omega_i+\omega_r)t\right)\right]$$

This term generates output if $\omega_i = \omega_r/3$

Makes the coherent detector sensitive to SUBHARMONICS in the input signal (and also to noise in these bands !)

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Coherent detection: Non-linearities

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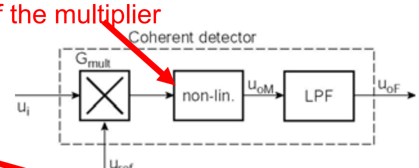
- o Non-linearities in the reference channel
- o Non-linearities in the input channel
- o Non-linearities at the output of the multiplier

In case of a quadratic distortion only:



$$u_{oM} = G_{mult}\hat{u}_r\hat{u}_i\left[\left(\cos(\omega_r t)\times\cos(\omega_i t)\right)+\delta_2\hat{u}_r\hat{u}_i\left(\cos(\omega_r t)\times\cos(\omega_i t)\right)^2\right]=$$

$$G_{mult}\frac{\hat{u}_r\hat{u}_i}{2}\left[\cos\left((\omega_r\pm\omega_i)t\right)+\frac{\delta_2\hat{u}_r\hat{u}_i}{2}\left(1+\cos(2\omega_r t)\right)\left(1+\cos(2\omega_i t)\right)\right]=$$

$$G_{mult}\frac{\hat{u}_r\hat{u}_i}{2}\left[\cos\left((\omega_r\pm\omega_i)t\right)+\frac{\delta_2\hat{u}_r\hat{u}_i}{2}\times\left(1+\cos(2\omega_r t)+\cos(2\omega_i t)+\underline{\frac{1}{2}\left(\cos\left(2(\omega_r\pm\omega_i)t\right)\right)}\right)\right]$$

So components at $\omega_i = \omega_r$ cause output

Self detection

Lock-in amplifier

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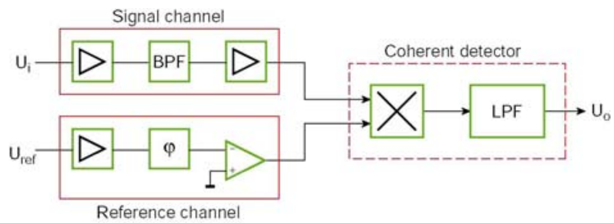
A lock-in amplifier is a coherent detector with all the extra features required of a general-purpose laboratory instrument.



DEMO: Coherent detection in an optical system

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Source: LED modulated with a sine wave

Detector: Photodiode

Instrument: Analog lock-in amplifier (PAR 186)

Interference:

Ambient light, DC (offset), 50 Hz mains (lamp)

Switching detector

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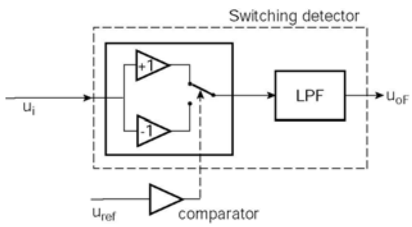
Simplifying multiplier at the expense of super-harmonic sensitivity



Use Fourier series of square wave signal, S(t):

$$U_{oM} = \hat{u}_i \cos(\omega t + \varphi)\left[S(t)\right] =$$

$$\hat{u}_i \cos(\omega t + \varphi)\left(\frac{4}{\pi}\left(\cos(\omega t) - \frac{\cos(3\omega t)}{3} + \frac{\cos(5\omega t)}{5} - \ldots\right)\right) =$$

$$\frac{2\hat{u}_i}{\pi}\left(\cos(\varphi) + \cos(2\omega t + \varphi) - \frac{\cos(2\omega t + \varphi)}{3} - \frac{\cos(4\omega t + \varphi)}{3} + \ldots\right)$$

$$U_{oF} = \frac{2\hat{u}_i}{\pi}\cos(\theta)$$

Similar transfer function as multiplier detector (apart from factor 2/π instead of ½).

Switching detector SNR considerations

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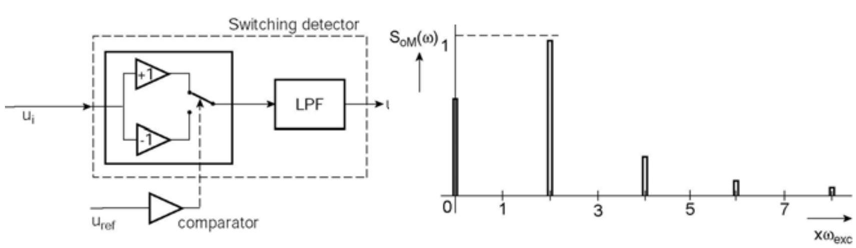
Simplifying multiplier at the expense of a slight decrease in SNR



$$u_{n,SD}^2 = N_o 2 f_{LPF} + \frac{N_o 2 f_{LDF}}{(-3)^2} + \frac{N_o 2 f_{LDF}}{5^2} + \frac{N_o 2 f_{LDF}}{(-7)^2} + \ldots =$$

$$N_o 2 f_{LDF}\left[1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2 + \ldots\right] = N_o 2 f_{LDF}\sum_{n=0}^{\infty}\left(\frac{1}{2n+1}\right)^2 = \frac{\pi^2}{8} N_o .2 f_{LDF}$$

Increase of noise due to super-harmonic sensitivity by a factor π^2 /8.

Phase detector

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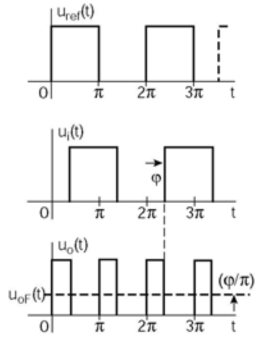
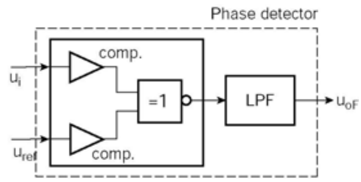
Further simplification of multiplier at the expense of super- and sub-harmonic sensitivity





Logic levels:
No amplitude information

Is often used in Phase Locked Loops (PLL).

PLL for recovery of excitation voltage

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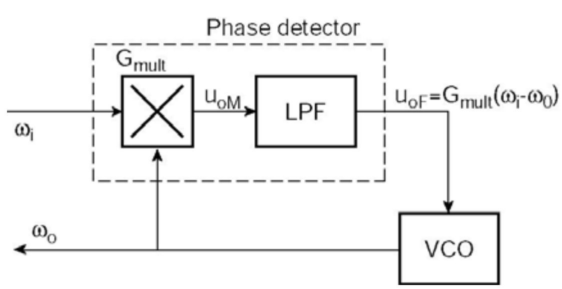
The excitation frequency of remote sensors is not always directly available, but can be retrieved from the sensor's output signal by a phase-locked loop (PLL).



The loop drives the VCO so as to minimize the error signal U_{oM}
As a result, $\omega_i \sim \omega_o$ with a small loop-gain-dependent phase error

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Digital lock-in amplifier

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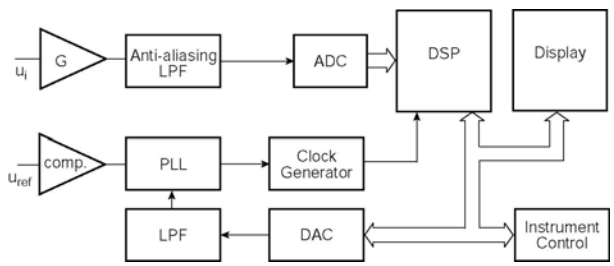
Analog functionality is replaced by digital signal processing (DSP)

⇒ Cost, Flexibility, User friendliness



ADC and DSP are critical (frequency limiting) components

Digital lock-in amplifiers are becoming more and more economical and common in test-setups in the lab and in industry

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Summary

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Coherent detection is a powerful technique to accurately measure low-level periodic signals in the presence of high levels of noise and interference.

Both amplitude and phase can be measured

The signal frequency should be known (reference channel) or it should be possible to regenerate it with a PLL.

The operating frequency should be in a frequency band where disturbing signals and noise are low.

Distortion in the input- or reference channel can lead to harmonic sensitivity

Modern (digital) lock-in amplifiers can also measure the amplitude and phase of harmonic signals (2f, 3f ,4f etc.) e.g. for the measurement of signal distortion.