



System Validation

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4. Sequential Processes: Theory



Sequential Processes

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Announcements

- ▶ There will be a guest lecturer on October 10, 2012.
- ▶ There will be no meetings on October 10, 2012.

Overview

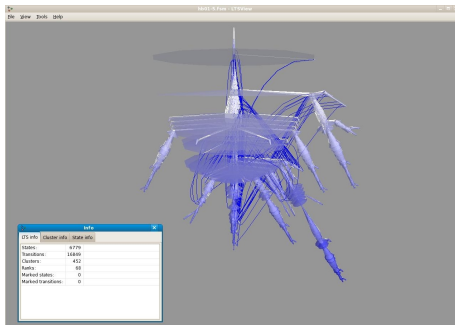
- ▶ Motivation
- ▶ Actions
- ▶ Multi-actions
- ▶ Alternative and sequential composition
- ▶ Deadlock
- ▶ Conditional and sum operator

From Processes to Their Algebra

Motivation

- ▶ in practice **graphical** representation is monstrously **big**

Example



From Processes to Their Algebra

Motivation

- ▶ in practice **graphical** representation is monstrously **big**
- ▶ **manipulating** and **analyzing** the graphical representation is virtually impossible

From Processes to Their Algebra

Motivation

- ▶ in practice **graphical** representation is monstrously **big**
- ▶ **manipulating** and **analyzing** the graphical representation is virtually impossible

Solution: use a compact **textual** presentation and **algebraic** rules for **manipulating** them.

Outline

Actions

Multi-Actions

Alternative and Sequential Composition

Conditional and Sum Operators

Actions

- ▶ **Atomic** building blocks of processes
- ▶ May represent: **internal** activities, **sending** messages, **receiving** messages and the result of a **synchronization**
- ▶ May take parameters, typically denoted by $a(d)$ (natural number, boolean, or any other **Abstract Data Type**)

Actions

- ▶ Examples:

- ▶ `act rcv_coin: Euro;`

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`(instance: snd_number(1))`

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- ▶ `act rcv_coin: Euro;`
- ▶ `act snd_number,rcv_number: Nat;`
`(instance: snd_number(1))`
- ▶ `act ack_number: Bool # Nat;`

Actions

- ▶ Examples:
 - ▶ `act rcv_coin: Euro;`
 - ▶ `act snd_number,rcv_number: Nat;`
`(instance: snd_number(1))`
 - ▶ `act ack_number: Bool # Nat;`
- ▶ N.B. actions are **not functions** (or procedures, in the programming languages' sense)

Outline

Actions

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Multi-Actions

- ▶ A number of actions happening **at the same time**
- ▶ Syntax:

$$\alpha ::= \tau \mid a \mid a(\vec{d}) \mid \alpha|\beta,$$

τ is the internal (**invisible**) action

- ▶ Auxiliary operators:
 - ▶ Removal of multi-actions $\alpha \setminus \beta$
 - ▶ Inclusion between multi-action $\alpha \sqsubseteq \beta$
 - ▶ Stripping data off $\underline{\alpha}$

Multi-Actions

Axioms (Part I: Basic axioms)

$$\text{MA1} \quad \alpha|\beta = \beta|\alpha$$

$$\text{MA2} \quad (\alpha|\beta)|\gamma = \alpha|(\beta|\gamma)$$

$$\text{MA3} \quad \alpha|\tau = \alpha$$

Multi-Actions

Axioms (Part II: Removal of multi-actions $\alpha \setminus \beta$)

$$\text{MD1} \quad \tau \setminus \alpha = \tau$$

$$\text{MD2} \quad \alpha \setminus \tau = \alpha$$

$$\text{MD3} \quad \alpha \setminus (\beta | \gamma) = (\alpha \setminus \beta) \setminus \gamma$$

$$\text{MD4} \quad (a(d) | \alpha) \setminus a(d) = \alpha$$

$$\text{MD5} \quad (a(d) | \alpha) \setminus b(e) = a(d) | (\alpha \setminus b(e)) \quad \text{if } a \neq b \text{ or } d \neq e$$

Multi-Actions

Axioms (Part III: Inclusion between multi-action $\alpha \sqsubseteq \beta$)

$$\text{MS1} \quad \tau \sqsubseteq \alpha = \text{true}$$

$$\text{MS2} \quad a \sqsubseteq \tau = \text{false}$$

$$\text{MS3} \quad a(d)|\alpha \sqsubseteq a(d)|\beta = \alpha \sqsubseteq \beta$$

$$\text{MS4} \quad a(d)|\alpha \sqsubseteq b(e)|\beta = a(d)|(\alpha \setminus b(e)) \sqsubseteq \beta \quad \text{if } a \not\equiv b \text{ or } d \not\equiv e$$

Multi-Actions

Axioms (Part IV: Stripping data off $\underline{\alpha}$)

$$\text{MAN1} \quad \underline{\tau} = \tau$$

$$\text{MAN2} \quad \underline{a(d)} = a$$

$$\text{MAN3} \quad \underline{\alpha|\beta} = \underline{\alpha}|\underline{\beta}$$

Exercises

$$(\alpha | \mathbf{a}(\mathbf{d})) \setminus \mathbf{a}(\mathbf{d}) = \alpha$$

4.2.2.1

4.2.2.3

Outline

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Multi-Actions

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Conditional and Sum Operators

Alternative composition

- ▶ Syntax: $p + q$
- ▶ Intuition: the process behaves as either p or q

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Axioms

$$\text{A1} \quad x + y = y + x$$

$$\text{A2} \quad x + (y + z) = (x + y) + z$$

$$\text{A3} \quad x + x = x$$

Alternative composition

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- ▶ Intuition: the process behaves as either p or q

Axioms

$$\text{A1} \quad x + y = y + x$$

$$\text{A2} \quad x + (y + z) = (x + y) + z$$

$$\text{A3} \quad x + x = x$$

Write $x \subseteq y$ for $x + y = y$.

Sequential composition

- ▶ Syntax: $p \cdot q$
- ▶ Intuition: the process behaves as p and upon termination of p , as q .

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- ▶ Syntax: $p \cdot q$
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Axioms

$$\text{A4} \quad (x + y) \cdot z = x \cdot z + y \cdot z$$

$$\text{A5} \quad (x \cdot y) \cdot z = x \cdot (y \cdot z)$$

(Delayable) Deadlock

- ▶ Syntax: δ
- ▶ Intuition: a process that cannot do anything but let the time pass

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Axioms

$$\text{A6} \quad \alpha + \delta = \alpha$$

$$\text{A7} \quad \delta \cdot x = \delta$$

Exercises

4.3.1

4.3.2

Outline

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Multi-Actions

Alternative and Sequential Composition

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Conditional operator

- ▶ Syntax: $c \rightarrow p \diamond q$, where c is of type `Bool`
- ▶ Intuition: behave as p if c is true, or otherwise, behave as q .

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Axioms

Cond1 $true \rightarrow x \diamond y = x$

Cond2 $false \rightarrow x \diamond y = y$

Sum operator

- ▶ Syntax: $\sum_{d:D} p(d)$
- ▶ Intuition: generalization of alternative composition (may behave as $p(d)$, for each value d of type D)

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Axioms

$$\text{SUM1} \quad \sum_{d:D} x = x$$

$$\text{SUM3} \quad \sum_{d:D} X(d) = \sum_{d:D} X(d) + X(d)$$

$$\text{SUM4} \quad \sum_{d:D} (X(d) + Y(d)) = \sum_{d:D} X(d) + \sum_{d:D} Y(d)$$

$$\text{SUM5} \quad (\sum_{d:D} X(d)) \cdot y = \sum_{d:D} X(d) \cdot y$$

Exercises

4.5.1

4.5.2