System Validation

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6. Parallel Processes: Theory

Parallel Processes

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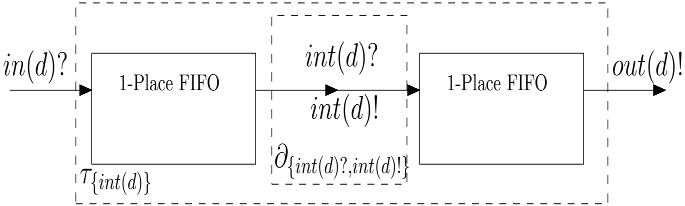
Overview

- Motivation
- Parallel Composition and Expansion Law
- Communication, Allow and Block
- Hiding

Outline

Motivating Examples

Decomposing a Two-Place Buffer



Parallel Composition

Semantics of $p \,||\, q$

Parallel Composition

Semantics of $p \,||\, q$

- $p \xrightarrow{\alpha} p'$, then $p \,||\, q \xrightarrow{\alpha} p' \,||\, q$,
- $q \xrightarrow{\alpha} q'$, then $p \,||\, q \xrightarrow{\alpha} p \,||\, q'$, and

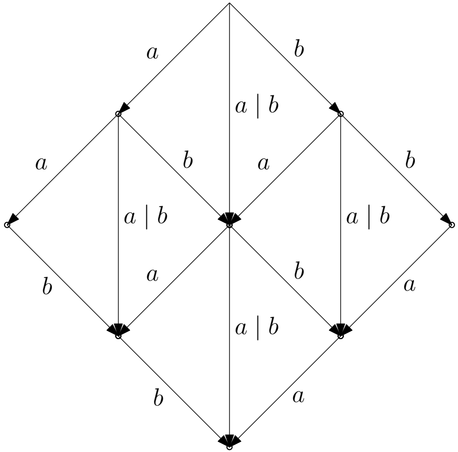
Parallel Composition

Semantics of $p \,||\, q$

- $p \xrightarrow{\alpha} p'$, then $p \,||\, q \xrightarrow{\alpha} p' \,||\, q$,
- $q \xrightarrow{\alpha} q'$, then $p \,||\, q \xrightarrow{\alpha} p \,||\, q'$, and
- $p \xrightarrow{\alpha} p'$ and $q \xrightarrow{\beta} q'$, then $p \,||\, q \xrightarrow{\alpha|\beta} p' \,||\, q'$.

Parallel Composition

$a \cdot a \,||\, b \cdot b$



The Saga of Axiomatizing Parallel Composition

Challenge

$$(Dish1 + Dish2) \parallel Coke \stackrel{?}{=} (Dish1 \parallel Coke) + (Dish2 \parallel Coke)$$

The Saga of Axiomatizing Parallel Composition

Challenge

$$(Dish1 + Dish2) \,||\, Coke \stackrel{?}{=} (Dish1 \,||\, Coke) + (Dish2 \,||\, Coke)$$

Faron Moller's Result

Parallel composition (without auxiliary operators) cannot be finitely axiomatized.

The Saga of Axiomatizing Parallel Composition

Bergstra and Klop's Invention

Axiomatize parallel composition using:

- Left merge ($\llfloor$): $p \stackrel{\alpha}{\to} p'$, then $p \llfloor q \stackrel{\alpha}{\to} p' \,||\, q$,
- Communication merge ($|$): $p \stackrel{\alpha}{\to} p'$ and $q \stackrel{\beta}{\to} q'$, then $p|q \stackrel{\alpha|\beta}{\to} p' \,||\, q'$.

The Saga of Axiomatizing Parallel Composition

Bergstra and Klop's Invention

Axiomatize parallel composition using:

- Left merge ($\lfloor\!\lfloor$): $p \xrightarrow{\alpha} p'$, then $p \lfloor\!\lfloor q \xrightarrow{\alpha} p' \,||\, q$,
- Communication merge ($|$): $p \xrightarrow{\alpha} p'$ and $q \xrightarrow{\beta} q'$, then $p|q \xrightarrow{\alpha|\beta} p' \,||\, q'$.

Expansion Law

$$p \,||\, q = (p \lfloor\!\lfloor q) + (q \lfloor\!\lfloor p) + (p|q)$$

The Saga of Axiomatizing Parallel Composition

$⌊⌊$ and $|$: Raisons d'être

$$(Dish1 + Dish2) ⌊⌊ Coke \underline{\leftrightarrow} (Dish1 ⌊⌊ Coke) + (Dish2 ⌊⌊ Coke)$$

$$(Dish1 + Dish2) | Coke \underline{\leftrightarrow} (Dish1 | Coke) + (Dish2 | Coke)$$

Parallel Composition: Axioms

Axioms for Untimed Processes

M	$x \parallel y = x \mathbin{\lfloor\!\lfloor} y + y \mathbin{\lfloor\!\lfloor} x + x \mid y$
LM1	$\alpha \mathbin{\lfloor\!\lfloor} x = \alpha \cdot x$
LM2	$\delta \mathbin{\lfloor\!\lfloor} x = \delta$
LM3	$\alpha \cdot x \mathbin{\lfloor\!\lfloor} y = \alpha \cdot (x \parallel y)$
LM4	$(x + y) \mathbin{\lfloor\!\lfloor} z = x \mathbin{\lfloor\!\lfloor} z + y \mathbin{\lfloor\!\lfloor} z$
LM5	$(\sum_{d:D} X(d)) \mathbin{\lfloor\!\lfloor} y = \sum_{d:D} X(d) \mathbin{\lfloor\!\lfloor} y$

Parallel Composition: Axioms

Axioms for Untimed Processes (Cont'd)

S1	$x \mid y = y \mid x$
S2	$(x \mid y) \mid z = x \mid (y \mid z)$
S3	$x \mid \tau = x$
S4	$\alpha \mid \delta = \delta$
S5	$(\alpha{\cdot}x) \mid \beta = \alpha \mid \beta{\cdot}x$
S6	$(\alpha{\cdot}x) \mid (\beta{\cdot}y) = \alpha \mid \beta{\cdot}(x \parallel y)$
S7	$(x + y) \mid z = x \mid z + y \mid z$
S8	$(\sum_{d:D} X(d)) \mid y = \sum_{d:D} X(d) \mid y$
TC1	$(x \mathbin{\lfloor\!\lfloor} y) \mathbin{\lfloor\!\lfloor} z = x \mathbin{\lfloor\!\lfloor} (y \parallel z)$
TC2	$x \mathbin{\lfloor\!\lfloor} \delta = x{\cdot}\delta$
TC3	$(x \mid y) \mathbin{\lfloor\!\lfloor} z = x \mid (y \mathbin{\lfloor\!\lfloor} z)$

The Saga of Axiomatizing Parallel Composition

Expanding Parallel Composition

$$(a \cdot a) \,||\, b \quad = \quad \mathsf{M}$$

The Saga of Axiomatizing Parallel Composition

Expanding Parallel Composition

$$
\begin{array}{lcr}
(a \cdot a) \,||\, b & = & \text{M} \\
((a \cdot a) \mathbin{\|\!\!\_} b) + (b \mathbin{\|\!\!\_} (a \cdot a)) + ((a \cdot a) | b) & = & \text{LM3,LM1,S5}
\end{array}
$$

The Saga of Axiomatizing Parallel Composition

Expanding Parallel Composition

$(a \cdot a) \parallel b$	$=$	M
$((a \cdot a) \mathbin{\|\!\lfloor} b) + (b \mathbin{\|\!\lfloor} (a \cdot a)) + ((a \cdot a) \mid b)$	$=$	LM3,LM1,S5
$(a \cdot (a \parallel b)) + (b \cdot a \cdot a) + ((a \mid b) \cdot a)$	$=$	M

The Saga of Axiomatizing Parallel Composition

Expanding Parallel Composition

$$
\begin{array}{lcr}
(a \cdot a) \parallel b & = & \text{M} \\
((a \cdot a) \mathbin{⌊\!⌊} b) + (b \mathbin{⌊\!⌊} (a \cdot a)) + ((a \cdot a) | b) & = & \text{LM3,LM1,S5} \\
(a \cdot (a \parallel b)) + (b \cdot a \cdot a) + ((a \mid b) \cdot a) & = & \text{M} \\
(a \cdot ((a \mathbin{⌊\!⌊} b) + (b \mathbin{⌊\!⌊} a) + (a|b))) + (b \cdot a \cdot a) + ((a \mid b) \cdot a) & = & \text{LM1,LM1}
\end{array}
$$

The Saga of Axiomatizing Parallel Composition

Expanding Parallel Composition

$$
\begin{array}{lcr}
(a \cdot a) \parallel b & = & \text{M} \\
((a \cdot a) \mathbin{\lfloor\!\lfloor} b) + (b \mathbin{\lfloor\!\lfloor} (a \cdot a)) + ((a \cdot a) | b) & = & \text{LM3,LM1,S5} \\
(a \cdot (a \parallel b)) + (b \cdot a \cdot a) + ((a \mid b) \cdot a) & = & \text{M} \\
(a \cdot ((a \mathbin{\lfloor\!\lfloor} b) + (b \mathbin{\lfloor\!\lfloor} a) + (a | b))) + (b \cdot a \cdot a) + ((a \mid b) \cdot a) & = & \text{LM1,LM1} \\
(a \cdot ((a \cdot b) + (b \cdot a) + (a \mid b))) + (b \cdot a \cdot a) + ((a \mid b) \cdot a) & &
\end{array}
$$

Outline

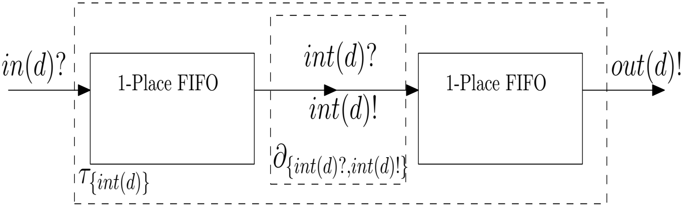
Parallel Composition

Communication and Allow

Hiding

Motivating Examples

Decomposing a Two-Place Buffer



Communication

Communication

- Idea: Give a name to the multi-action resulting from communication.
- Goal: To synchronize different parties.
- Example: $\Gamma_{\{snd|rcv \to comm\}}(snd(1) \,||\, \sum_{i:\mathbb{N}} rcv(i))$.

Communication

Axioms for Communication

C1	$\Gamma_C(\alpha) = \gamma_C(\alpha)$
C2	$\Gamma_C(\delta) = \delta$
C3	$\Gamma_C(x + y) = \Gamma_C(x) + \Gamma_C(y)$
C4	$\Gamma_C(x \cdot y) = \Gamma_C(x) \cdot \Gamma_C(y)$
C5	$\Gamma_C(\sum_{d:D} X(d)) = \sum_{d:D} \Gamma_C(X(d))$

Definition of γ_C

$$\gamma_{\emptyset}(\alpha) = \alpha$$

$$\gamma_{C_1 \cup C_2}(\alpha) = \gamma_{C_1}(\gamma_{C_2}(\alpha))$$

$$\gamma_{\{a_1 \mid \cdots \mid a_n \to b\}}(\alpha) = \begin{cases} b(d) \mid \gamma_{\{a_1 \mid \cdots \mid a_n \to b\}}(\alpha \setminus (a_1(d)|\cdots|a_n(d))) \\ \qquad \text{if } a_1(d)|\cdots|a_n(d) \sqsubseteq \alpha \text{ for some } d. \\ \alpha \qquad \text{otherwise.} \end{cases}$$

Definition of γ_C

$$
\begin{aligned}
\gamma_{\emptyset}(\alpha) &= \alpha \\
\gamma_{C_1 \cup C_2}(\alpha) &= \gamma_{C_1}(\gamma_{C_2}(\alpha)) \\
\gamma_{\{a_1 \mid \cdots \mid a_n \to b\}}(\alpha) &= \begin{cases} b(d) \mid \gamma_{\{a_1 \mid \cdots \mid a_n \to b\}}(\alpha \setminus (a_1(d) | \cdots | a_n(d))) \\ \qquad \text{if } a_1(d) | \cdots | a_n(d) \sqsubseteq \alpha \text{ for some } d. \\ \alpha \qquad \text{otherwise.} \end{cases}
\end{aligned}
$$

Examples

$$\gamma_{\{a|b \to c\}}((a(1) \mid b(1)) \mid a(2)) = c(1) \mid \gamma_{\{a|b \to c\}}(a(2)) = c(1) \mid a(2)$$

$$\gamma_{\{a|b \to c\}}(a(2) \mid b(2)) = c(2) \mid \gamma_{\{a|b \to c\}}\tau = c(2) \mid \tau = c(2)$$

$$\Gamma_{\{snd|rcv \rightarrow comm\}}(snd(1) \,||\, rcv(1)) \quad = \quad \mathsf{M}$$

$$\Gamma_{\{snd|rcv \to comm\}}(snd(1) \,||\, rcv(1)) \quad = \quad \text{M}$$

$$\Gamma_{\{snd|rcv \to comm\}}((snd(1) \,\|\!\!\_\, rcv(1)) + (rcv(1) \,\|\!\!\_\, snd(1)) + (snd(1)|rcv(1))) \quad = \quad \text{LM1,LM1}$$

$$
\begin{array}{lcr}
\Gamma_{\{snd|rcv \to comm\}}(snd(1) \,||\, rcv(1)) & = & \text{M} \\
\Gamma_{\{snd|rcv \to comm\}}((snd(1) \,⌊\!⌊\, rcv(1)) + (rcv(1) \,⌊\!⌊\, snd(1)) + & & \\
\quad (snd(1)|rcv(1))) & = & \text{LM1,LM1} \\
\Gamma_{\{snd|rcv \to comm\}}((snd(1) \cdot rcv(1)) + (rcv(1) \cdot snd(1)) + & & \\
\quad (snd(1)|rcv(1))) & = & \text{C3}
\end{array}
$$

$$
\begin{aligned}
&\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, rcv(1)) &= \quad & \text{M}\\
&\Gamma_{\{snd|rcv\to comm\}}((snd(1) \,⫾\, rcv(1)) + (rcv(1) \,⫾\, snd(1)) + \\
&\qquad (snd(1)|rcv(1))) &= \quad & \text{LM1,LM1}\\
&\Gamma_{\{snd|rcv\to comm\}}((snd(1)\cdot rcv(1)) + (rcv(1)\cdot snd(1)) + \\
&\qquad (snd(1)|rcv(1))) &= \quad & \text{C3}\\
&\Gamma_{\{snd|rcv\to comm\}}(snd(1)\cdot rcv(1)) + \\
&\qquad \Gamma_{\{snd|rcv\to comm\}}(rcv(1)\cdot snd(1)) + \\
&\qquad \Gamma_{\{snd|rcv\to comm\}}(snd(1)|rcv(1))) &= \quad & \text{C4}
\end{aligned}
$$

$$
\begin{array}{lr}
\Gamma_{\{snd|rcv \to comm\}}(snd(1) \,||\, rcv(1)) & = \quad M \\
\Gamma_{\{snd|rcv \to comm\}}((snd(1) \,\|\!\_\, rcv(1)) + (rcv(1) \,\|\!\_\, snd(1)) + & \\
\quad (snd(1)|rcv(1))) & = \quad LM1,LM1 \\
\Gamma_{\{snd|rcv \to comm\}}((snd(1) \cdot rcv(1)) + (rcv(1) \cdot snd(1)) + & \\
\quad (snd(1)|rcv(1))) & = \quad C3 \\
\Gamma_{\{snd|rcv \to comm\}}(snd(1) \cdot rcv(1)) + & \\
\quad \Gamma_{\{snd|rcv \to comm\}}(rcv(1) \cdot snd(1)) + & \\
\quad \Gamma_{\{snd|rcv \to comm\}}(snd(1)|rcv(1))) & = \quad C4 \\
(\Gamma_{\{snd|rcv \to comm\}}(snd(1)) \cdot \Gamma_{\{snd|rcv \to comm\}}(rcv(1))) + & \\
\quad (\Gamma_{\{snd|rcv \to comm\}}(rcv(1)) \cdot \Gamma_{\{snd|rcv \to comm\}}(snd(1))) + & \\
\quad \gamma_{\{snd|rcv \to comm\}}(snd(1)|rcv(1)) & = \quad C1{\times}3
\end{array}
$$

$$
\begin{array}{lcr}
\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, rcv(1)) & = & \text{M} \\
\Gamma_{\{snd|rcv\to comm\}}((snd(1) \,\lfloor\!\lfloor\, rcv(1)) + (rcv(1) \,\lfloor\!\lfloor\, snd(1)) + & & \\
\quad (snd(1)|rcv(1))) & = & \text{LM1,LM1} \\
\Gamma_{\{snd|rcv\to comm\}}((snd(1) \cdot rcv(1)) + (rcv(1) \cdot snd(1)) + & & \\
\quad (snd(1)|rcv(1))) & = & \text{C3} \\
\Gamma_{\{snd|rcv\to comm\}}(snd(1) \cdot rcv(1)) + & & \\
\quad \Gamma_{\{snd|rcv\to comm\}}(rcv(1) \cdot snd(1)) + & & \\
\quad \Gamma_{\{snd|rcv\to comm\}}(snd(1)|rcv(1))) & = & \text{C4} \\
(\Gamma_{\{snd|rcv\to comm\}}(snd(1)) \cdot \Gamma_{\{snd|rcv\to comm\}}(rcv(1))) + & & \\
\quad (\Gamma_{\{snd|rcv\to comm\}}(rcv(1)) \cdot \Gamma_{\{snd|rcv\to comm\}}(snd(1))) + & & \\
\quad \gamma_{\{snd|rcv\to comm\}}(snd(1)|rcv(1)) & = & \text{C1}\times 3 \\
(\gamma_{\{snd|rcv\to comm\}}(snd(1)) \cdot \gamma_{\{snd|rcv\to comm\}}(rcv(1))) + & & \\
\quad (\gamma_{\{snd|rcv\to comm\}}(rcv(1)) \cdot \gamma_{\{snd|rcv\to comm\}}(snd(1))) + & & \\
\quad \gamma_{\{snd|rcv\to comm\}}(snd(1)|rcv(1)) & = & \text{Def. of } \gamma
\end{array}
$$

$$
\begin{array}{lcr}
\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, rcv(1)) & = & \text{M} \\
\Gamma_{\{snd|rcv\to comm\}}((snd(1) \,⌊\!⌊\, rcv(1)) + (rcv(1) \,⌊\!⌊\, snd(1)) + & & \\
\quad (snd(1)|rcv(1))) & = & \text{LM1,LM1} \\
\Gamma_{\{snd|rcv\to comm\}}((snd(1) \cdot rcv(1)) + (rcv(1) \cdot snd(1)) + & & \\
\quad (snd(1)|rcv(1))) & = & \text{C3} \\
\Gamma_{\{snd|rcv\to comm\}}(snd(1) \cdot rcv(1)) + & & \\
\quad \Gamma_{\{snd|rcv\to comm\}}(rcv(1) \cdot snd(1)) + & & \\
\quad \Gamma_{\{snd|rcv\to comm\}}(snd(1)|rcv(1))) & = & \text{C4} \\
(\Gamma_{\{snd|rcv\to comm\}}(snd(1)) \cdot \Gamma_{\{snd|rcv\to comm\}}(rcv(1))) + & & \\
\quad (\Gamma_{\{snd|rcv\to comm\}}(rcv(1)) \cdot \Gamma_{\{snd|rcv\to comm\}}(snd(1))) + & & \\
\quad \gamma_{\{snd|rcv\to comm\}}(snd(1)|rcv(1)) & = & \text{C1}\times 3 \\
(\gamma_{\{snd|rcv\to comm\}}(snd(1)) \cdot \gamma_{\{snd|rcv\to comm\}}(rcv(1))) + & & \\
\quad (\gamma_{\{snd|rcv\to comm\}}(rcv(1)) \cdot \gamma_{\{snd|rcv\to comm\}}(snd(1))) + & & \\
\quad \gamma_{\{snd|rcv\to comm\}}(snd(1)|rcv(1)) & = & \text{Def. of } \gamma \\
(snd(1) \cdot rcv(1)) + (rcv(1) \cdot snd(1)) + comm(1) & &
\end{array}
$$

Allow

- Idea: To enforce synchronization by only allowing for the results of communication.
- Goal: To synchronize different parties.
- Example: $\nabla_{comm}\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, \sum_{i:\mathbb{N}} rcv(i))$.

Allow

Axioms for Allow

V1	$\nabla_V(\alpha) = \alpha$	if $\underline{\alpha} \in V \cup \{\tau\}$
V2	$\nabla_V(\alpha) = \delta$	if $\underline{\alpha} \notin V \cup \{\tau\}$
V3	$\nabla_V(\delta) = \delta$	
V4	$\nabla_V(x + y) = \nabla_V(x) + \nabla_V(y)$	
V5	$\nabla_V(x \cdot y) = \nabla_V(x) \cdot \nabla_V(y)$	
V6	$\nabla_V(\sum_{d:D} X(d)) = \sum_{d:D} \nabla_V(X(d))$	
TV1	$\nabla_V(\nabla_W(x)) = \nabla_{V \cap W}(x)$	

$$\nabla_{\{comm\}}\Gamma_{\{snd|rcv\to comm\}}(snd(1)\,||\,rcv(1)) \qquad = \text{Prev. Ex.}$$

$$\nabla_{\{comm\}}\Gamma_{\{snd|rcv \to comm\}}(snd(1) \,||\, rcv(1)) \quad = \text{Prev. Ex.}$$

$$\nabla_{\{comm\}}((snd(1) \cdot rcv(1)) + (rcv(1) \cdot snd(1)) + comm(1)) \quad = \quad \text{V4}$$

$$\nabla_{\{comm\}}\Gamma_{\{snd|rcv \to comm\}}(snd(1) \,||\, rcv(1)) \quad = \text{Prev. Ex.}$$

$$\nabla_{\{comm\}}((snd(1)\cdot rcv(1)) + (rcv(1)\cdot snd(1)) + comm(1)) \quad = \quad V4$$

$$\nabla_{\{comm\}}(snd(1)\cdot rcv(1)) + \nabla_{\{comm\}}(rcv(1)\cdot snd(1)) + \nabla_{\{comm\}}(comm(1)) \quad = \quad V5$$

$$
\begin{aligned}
&\nabla_{\{comm\}}\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, rcv(1)) &&= \text{Prev. Ex.}\\
&\nabla_{\{comm\}}((snd(1)\cdot rcv(1))+ \\
&\quad (rcv(1)\cdot snd(1)) + comm(1)) &&= \quad \text{V4}\\
&\nabla_{\{comm\}}(snd(1)\cdot rcv(1))+ \\
&\quad \nabla_{\{comm\}}(rcv(1)\cdot snd(1)) + \nabla_{\{comm\}}(comm(1)) &&= \quad \text{V5}\\
&(\nabla_{\{comm\}}(snd(1))\cdot \nabla_{\{comm\}}(rcv(1)))+ \\
&\quad (\nabla_{\{comm\}}(rcv(1))\cdot \nabla_{\{comm\}}(snd(1))) + \nabla_{\{comm\}}(comm(1)) &&= \quad \text{V1,V5}
\end{aligned}
$$

$$
\begin{aligned}
&\nabla_{\{comm\}}\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, rcv(1)) &&= \text{Prev. Ex.}\\
&\nabla_{\{comm\}}((snd(1)\cdot rcv(1))+ \\
&\quad (rcv(1)\cdot snd(1)) + comm(1)) &&= \text{V4}\\
&\nabla_{\{comm\}}(snd(1)\cdot rcv(1))+ \\
&\quad \nabla_{\{comm\}}(rcv(1)\cdot snd(1)) + \nabla_{\{comm\}}(comm(1)) &&= \text{V5}\\
&(\nabla_{\{comm\}}(snd(1))\cdot \nabla_{\{comm\}}(rcv(1)))+ \\
&\quad (\nabla_{\{comm\}}(rcv(1))\cdot \nabla_{\{comm\}}(snd(1))) + \nabla_{\{comm\}}(comm(1)) &&= \text{V1,V5}\\
&\delta\cdot\delta + \delta\cdot\delta + comm(1) &&= \text{A7,A3}
\end{aligned}
$$

$$
\begin{array}{lr}
\nabla_{\{comm\}}\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, rcv(1)) & =\text{Prev. Ex.} \\
\nabla_{\{comm\}}((snd(1)\cdot rcv(1))+ & \\
\quad (rcv(1)\cdot snd(1)) + comm(1)) & = \quad \text{V4} \\
\nabla_{\{comm\}}(snd(1)\cdot rcv(1))+ & \\
\quad \nabla_{\{comm\}}(rcv(1)\cdot snd(1)) + \nabla_{\{comm\}}(comm(1)) & = \quad \text{V5} \\
(\nabla_{\{comm\}}(snd(1))\cdot \nabla_{\{comm\}}(rcv(1)))+ & \\
\quad (\nabla_{\{comm\}}(rcv(1))\cdot \nabla_{\{comm\}}(snd(1))) + \nabla_{\{comm\}}(comm(1)) & = \quad \text{V1,V5} \\
\delta\cdot\delta+\delta\cdot\delta+comm(1) & = \quad \text{A7,A3} \\
\delta+comm(1) & = \quad \text{A1,A6}
\end{array}
$$

$$
\begin{array}{lr}
\nabla_{\{comm\}}\Gamma_{\{snd|rcv\to comm\}}(snd(1) \,||\, rcv(1)) & =\text{Prev. Ex.} \\
\nabla_{\{comm\}}((snd(1)\cdot rcv(1))+ & \\
\quad (rcv(1)\cdot snd(1)) + comm(1)) & = \quad \text{V4} \\
\nabla_{\{comm\}}(snd(1)\cdot rcv(1))+ & \\
\quad \nabla_{\{comm\}}(rcv(1)\cdot snd(1)) + \nabla_{\{comm\}}(comm(1)) & = \quad \text{V5} \\
(\nabla_{\{comm\}}(snd(1))\cdot \nabla_{\{comm\}}(rcv(1)))+ & \\
\quad (\nabla_{\{comm\}}(rcv(1))\cdot \nabla_{\{comm\}}(snd(1))) + \nabla_{\{comm\}}(comm(1)) & = \quad \text{V1,V5} \\
\delta\cdot\delta+\delta\cdot\delta+comm(1) & = \quad \text{A7,A3} \\
\delta+comm(1) & = \quad \text{A1,A6} \\
comm(1) &
\end{array}
$$

Outline

Hiding

- Idea: To rename the irrelevant actions to τ.
- Example:
  $\tau_{\{comm\}} \nabla_{comm,show} \Gamma_{\{snd|rcv \to comm\}}(snd(1) \,||\, \sum_{i:\mathbb{N}} rcv(i).show(i))$.
- N.B.: Use communication, allow and hiding only the above-specified order.

Hiding

Axioms for Hiding

H1	$\tau_I(\tau) = \tau$	
H2	$\tau_I(a(d)) = \tau$	if $a \in I$
H3	$\tau_I(a(d)) = a(d)$	if $a \notin I$
H4	$\tau_I(\alpha\|\beta) = \tau_I(\alpha)\|\tau_I(\beta)$	
H5	$\tau_I(\delta) = \delta$	
H6	$\tau_I(x+y) = \tau_I(x) + \tau_I(y)$	
H7	$\tau_I(x\cdot y) = \tau_I(x)\cdot\tau_I(y)$	
H8	$\tau_I(\sum_{d:D} X(d)) = \sum_{d:D} \tau_I(X(d))$	
H10	$\tau_I(\tau_{I'}(x)) = \tau_{I\cup I'}(x)$	