Flight and Orbital Mechanics

Solutions

Solutions - Exam flight and orbital mechanics – February 2012

Question 1

The decision speed is selected such that when a this speed an engine failure is recognized, the pilot is able to abort the take-off and make a full stop on the runway (accelerate stop), or to continue take-off to the screen height with one engine out (accelerate climb), in the same distance. If the runway is wet, then the friction coefficient is lower. This means that the accelerate stop distance decreases. The accelerate climb distance however is not affected. A lower decision speed will therefore result in equal accelerate climb and accelerate stop distances. So, the decision speed should be lower.

Question 2

$$L = W \Rightarrow$$

$$P_r = DV = \frac{C_D}{C_L} W \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{W^3}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

So minimum power occurs when C_L^3 / C_D^2 is at its maximum.

$$\frac{d\left(\frac{C_L^3}{C_D^2}\right)}{dC_L} = 0$$

$$\frac{C_D^2 \cdot 3C_L^2 - C_L^3 \cdot 2C_D \frac{dC_D}{dC_L}}{C_D^4} = 0$$

$$\left.\begin{array}{l} \frac{3}{2}\frac{C_D}{C_L} = \frac{dC_D}{dC_L} \\ C_D = C_{D_0} + k_1 C_L + k_2 C_L^2 \\ \frac{dC_D}{dC_L} = k_1 + 2k_2 C_L \end{array}\right\} \Rightarrow 3C_{D_0} + 3k_1 C_L + 3k_2 C_L^2 = 2k_1 C_L + 4k_2 C_L^2$$

$$C_L^2 - \frac{k_1}{k_2} C_L - 3\frac{C_{D_0}}{k_2} = 0$$

$$C_L = \frac{\frac{k_1}{k_2} \pm \sqrt{\frac{k_1^2}{k_2^2} + 12\frac{C_{D_0}}{k_2}}}{2}$$

Check, if k_1 is equal to zero then this equation simplifies to the well-known relation:

$$C_L = \sqrt{3\frac{C_{D_0}}{k_2}} = \sqrt{3C_{D_0}\pi Ae}$$

Question 3

The horizontal airspeed $V_{a,h}$ follows from Pythagoras:

$$V_{a,h} = \sqrt{100^2 - 20^2} = 98 \text{ [m/s]}$$

The ground speed equals the horizontal airspeed minus the headwind

$$V_g = V_{a,h} - V_w = 98 - 15 = 83 \text{ [m/s]}$$

Question 4

a.

quasi-rectilinear; flight in which the center of gravity of the airplane travels almost along a straight line ($d\gamma/dt \cong 0$)

Symmetric flight; flight in which both the angle of sideslip is zero and the plane of symmetry of the airplane is perpendicular to the horizontal plane of the earth.

b.

See lecture 1 slide 19

c.

The general equations of motion can be derived from the FBD and KD of question b.

$$\frac{W}{g}\frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma$$
$$\frac{W}{g}V\frac{d\gamma}{dt} = L + T\sin\alpha_T - W\cos\gamma$$

The following assumptions can be applied.

$\cos\gamma \approx 1$ (small flight path angle)
$\sin\gamma \neq 0$ (small flight path angle)
$\frac{d\gamma}{dt} \approx 0$ (quasi-rectilinear flight)
$\alpha_T = 0$ (assumption)

Thus:

$$\frac{W}{g}\frac{dV}{dt} = T - D - W\sin\gamma$$
$$L = W$$

d.

Multiply the first equation of motion with V.

$$\frac{W}{g}V\frac{dV}{dt} = TV - DV - WV\sin\gamma$$

Simplify with the following relations:

$$P_a = TV$$
$$P_r = DV$$
$$V \sin\gamma = RC$$

This yields the power equation

$$\frac{P_a - P_r}{W} = RC + \frac{V}{g}\frac{dV}{dt}$$

e.

In steady flight dV/dt is equal to zero. Hence;

$$\frac{P_a - P_r}{W} = RC_{st}$$

The term dV/dt can be rewritten:

$$\frac{V}{g}\frac{dV}{dt} = \frac{V}{g}\frac{dV}{dh}\frac{dh}{dt} = \frac{V}{g_0}\frac{dV}{dH}\frac{dH}{dt} = \frac{V}{g_0}\frac{dV}{dH}RC$$

Combine everything:

$$\frac{P_a - P_r}{W} = RC + \frac{V}{g}\frac{dV}{dt}$$
$$RC_{st} = RC + \frac{V}{g_0}\frac{dV}{dH}RC$$
$$\frac{RC}{RC_{st}} = \frac{1}{1 + \frac{V}{g_0}\frac{dV}{dH}}$$

f.

Introduce the Mach number in the equation

$$V = M \cdot a = M\sqrt{\gamma RT}$$

$$\frac{RC}{RC_{st}} = \frac{1}{1 + \frac{V}{g_0}\frac{dV}{dH}} = \frac{1}{1 + \frac{1}{2g_0}\frac{dV^2}{dH}} = \frac{1}{1 + \frac{1}{2g_0}\frac{d\left(M^2\gamma RT\right)}{dH}}$$

The Mach number, the ratio of specific heats of air (γ) and the specific gas constant of air (R) are all constant.

$$\frac{RC}{RC_{st}} = \frac{1}{1 + \frac{M^2\gamma R}{2g_0}\frac{dT}{dH}} = \frac{1}{1 + \frac{M^2 \cdot 1.4 \cdot 287.05}{2 \cdot 9.80665} \cdot -0.0065} = \frac{1}{1 - 0.133 \cdot M^2}$$

g.

$M = 0.8$

$$\frac{RC}{RC_{st}} = \frac{1}{1 - 0.133 \cdot M^2} = \frac{1}{1 - 0.133 \cdot 0.8^2} = 1.093$$

h.

A climbing flight at constant Mach number is a decelerating flight since the speed of sound decreases with altitude in the troposphere. Hence, additional energy is available to climb. The rate of climb is therefore 9.3% higher than the maximum rate of climb in steady flight. (kinetic energy decreases and potential energy increases)

Questions and answers exam ae2-104 d.d. February 2, 2012.

Question Space-1:

a) (2 points) Compute the dimension of the Sphere of Influence of the Earth (when the Sun is considered as the perturbing body). The SoI is given by the following general equation:

$$r_{SoI} = r_{3rd} \left(\frac{M_{main}}{M_{3rd}} \right)^{0.4}$$

b) (4 points) What is the value of the radial attraction exerted by the Earth at this distance? (if you were unable to make the question (a), use a value of 1×10^6 km for this position).

c) (4 points) What is the effective gravitational acceleration by the Sun at this position? Assume that Earth, Sun and this point on the SoI are on a straight line.

d) (4 points) What is the relative perturbation of the solar attraction, compared to that of the main attraction of the Earth?

Data: 1 AU = 149.6×10^6 km, $M_{Sun} = 2.0 \times 10^{30}$ kg, $M_{Earth} = 6.0 \times 10^{24}$ kg, $\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2, $\mu_{Earth} = 398600$ km^3/s^2.

Answers

a) 924230 km

b) acc=μ/r^2 -> acc=4.666×10^{-7} km/s^2 = 4.666×10^{-4} m/s^2

c) $acc_{direct} = \mu/(1AU\text{-}SoI)^2 = 6.00376 \times 10^{-6}$ km/s^2; $acc_{onEarth} = \mu/(1AU)^2 = 5.92981 \times 10^{-6}$ km/s^2; $acc_{effective} = acc_{direct} - acc_{onEarth} = 7.4 \times 10^{-8}$ km/s^2 = 7.4×10^{-5} m/s^2

d) 15.8 %

Question Space-2:

One of the main issues for designing a space mission is the occurrence of eclipses.

a) (3 points) What is the definition of an eclipse?

b) (4 points) An eclipse has consequences for at least 3 subsystems of the satellite. What are these, and discuss the consequences for each one briefly (about 2 lines each).

c) (4 points) What are the two conditions that determine whether an Earth satellite is in eclipse or not? Give the mathematical conditions in a sketch and discuss each one briefly.

d) (3 points) One of the conditions can translate into the so-called shadow function as given below. Discuss the meaning of the various elements in the equation, and discuss the use of this equation.

$$S(\theta) = R_e^2 (1 + e\cos\theta)^2 + p^2 (\bar{\alpha}\cos\theta + \bar{\beta}\sin\theta)^2 - p^2$$

Answers:

a) situation in which satellite not directly illuminated by Sun
b) power system, thermal control, attitude control, operations instruments, optical tracking from Earth, ….
c) satellite in front of Earth, and component satellite position perpendicular to direction to Sun must be smaller than Earth radius. See sheets for sketch.
d) see sheets for explanation of R_e, e, θ, p, αbar, βbar. Used to assess whether satellite is in eclipse or not.

Question Space-3:

Consider a transfer from a circular parking orbit at 185 km and i =5° (i.e., launch from Kourou) to the geostationary orbit (GEO; $T=23^h56^m4^s$, e=0, i=0°).

a) (2 points) Compute the radius of the geostationary orbit.
b) (2 points) Compute the circular velocities in the original orbit and in the target orbit.
c) (4 points) Compute the velocities in the pericenter and the apocenter of a Hohmann transfer orbit (i.e., assuming that the initial and target orbits are coplanar – which they are not in reality).
d) (2 points) Compute the total ΔV that would be required for the orbit raising, assuming that we fly a Hohmann transfer (again, the two orbits would be assumed to be coplanar).
e) (4 points) Compute the ΔV that would be required to only change the inclination of the original parking orbit to that of the GEO.
f) (2 points) Compute the total ΔV if the sequence of maneuvers was (1) full inclination change (only) in initial orbit, and (2) Hohmann transfer to GEO.
g) (2 points) Compute the total ΔV if the sequence of maneuvers was (1) Hohmann transfer to GEO altitude (without changing inclination), and (2) full inclination change at GEO altitude.
h) (4 points) Compute the total ΔV if the full inclination change is combined with the 2nd step of the Hohmann transfer (i.e., the Hohmann transfer orbit still has an inclination of 5°).
i) (2 points) Discuss the results of the computations of total ΔV (i.e., subquestions (f)-(h)).

Data: $\mu_{Earth} = 398600\ km^3/s^2$, $R_{Earth} = 6378.137$ km

Answers

a) $T=2\pi\sqrt{(a^3/\mu)}$ -> $a_{GEO}=r_{GEO}=42164.14$ km.
b) $V_c=\sqrt{(\mu/r)}$ -> $V_{c,185} = 7.793$ km/s, $V_{c,GEO} = 3.075$ km/s.
c) $a_H=(r_a+r_p)/2$ -> $a_H=24363.64$ km. $V^2/2-\mu/r=-\mu/(2a)$ -> $V_{H,per}=10.252$ km/s, $V_{H,apo}=1.596$ km/s.
d) $\Delta V_1=V_{H,per}-V_{c,185}=2.459$ km/s; $\Delta V_2=V_{c,geo}-V_{H,apo}=1.479$; $\Delta V_H=\Delta V_1+\Delta V_2=3.938$ km/s.
e) $(\Delta V)^2=V_1^2+V_2^2-2V_1V_2\cos(\Delta i)$ -> $\Delta V_{dogleg,185} = 0.680$ km/s.
f) $\Delta V_{total} = \Delta V_H+\Delta V_{dogleg,185} = 4.618$ km/s.
g) $\Delta V_{dogleg,geo}=0.268$ km/s; $\Delta V_{total} = \Delta V_H+\Delta V_{dogleg,geo} = 4.206$ km/s.
h) $\Delta V_1=V_{H,per}-V_{c,185}=2.459$ km/s; $(\Delta V_2)^2=V_{H,apo}^2+V_{c,geo}^2-2V_{H,apo}V_{c,geo}\cos(\Delta i)$ -> $\Delta V_2 = 1.492$ km/s; $\Delta V_{tot}=\Delta V_1+\Delta V_2=3.951$ km/s.

i) performing inclination changes at high altitudes (where velocities are low) is more advantageous than at low altitudes (where velocities are high); doing it in combination with other in-plane maneuvers is even better.