Flight and Orbital Mechanics

Solutions

TU Delft

Solutions Exam AE2104 – January 2012

Question 1: b

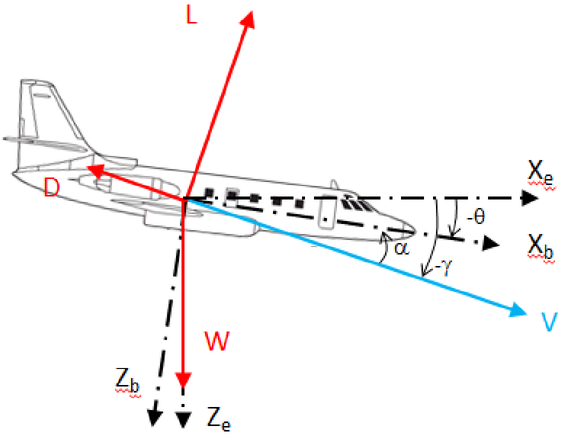
Question 2: b

Question 3: a

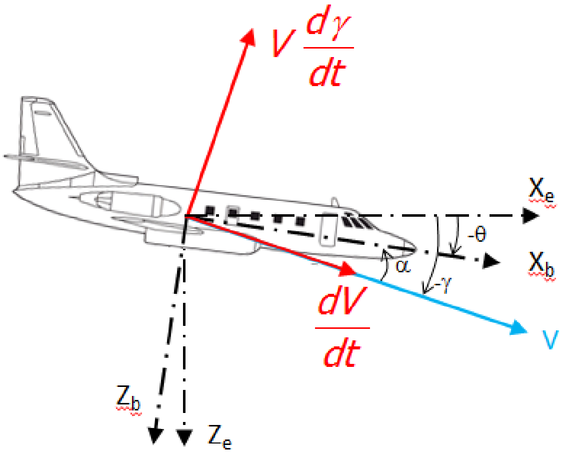
Question 4: d

Question 5a, b, c.:

Free Body Diagram



Kinetic Diagram



Note:

1. the pitch attitude and flight path angle are positive upwards, hence the minus sign in the diagrams)
2. It is also allowed to draw a climbing flight (with positive flight path angle and pitch attitude). This will result in the same equations of motion

Equations of motion

$$\sum F_{//V}: \frac{W}{g}\frac{dV}{dt} = -D + W\sin(-\gamma)$$

$$\sum F_{\perp V}: \frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos(-\gamma)$$

Hence

$$\frac{W}{g}\frac{dV}{dt} = -D - W\sin\gamma$$

$$\frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma$$

The only assumptions made are:

- Thrust equals zero (gliding flight)
- Symmetric flight (no turns and no sideslip angle, hence only two equations of motion)

Power equation

$$\frac{W}{g}\frac{dV}{dt}V = -DV - WV\sin\gamma$$
$$\frac{W}{g}V\frac{dV}{dt} = -P_r - WRC$$
$$\frac{-P_r}{W} = \frac{V}{g}\frac{dV}{dt} + RC$$

Question 5d:

Unsteady flight:

$$\frac{-P_r}{W} = \frac{V}{g}\frac{dV}{dt} - RD \qquad (1)$$

Steady flight:

$$\frac{-P_r}{W} = -RD_{steady} \qquad (2)$$

Combine these two equations:

$$-RD_{steady} = \frac{V}{g}\frac{dV}{dt} - RD$$
$$-RD_{steady} = \frac{V}{g}\frac{dV}{dH}\frac{dH}{dt} - RD$$
$$-RD_{steady} = -\frac{V}{g}\frac{dV}{dH}RD - RD$$
$$-RD_{steady} = -RD\left(1 + \frac{V}{g}\frac{dV}{dH}\right)$$

This gives the final result:

$$\frac{RD}{RD_{steady}} = \frac{1}{1 + \frac{V}{g}\frac{dV}{dH}}$$

Question 5e:

Equivalent airspeed is defined as follows:

$$\tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho_0 V_E^2$$

$$V_E^2 = \frac{\rho}{\rho_0}V^2$$

With this definition, the ratio can be rewritten

$$\frac{RD}{RD_{steady}} = \frac{1}{1+\frac{1}{2g}\frac{dV^2}{dH}} = \frac{1}{1+\frac{1}{2g}\frac{d\left(\frac{\rho_0}{\rho}V_E^2\right)}{dH}} = \frac{1}{1+\frac{\rho_0 V_E^2}{2g}\frac{d\left(\frac{1}{\rho}\right)}{dH}}$$

Question 5f:

$$\frac{RD}{RD_{steady}} = \frac{1}{1+\frac{\rho_0 V_E^2}{2g}\frac{d\left(\frac{1}{\rho}\right)}{dH}} = \frac{1}{1+\frac{1.225\cdot 150^2}{2\cdot 9.80665}\cdot 1.45\cdot 10^{-4}} = 0.83$$

So, the actual rate of descent is smaller than the steady rate of descent. Flying a descent at constant equivalent airspeed results in a decreasing true airspeed and thus decelerating flight. This means that part of the kinetic energy is used to increase potential energy. Hence, the plane will stay longer in the air than flight at a constant true airspeed.

Question 5g.

$$s = \int_{H_1}^{H_2} \frac{dH}{\sin\gamma}$$

$$\frac{RD}{RD_{steady}} = \frac{1}{1+\frac{V}{g}\frac{dV}{dH}} = \frac{\sin\gamma}{\sin\gamma_s}$$

Combine these two equations:

$$ds = \frac{dH}{\sin\gamma} = \frac{1+\frac{V}{g}\frac{dV}{dH}}{\sin\gamma_s}dH = \frac{1}{\sin\gamma_s}\left(dH + \frac{1}{2g}dV^2\right)$$

Energy height principle

$$H_e = H + \frac{V^2}{2g}$$

Hence

$$dH_e = dH + \frac{1}{2g}dV^2$$

Thus:

$$ds = \frac{dH_e}{\sin\gamma_s}$$

Integration of this final equation results in the equation provided:

$$s = \int_{H_{e_1}}^{H_{e2}} \frac{dH_e}{\sin\gamma_s}$$

Question 5h.

The aircraft flies at constant equivalent airspeed. The equation of motion parallel to the airspeed vector states:

$$-D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

In steady flight, this equation becomes:

$$-D - W\sin\gamma_s = 0$$

$$\sin\gamma_s = -\frac{D}{W} = -\frac{D}{W}\frac{L}{L} = -\frac{D}{W}\frac{W\cos\gamma_s}{L} = -\frac{C_D}{C_L}\cos\gamma_s \approx -\frac{C_D}{C_L}$$

The assumption made here is the 'small angle approximation' (the cosine of the flight path angle is approximately zero, the sine of the flight path angle is nonzero!)

$$s = \int_{H_{e_1}}^{H_{e2}} \frac{dH_e}{\sin\gamma_s} = \int_{H_{e_1}}^{H_{e2}} -\frac{C_D}{C_L} dH_e$$

When flying at constant equivalent airspeed, the lift coefficient (and drag coefficient) is constant. See proof below:

$$L = W\cos\gamma \approx W$$

$$C_L \tfrac{1}{2}\rho V^2 S = W$$

$$C_L = \frac{W}{\tfrac{1}{2}\rho V^2 S} = \frac{W}{\tfrac{1}{2}\rho_0 V_E^2 S}$$

Thus, the distance can be calculated:

$$s = \int_{H_{e_1}}^{H_{e2}} -\frac{C_D}{C_L} dH_e = -\frac{C_D}{C_L}\int_{H_{e_1}}^{H_{e2}} dH_e = -\frac{C_D}{C_L}\left(H_{e_2} - H_{e_1}\right) = -\frac{C_D}{C_L}\left(H_2 + \frac{V_2^2}{2g} - H_1 - \frac{V_1^2}{2g}\right)$$

The initial and final altitude are fixed and C_L and C_D are constant. Flying a descent at constant equivalent airspeed results in a decreasing true airspeed and thus decelerating flight. Thus $V_2 < V_1$

Thus, the distance flown with a constant equivalent airspeed is larger than for a steady symmetric descent. In essence, part of the kinetic energy is used to stay in the air longer

Question 6a:

$i = 17.4°$

Question 6b:

$i = 147.5°$

Question 6c:

$T_{orbit} = 6207$ s = 103.45 min.

Question 6d:

Question a): $T_{repeat} = 478\ T_{orbit}$ = 49450.6 min = 824.18 hr = 34.34 days. Question b): $T_{repeat} = 493\ T_{orbit}$ = 51002.4 min = 850.04 hr = 35.42 days. Earth's flattening has opposite effects on orbits with inclination < 90° (J_2 precession acts in Westward direction) and orbits with inclination > 90° (J_2 precession acts in Eastward direction).

Question 7:

1. The decay rate of both satellites is more or less equal, since they orbit at about the same altitude, and their ballistic coefficient is more or less identical.
2. Solar maximum in beginning and at end of interval causes high decay rate; solar minimum in center causes small decay rate.
3. 3. Mir is active spacecraft, regular boosts to increase altitude again. GFZ-1 is a fully passive satellite -> no corrections for orbit loss.
4. Mir is purposely lowered in about 1996, to allow for a rendezvous with a fully loaded vehicle from Earth.

Question 8a:
$a = 0.6935$ AU = 103.7×10^6 km; $e = 0.442$

Question 8b:
$T_H = 9.11\times10^6$ sec = 0.289 yrs

Question 8c:
$V_{Earth} = 29.784$ km/s; $V_{Mercury} = 47.877$ km/s

Question 8d:
$V_{sat,helio,Earth} = 22.249$ km/s, $V_{sat,helio,Mercury} = 57.492$ km/s. → V_∞(Earth) = 7.535 km/s; V_∞(Mercury) = 9.615 km/s

Question 8e:
V_c(in Earth parking orbit) = 7.613 km/s; V_c(in Mercury parking orbit) = 2.738 km/s

Question 8e:
$V_{peri,hyperbola,Earth} = 13.141$ km/s, $V_{peri,hyperbola,Mercury} = 10.365$ km/s. ΔV(Earth) = 5.528 km/s; ΔV(Mercury) = 7.627 km/s; $\Delta V_{tot} = 13.155$ km/s