Flight and Orbital Mechanics

Solutions

TU Delft

Solutions exam Flight and Orbital Mechanics (November 2011)

<u>Question 1</u>

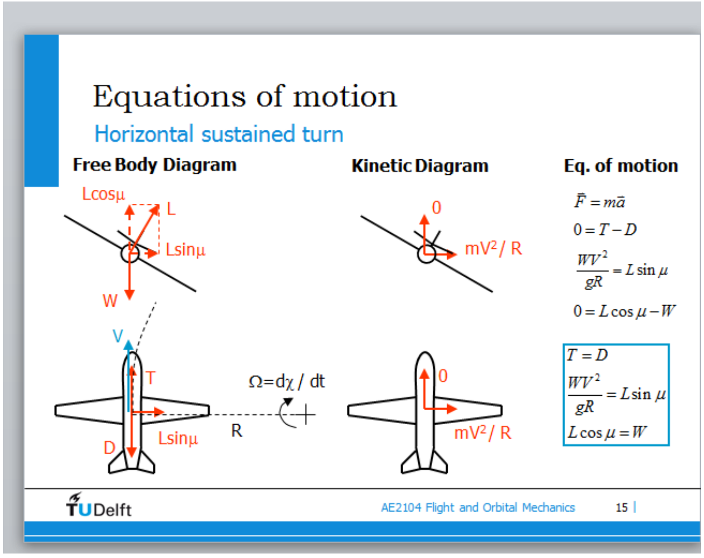
F

<u>Question 2</u>

D

<u>Question 3</u>

a and b.



c.

$$T = D$$

$$L = nW$$

$$D = D\frac{L}{L} = \frac{D}{L}nW = \frac{C_D}{C_L}nW$$

$$T = \frac{C_D}{C_L}nW$$

$$n_{\max} = \frac{T_{\max}}{W}\frac{C_L}{C_D}$$

d.

$$V = M \cdot a = M\sqrt{\gamma RT} = 0.25\sqrt{1.4 \cdot 287.05 \cdot 288.15} = 85.1 \text{ [m/s]}$$

$$L = n_{\max} W$$

$$V = \sqrt{\frac{n_{\max} W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{\frac{T_{\max}}{W}\frac{C_L}{C_D}W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{T_{\max}}{S}\frac{2}{\rho}\frac{1}{C_D}} = \sqrt{\frac{T_{\max}}{S}\frac{2}{\rho}\frac{1}{\left(C_{D_0} + \frac{C_L^2}{\pi A e}\right)}}$$

$$P_{\max} = T_{\max} V$$

$$1000000 = T_{\max} \cdot 85.1 \Rightarrow T_{\max} = 11755 \text{ [N]}$$

$$A = \frac{b^2}{S} = \frac{14.6^2}{23.3} = 9.15$$

$$V = \sqrt{\frac{T_{\max}}{S}\frac{2}{\rho}\frac{1}{\left(C_{D_0} + \frac{C_L^2}{\pi A e}\right)}}$$

$$85.1 = \sqrt{\frac{11755}{23.3}\frac{2}{1.225}\frac{1}{\left(0.04 + \frac{C_L^2}{\pi \cdot 9.14 \cdot 0.6}\right)}} \Rightarrow C_L = 1.13$$

e.

$$\left.\begin{aligned} L\cos\mu &= W \\ L\sin\mu &= \frac{W}{g}\frac{V^2}{R} \\ \sin^2\mu + \cos^2\mu &= 1 \end{aligned}\right\} \Rightarrow R = \frac{V^2}{g\sqrt{n^2 - 1}}$$

$$C_L = 1.13$$

$$C_D = 0.04 + 0.058 \cdot 1.13^2 = 0.114$$

$$n = \frac{11755}{100000}\frac{1.13}{0.114} = 1.17$$

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} = \frac{85.1^2}{9.80665\sqrt{1.17^2 - 1}} = 1235 \text{ [m]}$$

$$T_\pi = \frac{\pi R}{V} = \frac{\pi \cdot 1235}{85.1} = 46 \text{ [s]}$$

<u>Question 4</u>

a.

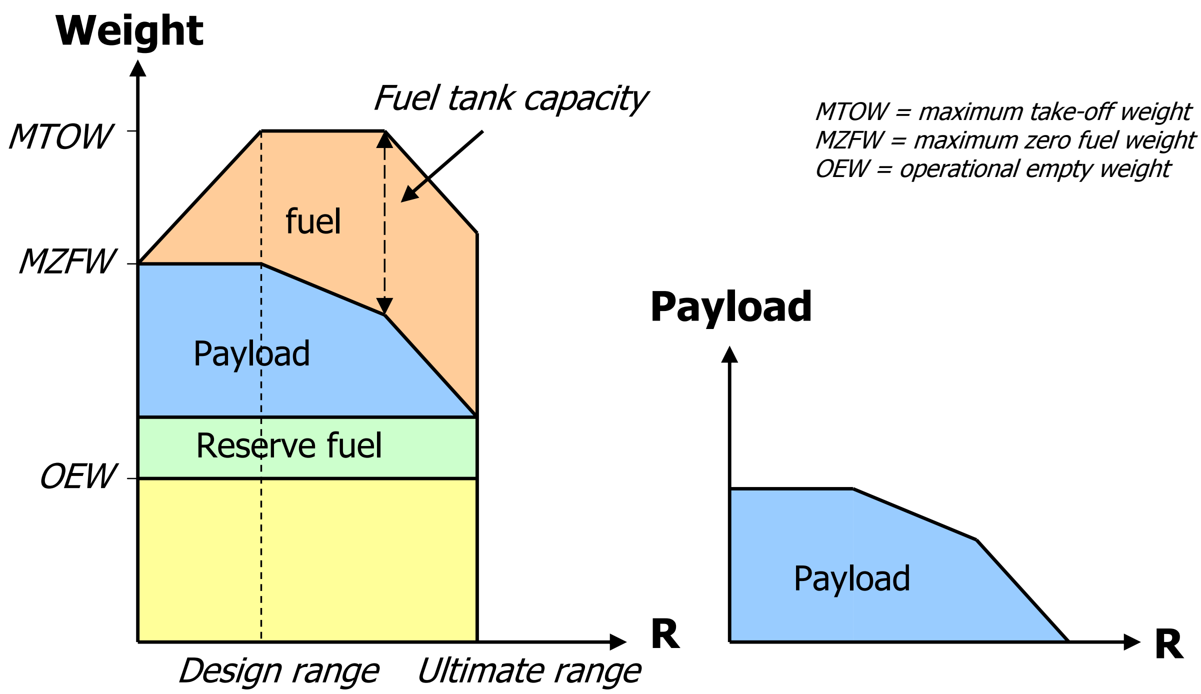
$$\left.\begin{array}{l} R = \int_{t_0}^{t_1} V dt \\ \dfrac{dW}{dt} = -F \Rightarrow dt = -\dfrac{dW}{F} \end{array}\right\} R = \int_{W_1}^{W_0} \frac{V}{F} dW$$

$$R = \int_{W_1}^{W_0} \frac{V}{F} dW = \int_{W_1}^{W_0} \frac{V}{c_T T} dW = \int_{W_1}^{W_0} \frac{V}{c_T} \frac{C_L}{C_D} \frac{1}{W} dW$$

Angle of attack, c_T and airspeed are constant, hence:

$$R = \frac{V}{c_T} \frac{C_L}{C_D} \int_{W_1}^{W_0} \frac{1}{W} dW = \frac{V}{c_T} \frac{C_L}{C_D} \ln\left(\frac{W_0}{W_1}\right)$$

B



Question 5:

The ESA satellite GOCE (altitude 250 km, circular orbit) observes the gravity field of the Earth at this very moment. One of the elements of interest is the term $J_{3,1}$. The gravity potential of the Earth is given by the following equation:

$$U \quad = \quad -\frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\delta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m} \left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda - \lambda_{n,m}))\right]$$

Here, $P_n(\sin\delta)$ and $P_{n,m}(\sin\delta)$ represent the Legendre polynomials and functions, respectively:

$$P_n(x) = \frac{1}{(-2)^n\ n!}\frac{d^n}{dx^n}(1-x^2)^n$$

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

a) (2 points) What do the parameters J_n, $J_{n,m}$ and $\lambda_{n,m}$ represent?
b) (2 points) How are the parameters r, δ and λ defined?
c) (2 points) Give the general expression to derive the radial acceleration from the potential formulation for the gravity field.
d) (6 points) Derive the general equation for the radial acceleration due to the term $J_{3,1}$ for an arbitrary satellite (i.e., without substituting any numbers).
e) (3 points) What is the equation for the radial acceleration due to $J_{3,1}$ as experienced at GOCE' altitude (expressed in numbers, still for arbitrary latitude and longitude)?
f) (3 points) What are the values for δ for which this radial acceleration changes sign?

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_{3,1}$ = -1.72×10^{-6}; $\lambda_{3,1}$ = -1.0°.

Answers:

a) J-terms are scaling factors of the irregularities in the gravity field; $\lambda_{n,m}$ defines the orientation of a particular irregularity
b) r = radial distance between position of interest and center of Earth; δ = latitude w.r.t. Earth equator; λ = longitude w.r.t. Earth-fixed reference meridian
c) $a_r = -\partial U/\partial r$
d) $a_r = -4\ \mu\ J_{3,1}\ R_e^3\ r^{-5}\cos\delta\ ((15/2)\sin^2\delta - (3/2))\cos(\lambda-\lambda_{3,1})$
e) $a_r = 5.562\ 10^{-5}\cos\delta\ ((15/2)\sin^2\delta - (3/2))\cos(\lambda+1.0)$ [m/s^2]
f) zero values for δ=-90° and +90° -> do not play a role. Other zero values for δ=-26.6° and δ=+26.6°

Question 6:

Consider a sample return mission to an asteroid (i.e., from Earth to the asteroid and then back to Earth again).

a) (3 points) Compute the 1-way travel time for a Hohmann transfer from Earth to the asteroid.
b) (2 points) The synodic period can be computed with the following equation:

$$\frac{1}{T_{syn}} = \left|\frac{1}{T_1} - \frac{1}{T_2}\right|$$

What is the definition of a synodic period?

c) (2 points) Compute the synodic period for the system Sun-Earth-asteroid.
d) (2 points) Compute the mean motion of Earth around the Sun (normally labeled "n", here labeled ω_E), and of the asteroid around the Sun (labeled ω_{ast}) (both in [rad/s]).

e) (2 points) The following general equations give the total round-trip time T and the time to be spent at the asteroid t_{stay} (provided we do both transfers by means of a Hohmann trajectory, and the departure and target objects orbit the Sun in circular orbits):

$$t_{stay} = T - 2T_H = \frac{2\pi\,(N+1) - 2\omega_E T_H}{\omega_E - \omega_{ast}}$$

$$T = \frac{2\pi\,(N+1) - 2\omega_{ast} T_H}{\omega_E - \omega_{ast}}$$

Discuss the meaning of the two equations and the parameter N (i.e., give a physical interpretation).

f) (3 points) What are the values for the parameter N, the stay time, and the total trip time for a mission to this asteroid?

g) (2 points) Can we reduce the total trip time? If so, how? A qualitative answer is sufficient.

Data: μ_{Sun}=1.3271×10^{11} km^3/s^2; distance Earth-Sun = 1 AU; distance asteroid-Sun = 0.95 AU; 1 AU = 149.6×10^6 km.

Answers:

a) $a_{tr} = (a_E+a_{ast})/2$ = 0.975 AU; T = $\pi\sqrt{(a_{tr}^3/\mu)}$ = 15191560 s = 175.83 days
b) time interval after which relative geometry of 2 objects (planets, asteroids) repeats
c) $T_E = 2\pi\sqrt{(a_E^3/\mu)}$ = 31559160 s = 365.27 days; T_{ast} = 29222061 s = 338.22 days; T_{syn} = 3.946 10^8 s = 4567.15 days = 12.504 years
d) $\omega_E = \sqrt{(\mu/a_E^3)}$ = 1.9909 10^{-7} rad/s; $\omega_{ast} = \sqrt{(\mu/a_{ast}^3)}$ = 2.1502 10^{-7} rad/s
e) Equation for stay times gives time needed to wait for favorable geometry for Hohmann return trip; since Earth and asteroid have different mean motions around Sun one of them has to orbit the Sun an integer number of orbits N more/less than the other. T: total trip time, based on Hohmann transfer “out”, stay/wait time, and Hohmann transfer ”back”
f) $t_{stay} > 0$ → minimum value for N = -1 (must be an integer) → t_{stay} = 3.7901 10^8 sec = 4386.68 days = 12.01 years → T = 4.0939 10^8 sec = 4738.33 days = 12.97 years
g) Fly faster leg to asteroid, stay for short time, fly faster leg back to Earth; will cost more energy

Question 7

a) (6 points) Consider a single-stage rocket. Using the definitions of the payload fraction p ($p=M_{payload}/M_{begin}$) and the structural mass fraction σ ($\sigma=M_{structure}/M_{propellant}$), derive the following relation:

$$\frac{M_{begin}}{M_{end}} = \frac{1+\sigma}{p+\sigma}$$

Here, M_{begin} is the total mass of the launcher before ignition, and M_{end} is the total mass after engine burnout.

b) (4 points) Now consider a multi-stage launcher, where the payload fractions of the individual stages are identical (i.e. $p_{tot} = p_i^N$, where p_{tot} is the payload fraction of the entire launcher, p_i is the payload fraction of an individual stage i, and N is the total number of stages), the specific impulse is identical for the rocket engines of all stages, and the structural mass fraction σ is also the same for all stages.
Derive the following equation for the total velocity gain provided by the launcher:

$$\Delta V_{tot} = I_{sp}\, g_0\, N\, \{\ln(1+\sigma) - \ln(\sigma + \sqrt[N]{p_{tot}})\}$$

c) (3 points) Compute the total velocity gain for a rocket with I_{sp} = 400 s, p_{tot} = 0.02 and σ=0.08, for N=1, 2 and 3.
d) (3 points) Discuss the advantages and disadvantages of replacing a single-stage launcher with a multi-stage one. What number of stages would you choose?

Data: g_0 = 9.81 m/s^2.

Answers:

a) see sheets
b) see sheets
c) N=1 → ΔV_{tot} = 9337 m/s; N=2 → ΔV_{tot} = 12436 m/s; N=3 → ΔV_{tot} = 13216 m/s
d) Advantage: better performance (larger ΔV, more payload, combination). Disadvantage: more complexity (jettison stages, ignition) -> more risk. Also: higher development and construction costs. E.g. N=10 → ΔV_{tot} = 13983 m/s, marginal gains. In practice select N=3 or N=4, complexity not worth effort.