Flight and Orbital Mechanics

Solutions

Solution question 1

a.

$$T = D$$
$$L = W$$
$$R = \int V dt = \int_{W_{start}}^{W_{end}} -\frac{V}{F} dW = \int_{W_{end}}^{W_{start}} \frac{V}{F} dW$$
$$R = \int_{W_{end}}^{W_{start}} \frac{V}{c_T T} dW = \int_{W_{end}}^{W_{start}} \frac{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}}{c_T D} dW \int_{W_{end}}^{W_{start}} \frac{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}}{c_T \frac{C_D}{C_L} W} dW$$

All factors within the integral, except W, are constant. Hence,

$$R = \sqrt{\frac{1}{S}\frac{2}{\rho}\frac{1}{c_T^2}\frac{C_L}{C_D^2}} \int_{W_{end}}^{W_{start}} \frac{1}{\sqrt{W}} dW$$

The lift coefficient can be calculated from the initial condition

$$L = W$$
$$C_L = \frac{W}{S}\frac{2}{\rho}\frac{1}{V_{start}^2} = \frac{3500000}{520}\frac{2}{0.3648}\frac{1}{(936/3.6)^2} = 0.55$$

Fill in the known values

$$10000000 = \sqrt{\frac{1}{520}\frac{2}{0.3648}\frac{3600^2}{0.65^2}} \sqrt{\frac{0.55}{C_D^2}} \int_{W_{end}}^{W_{start}} \frac{1}{\sqrt{W}} dW$$
$$10000000 = \frac{420.2}{C_D} \int_{W_{end}}^{W_{start}} \frac{1}{\sqrt{W}} dW = \frac{420.2}{C_D} 2\left(\sqrt{W_{start}} - \sqrt{W_{end}}\right) = \frac{420.2}{C_D} 2 \cdot (1871 - 1470)$$
$$C_D = 0.0337$$

The aspect ratio follows from the parabolic lift drag polar.

$$C_D = 0.018 + \frac{0.55^2}{\pi A \cdot 0.85} = 0.018 + \frac{0.113}{A} = 0.0337$$

Hence,

$$A = 7.10$$

b.

The span can be derived from the aspect ratio.

$$A = \frac{b^2}{S}$$

$$b = \sqrt{AS} = \sqrt{7.1 \cdot 520} = 60.77 \text{ [m]}$$

Finally, the Mach number at the start and end of the cruise phase can be obtained:

$$a = \sqrt{1.4 \cdot 287.05 \cdot 216.7} = 295.1$$

$$M_{start} = V_{start} \cdot a = \frac{936}{3.6} = 0.88$$

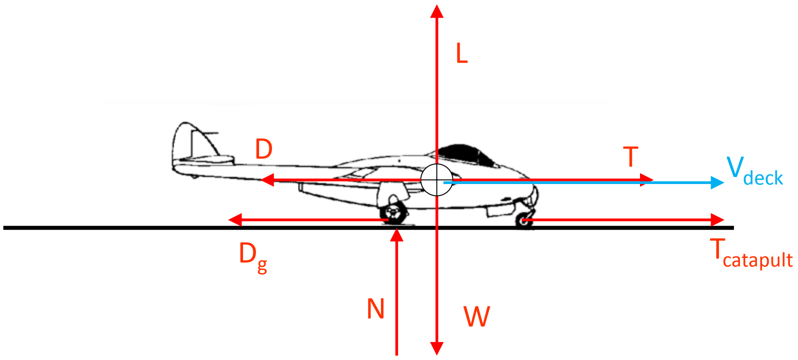
$$V_{end} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{2160000}{520}\frac{2}{0.3648}\frac{1}{0.55}} = 204.2 \text{ [m/s]}$$

$$M_{end} = V_{end} \cdot a = 0.69$$

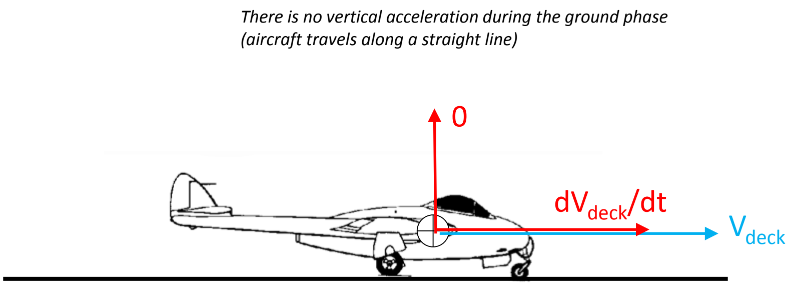
Solution question 2

a.

Free body diagram



Kinetic diagram



Note that the velocities and accelerations are expressed relative to the ship deck

b.

$$\frac{W}{g}\frac{dV}{dt} = T + T_{catapult} - D - D_g$$
$$0 = L - W + N$$
$$\left(D_g = \mu N\right)$$
$$\Rightarrow \begin{cases} N = W - L \\ \frac{W}{g}\frac{dV}{dt} = T + T_{catapult} - D - \mu\left(W - L\right) \end{cases}$$

c.

$$s_{deck} = \int_0^{V_{end,deck}} Vdt = \int_0^{V_{end,deck}} \frac{V}{a}dV = \frac{1}{\bar{a}}\int_0^{V_{end,deck}} VdV = \frac{V_{end,deck}^2}{2\bar{a}}$$
$$s_{deck} = \frac{V_{end,deck}^2}{2\bar{a}}$$

The velocity in this equation is expressed relative to the ship deck

d.

$$\frac{dV}{dt} = a = \frac{g}{W}\left(T + T_{catapult} - D - \mu\left(W - L\right)\right)$$
$$\bar{a} = \frac{g}{W}\left(\bar{T} + \bar{T}_{catapult} - \bar{D} - \mu\left(W - \bar{L}\right)\right)$$
$$\bar{a} = \frac{9.80665}{200000}\left(120000 + 100000 - \bar{D} - 0.04\left(200000 - \bar{L}\right)\right)$$

So, we should find the average lift and drag. **Both lift and drag depend on the airspeed**.

$$C_L = 0.1$$
$$C_D = C_{D_0} + kC_L^2 = 0.024 + 0.095 \cdot 0.1^2 = 0.025$$

$$D = C_D \tfrac{1}{2}\rho V_{air}^2 S$$
$$\bar{D} = C_D \tfrac{1}{2}\rho\left(\frac{V_{end,air}}{\sqrt{2}}\right)^2 S = 0.025 \cdot \tfrac{1}{2} \cdot 1.225 \cdot \tfrac{1}{2} \cdot V_{end,air}^2 \cdot 46.5$$
$$\bar{D} = 0.356 V_{end,air}^2$$
$$\bar{L} = C_L \tfrac{1}{2}\rho\left(\frac{V_{end,air}}{\sqrt{2}}\right)^2 S = 0.1 \cdot \tfrac{1}{2} \cdot 1.225 \cdot \tfrac{1}{2} \cdot V_{end,air}^2 \cdot 46.5$$
$$\bar{L} = 1.425 V_{end,air}^2$$

Now we can combine all results

$$\bar{a} = 10.4 - 1.47 \cdot 10^{-5} V_{end,air}^2$$

e.

The aircraft is experiencing headwind and an additional airspeed due to the motion of the ship.

$$V_{end,air} = V_{end,deck} + V_{ship} + V_{wind}$$
$$V_{end,deck} = V_{end,air} - 23.1$$

f.

$$s_{deck} = \frac{V_{end,deck}^2}{2\overline{a}}$$
$$100 = \frac{\left(V_{end,air} - 23.1\right)^2}{2 \cdot \left(10.4 - 1.47 \cdot 10^{-5} V_{end,air}^2\right)}$$
$$-1.0029V_{end,air}^2 + 46.3V_{end,air} + 1543 = 0$$
$$V_{end,air} = 68.6 \text{ [m/s]}$$
$$V_{end,deck} = 68.6 - 23.1 = 45.5 \text{ [m/s]}$$

g.

The velocity relative to the deck in question f equals 45.5 m/s. The wind speed affects both lift and drag (more aerodynamic drag and less ground friction) but these terms work in opposite sense and are very small compared to the aircraft thrust and the catapult force. It can in fact be observed from the equation at the end of question d. that the velocity has a negligible contribution to the acceleration relative to the ship deck.

($10.4 - 1.47 \cdot 10^{-5} \cdot 45.5^2 = 10.37$)

Thus, in case there is no wind and no ship motion, the average acceleration will be almost identical and the airspeed at the end of the deck will be almost the same as 45.5 m/s.

(You can also simply redo the calculation for zero wind and zero ship speed but that will take much more time!)

Exam ae2-104 d.d. November 2, 2012 (orbital mechanics part only)

Question 3 (18 points)

An essential aspect of a satellite mission is the occurence of solar eclipses.

a) (5 points) For what (3) subsystems plays the presence (and absence) of solar radiation a role? Discuss each situation briefly (2-3 lines each).
b) (4 points) For an eclipse to occur, 2 conditions have to be satisfied. What are they? Illustrate in a sketch.
c) (4 points) Consider a 2-dimensional situation, where the Sun is located in the orbital plane of the satellite. What is the length of the eclipse period for a satellite in a circular orbit at 800 km altitude (expressed as percentage of the orbital period)? What is it for a geostationary satellite?
d) (2 points) Give the definition of a sun-synchronous orbit.
e) (3 points) Is it possible to select a satellite orbit such that the satellite is in full sunlight throughout its mission lifetime (*e.g.,* 10 years)? If so, what are the (3) conditions for this? Is this solution applied?

Data: R_e = 6378.137 km, μ_{earth} = 398600.4415 km^3/s^2, h_{GEO} = 35786 km.

Answers:

a) Power, thermal, attitude control, optical instruments.
b) night side of the Earth + perpendicular position component smaller thanEarth radius.
c) $\sin(\lambda)=R_e/a$, and $T_{eclipse}=2\lambda/360\times100$ ➔ $T_{eclipse}$=34.8% (800 km); $T_{eclipse}$=4.8% (GEO).
d) A sun-synchronous orbit keeps a stable orientation w.r.t. the direction to the Sun (effects of North-South motion Sun excluded).
e) Yes. Conditions: (1) sun-synchronous orbit, (2) line of nodes perpendicular to direction to Sun, (3) inclination in range that gives “cross” component of satellite position which is larger than Earth radius always (101°<inc<115°; no need to give these numbers). Solution is not applied, since it will lead to altitudes above 1400 km (bad for resolution earth observation) and take the satellite through the Van Allen Belts (exposure to high radiation levels).

Question 4 (17 points)

The gravity field of the Earth is dominant for the motion of spacecraft orbiting Earth. It is given by the following equation:

$$U = -\frac{\mu}{r}[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\delta)$$

$$+\sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m} \left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda-\lambda_{n,m}))\,]$$

where

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

$$P_n(x) = \frac{1}{(-2)^n\, n!}\frac{d^n}{dx^n}(1-x^2)^n$$

a) (2 points) What is the general definition for the East-West acceleration, based on the potential formulation for the gravity field of the Earth?
b) (7 points) Derive the general equation for the East-West acceleration due to the term (2,2) for an arbitrary satellite.
c) (2 points) Compute the orbit radius of a geostationary satellite.
d) (3 points) What is the (numerical) expression for the East-West acceleration due to $J_{2,2}$ for a geostationary satellite?
e) (3 points) The resulting equation has the following form: $acc_{EW;2,2}$ = constant × $\sin(2(\lambda-\lambda_{2,2}))$. What are the locations of the equilibrium points?

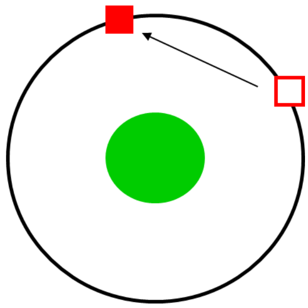
Data: $\mu_{Earth} = 398600.4415\ km^3/s^2$; $T_E = 23^h56^m4^s$; $J_{2,2} = 1.816\times10^{-6}$; $\lambda_{2,2} = -14.9°$; $R_e = 6378.137$ km.

Answers:

a) $acc_{EW} = -1/(r\cos\delta)\ \partial U/\partial\lambda$
b) $U_{2,2} = -\mu/r\ [J_{2,2}(R_e/r)^2 P_{2,2}(\sin\delta)\cos(2(\lambda-\lambda_{2,2}))$; $P_2(x) = 1.5x^2 - 0.5$; $P_{2,2}(x) = 3(1-x^2)$; $acc_{EW;2,2} = -6\mu J_{2,2} R_e^2 r^{-4}\sin(2(\lambda-\lambda_{2,2}))$
c) $T=2\pi\sqrt{a^3/\mu}=T_E \rightarrow a = 42164.140$ km
d) $acc_{EW;2,2} = -5.59\times10^{-11}\sin(2(\lambda-\lambda_{2,2}))\ km/s^2$
e) λ = 75.1°, 165.1°, 255.1° and 345.1°.

Question 5 (15 points)

Consider the situation where the cargo vehicle Dragon is to dock with the International Space Station (ISS). Both are orbiting Earth in a circular (coplanar) orbit at 400 km altitude. The ISS is 90° ahead of the Dragon vehicle. You are responsible for designing the transfer between the two, where Dragon is the active vehicle and the ISS the (passive) target. In order to catch up with the ISS, Dragon will be taken to an elliptical (*i.e*. non-circular) transfer orbit with a different orbital period.



a) (1 point) In order to rendezvous with the ISS, would you lower or raise the orbit of the Dragon vehicle? Give an argumentation on your choice.
b) (1 point) What is the orbital period at an altitude of 400 km?
c) (2 points) What is the circular velocity at an altitude of 400 km?
d) (2 points) Consider the situation that one wants to have Dragon complete 15 revolutions in its elliptical transfer orbit, before it does the rendezvous with the ISS. What is the required shift per revolution of Dragon in its transfer orbit, w.r.t. the ISS?
e) (3 points) What is the orbital period of this transfer orbit, where the total transfer is to be completed after exactly 15 revolutions? If you were unable to answer question (b), use a value of 100 minutes for the orbital period in the original orbit.
f) (2 points) What is the semi-major axis of this transfer orbit?
g) (2 points) What is the velocity in the transfer orbit at the original altitude of 400 km? In case you could not compute an answer for question (f), use a value of the semi-major axis of 6700.0 km.
h) (2 points) What is the required total velocity change for this transfer scenario?

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km.

Answers:

a) Dragon has to catch up, so has to have a smaller orbital period → lower its orbit.
b) $T=2\pi\sqrt{a^3/\mu}$ → T = 5553.6 sec = 92.56 min.
c) $V_{circ} = \sqrt{\mu/r}$ = 7.669 km/s.
d) 90°/15=6° per orbit.
e) $T_{transfer}$ = 354/360 × T_{ISS} = 5461.1 sec = 91.02 min.
f) $T=2\pi\sqrt{a^3/\mu}$ → a = 6702.602 km.
g) Energy equation: $\frac{1}{2}V^2 - \mu/r = -\mu/(2a)$ → V_{apo} = 7.625 km/s.
h) Lower the pericenter with $\Delta V = V_{circ,ISS} - V_{apo}$; this is to be followed (after 15 revs) by a similar maneuver but now to increase the velocity → ΔV = 2(7.669-7.626) = 0.088 km/s.