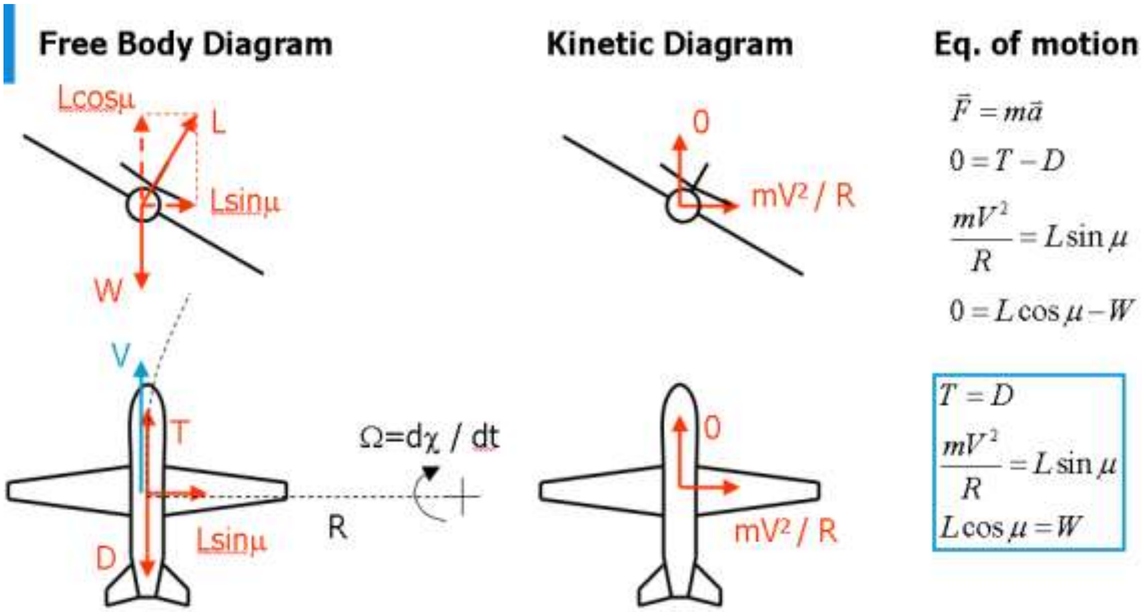
Flight and Orbital Mechanics

Solutions

TU Delft

AE2104 EXAM

Question 1a and b



Solution 2c

Steady climbing flight:

$$\frac{P_a - P_r}{W} = RC$$

$$\frac{85000 - P_r}{14000} = 4$$

$$P_r = 29000 \text{ [J/s]}$$

With the required power, the lift over drag ratio can be calculated.

$$P_r = DV = \frac{C_D}{C_L} W \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} = \sqrt{\frac{W^3}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

$$P_r = \sqrt{\frac{W^3}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}$$

$$29000 = \sqrt{\frac{14000^3}{13}\frac{2}{1.0065}\frac{C_D^2}{C_L^3}} \Rightarrow \frac{C_D^2}{C_L^3} = 0.002$$

Angle of attack remains constant, which means that C_L and C_D also remain constant. (this is the minimum power condition). So the turn is flown at:

$$\frac{C_D^2}{C_L^3} = 0.002$$

Power available must equal power required in the turn.

$$P_a = P_r$$
$$L = nW$$
$$P_a = P_r = DV = \frac{C_D}{C_L} nW \sqrt{\frac{nW}{S} \frac{2}{\rho} \frac{1}{C_L}} = \sqrt{\frac{n^3 W^3}{S} \frac{2}{\rho} \frac{C_D^2}{C_L^3}}$$
$$P_a = \sqrt{\frac{n^3 W^3}{S} \frac{2}{\rho} \frac{C_D^2}{C_L^3}}$$
$$85000 = \sqrt{\frac{n^3 14000^3}{13} \frac{2}{1.0065} 0.002} \Rightarrow n = 2.05$$

$$\mu = \cos^{-1}\left(\frac{1}{2.05}\right) = 60.8 \text{ [deg]}$$

Solution question 2

B

Solution question 3a

In cruise flight, lift equals weight.

$$L = W$$

This results in an equation for the lift coefficient

$$C_L = \frac{W}{S} \frac{2}{\rho} \frac{1}{V^2}$$

Thus,

$$\frac{\left(C_L\right)_{V_{D_{\min}}}}{\left(C_L\right)_{1.1V_{D_{\min}}}}=\frac{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{V_{D_{\min}}^2}}}{\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{\left(1.1V_{D_{\min}}\right)^2}}}=1.1^2$$

$$\left(C_L\right)_{1.1V_{D_{\min}}}=\frac{\left(C_L\right)_{V_{D_{\min}}}}{1.1^2}$$

In the minimum drag condition, the ratio of C_L over C_D should be maximal.

$$D=\frac{C_D}{C_L}W$$

The corresponding lift coefficient can be calculated with the lift drag polar

$$\frac{d}{dC_L}\left(\frac{C_L}{C_D}\right)=0\Rightarrow C_L=\sqrt{C_{D_0}\pi Ae}\text{ (you must show the full derivation)}$$

Therefore,

$$\left(C_L\right)_{V_{D_{\min}}}=\frac{1}{1.1^2}\sqrt{C_{D_0}\pi Ae}$$

Solution question 3b

$$\left.\begin{aligned}&R=\int_{t_0}^{t_1}Vdt\\&F=-\frac{dW}{dt}\\&P_a=P_r=DV=\frac{C_D}{C_L}WV\\&F=c_pP_{br}=\frac{c_p}{\eta}P_a\end{aligned}\right\}R=-\int_{W_0}^{W_1}\frac{V}{F}dW=\int_{W_1}^{W_0}\frac{V}{F}dW=\int_{W_1}^{W_0}\frac{V}{V}\frac{\eta}{c_p}\frac{C_L}{C_D}\frac{dW}{W}=\frac{\eta}{c_p}\frac{C_L}{C_D}\ln\left(\frac{W_0}{W_1}\right)$$

The angle of attack (and thus lift and drag coefficient) are constant (see question a). Therefore, they can be taken outside the integral. This is also true for the specific fuel consumption and the efficiency.

Solution question 3c

The lift coefficient is known from question a.

$$C_L = \frac{1}{1.1^2}\sqrt{C_{D_0}\pi Ae} = \frac{1}{1.1^2}\sqrt{0.018\cdot\pi\cdot A\cdot 0.82} = 0.178\sqrt{A}$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi Ae} = 0.018 + \frac{0.178^2 A}{\pi Ae} = 0.0303$$

The fuel weight at the start is 40% of the total weight

$$W_0 = W_f + W_1$$

$$1 = \frac{W_f}{W_0} + \frac{W_1}{W_0} = 0.4 + \frac{W_1}{W_0} \Rightarrow \frac{W_1}{W_0} = 0.6$$

Combine all results:

$$R = \frac{\eta}{c_p}\frac{C_L}{C_D}\ln\left(\frac{W_0}{W_1}\right)$$

$$4000000 = \frac{0.35}{0.93\cdot 10^{-7}\cdot 9.81}\frac{0.178\sqrt{A}}{0.0303}\ln\left(\frac{1}{0.6}\right)$$

$$A = 8.25$$

Solution question 3d

The aircraft is flying at $1.1V_{Dmin}$. Thus, it is flying at a constant angle of attack (see question a). The aircraft weight is decreasing. The altitude (air density) is constant. Hence, the airspeed will have to decrease. $V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$

Questions and answers exam ae2-104 dated October 29, 2010 (space part)

Question 4:

The gravity potential of the Earth is given by the following equation:

$$U = -\frac{\mu}{r}\left[1-\sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\delta) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cos(m(\lambda-\lambda_{n,m}))\right]$$

Here, $P_n(\sin\delta)$ and $P_{n,m}(\sin\delta)$ represent the Legendre polynomials and functions, respectively:

$$P_n(x) = \frac{1}{(-2)^n\ n!}\frac{d^n}{dx^n}(1-x^2)^n$$

$$P_{n,m}(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}$$

a) (2 points) Give the general expression to derive the North-South acceleration from the potential formulation for the gravity field.
b) (4 points) Derive the general equation for the North-South acceleration due to the term J_2 for an arbitrary satellite.
c) (2 points) What is the equation for the North-South acceleration due to J_2 for a satellite at 500 km altitude (expressed in numbers, still for arbitrary latitude and longitude)?
d) (2 points) Make a sketch of this acceleration as a function of latitude ($-90° \leq \delta \leq 90°$).

Data: μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; $J_2 = 1082\times10^{-6}$

Answers:

a) $a_{NS} = -(1/r)\ \partial U/\partial\delta$
b) $a_{NS} = -3\ \mu\ J_2\ R_e^2\ r^{-4}\ \sin\delta\ \cos\delta$
c) $a_{NS} = -0.0235\ \sin\delta\ \cos\delta$ [m/s^2]
d) sketch, zero values for δ=-90°, 0° and +90°. Negative values over northern hemisphere, positive values over southern hemisphere

<u>Question 5:</u>

The parameters of an Earth-repeat orbit have to satisfy the following equation:

$$j \; |\Delta L_1 + \Delta L_2| \quad = \quad k \; 2 \, \pi$$

where

$$\Delta L_1 \quad = \quad -2 \, \pi \, \frac{T}{T_E} \qquad \text{[rad/rev]}$$

$$\Delta L_2 \quad = \quad -\frac{3 \, \pi \; J_2 \; R_e^2 \; \cos i}{a^2 \; (1 - e^2)^2} \qquad \text{[rad/rev]}$$

a) (1 point) What is the main characteristic of an Earth-repeat orbit?
b) (1 point) What is the main characteristic of a Sun-synchronous orbit?
c) (4 points) Derive a general equation for the orbital period of a satellite that satisfies the requirements on Earth-repeat and Sun-synchronous orbits simultaneously.
d) (2 points) Compute the value for the semi-major axis for such an orbit for the Earth-repeat conditions (43,3).
e) (2 points) Compute the corresponding orbital inclination.

Data: T_E = $23^h 56^m 4^s$; T_{ES} = 365.25 days; μ_{Earth} = 398600.4415 km^3/s^2; R_e = 6378.137 km; J_2 = 1082×10^{-6}

<u>Answers:</u>

a) ground track repeats after integer #orbits and integer #Earth revolutions.
b) relative orientation orbit w.r.t. Sun remains constant.
c) Sun-synch: $\Delta L_2 = (2\pi/T_{ES})\,T$. Substitute → $T = k / ([\, j\,(1/T_E - 1/T_{ES})] = k\, T_E\, T_{ES} / [\, j\,(T_{ES} - T_E)]$
d) T = 6027.9 sec → a = 7158.74 km
e) (e=0) → i = 98.53°.

Question 6:

Consider a transfer from a circular parking orbit at 185 km and i =29.8° (i.e., launch from Kennedy Space Center) to the International Space Station (h=400 km, e=0, i=55.6°).

a) (3 points) When is an in-plane maneuver (i.e., ΔV) most efficient?
b) (3 points) When is an out-of-plane maneuver most efficient?
c) (2 points) Compute the velocities in the original orbit and in the target orbit.
d) (6 points) Compute the total ΔV that would be required for the orbit raising, assuming that the two orbits are coplanar (Hohmann transfer).
e) (2 points) Compute the ΔV that would be required to only change the inclination of the original parking orbit to that of the ISS orbit.
f) (2 points) Compute the total ΔV if the sequence of maneuvers was (1) dog-leg maneuver (only) in initial orbit, and (2) Hohmann transfer.
g) (2 points) Compute the total ΔV if the Hohmann transfer to 400 km altitude is done first, and the inclination change is done next (i.e., at 400 km) in an independent maneuver.

Data: μ_{Earth} = 398600.4415 km^3/s^2, R_{Earth} = 6378.137 km

Answers:

a) When done parallel to original velocity, and at point where original velocity is largest.
b) When done where original velocity is smallest.
c) V_{c1} = 7.793 km/s. V_{c2} = 7.669 km/s.
d) ΔV_1 = 0.063 km/s. ΔV_2 = 0.062 km/s. $\Delta V_{tot} = \Delta V_1 + \Delta V_2$ = 0.125 km/s.
e) ΔV_{dl185} = 3.480 km/s
f) ΔV_{tot} = 3.480 + ΔV_1 + ΔV_2 = 3.604 km/s.
g) ΔV_{tot} = 3.549 km/s.

Question 7:

Consider a Hohmann transfer from an inner planet 1 to an outer planet 2.

a) (2 points) Derive the following general equations for the epoch of departure t_1 and the epoch of arrival t_2:

$$t_1 = t_0 + \frac{\theta_2(t_0) - \theta_1(t_0) + n_2\, T_H - \pi}{n_1 - n_2}$$

$$t_2 = t_1 + T_H$$

Here, t_0 is a common reference epoch, T_H is the transfer time in a Hohmann orbit, n_1 and n_2 are the mean motion of the two planets, and θ_1 and θ_2 are the true anomalies of the planetary positions, respectively. Assume circular orbits for both planets.

b) (2 points) Consider a Hohmann transfer from Earth to Neptune. What is the transfer period?

c) (2 points) Assuming that on January 1, 2010, θ_{Earth}=70° and $\theta_{Neptune}$=240°, what would be the epoch of departure (expressed in days w.r.t. this January 1)?

d) (2 points) What would be the arrival epoch?

e) (2 points) Can we change the launch window? If so, how? A qualitative answer is sufficient.

Data: μ_{Sun}=1.3271×10^{11} km^3/s^2; distance Earth-Sun = 1 AU; distance Neptune-Sun = 30.1 AU; 1 AU = 149.6×10^6 km.

Answers:

a) See sheets.

b) T_H = 9.676 10^8 sec = 30.66 yrs.

c) t_1-t_0 = 5013056.3 sec = 58.02 days (after Jan 1, 2010).

d) t_2=t_1+T_H = 11256.9 days = 30.820 yrs.

e) Yes, shift both t_1 and t_2 by the synodic period. Or: fly a faster mission (costing more energy).