Spectral Analysis of Noise in Switching LC-Oscillators
Sub-Outline

- Duty Cycle of $g_m$-cell Small-Signal Gain
- Oscillation Condition
- LC-Tank Noise
- $g_m$-cell Noise
- Tail-Current Source Noise
- (Phase) Noise Factor – Bipolar VCO
- (Phase) Noise Factor – CMOS VCO
- Bipolar vs. CMOS VCO
Is it Indeed so Simple?

- noise from the transistors $Q_1$ and $Q_2$ is switched ON and OFF
- noise from current source $Q_{CS}$ is modulated by oscillator switching

LC-tank noise

transconductor noise

tail-current source noise
The $g_m$-cell Transfer Function

- small-signal gain in the presence of a large signal

- Fourier domain – (magnitude) complex harmonic components

\[ g_{2i} = \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} g(t)e^{-j2\omega t} dt \]

\[ g_{2i} = gd \frac{\sin(i\pi d)}{i\pi d} \]

\[ d = \frac{1}{2k} \]
LC-Tank Noise Contribution

• phase-modulating noise component:
  \[ i_{PM} = \frac{1}{2} \left[ i_{N,O}(f_0 + \Delta) + i_{N,O}(-f_0 + \Delta) + i_{N,O}(-f_0 - \Delta) + i_{N,O}(f_0 - \Delta) \right] \]
  \[ i_{PM} = (g_0 + g_2) v_N (f_0 + \Delta) + (g_0 + g_2) v_N (f_0 - \Delta) \]

• phase-related noise power \((k >> 1, g = g_0 = g_{\pm 2})\):
  \[ i_{PM}^2 (R_{TK}) = (g_0 + g_2)^2 v_N^2 (R_{TK}) \]

• LC-tank noise transfer function:
  \[ g^2 (R_{TK}) = (g_0 + g_2)^2 = G_{TK}^2 \]

• LC-tank noise factor:
  \[ F(R_{TK}) = \frac{2i_{PM}^2 (R_{TK})}{4KTG_{TK}} = \frac{2(g_0 + g_2)^2 v_N^2 (R_{TK})}{4KTG_{TK}} = \frac{2G_{TK}^2 v_N^2 (R_{TK})}{4KTG_{TK}} = \frac{4K TG_{TK}}{4KTG_{TK}} = 1 \]
Collector-Current Noise Transfer Function in Limiting Region

\[
i_{C1} = \frac{Q_1}{2} i_{C1}
\]

\[
V_{CC} - C_V - L - C_V
\]

\[
I_{TAIL}
\]
Collector-Current Noise Transfer Function in Limiting Region

\[ i_{C1} \]

\[ R_{TK}/2 \]

\[ Q_1 \]

\[ i_{C1} \]

\[ i_{C1} \]

\[ R_{TK}/2 \]

\[ Q_1 \]

\[ i_{C1} \]

\[ i_{C1} \]

\[ I_{TAIL} \]

\[ i_{C1} \]

\[ R_{TK}/2 \]

\[ R_{TK}/2 \]

\[ i^2_{PM} (I_C) = 0 \]
Base-Resistance Noise Transfer Function in Limiting Region

\[ v_{N} (g_{m-IN}) = 2(v_{N}(r_{B}) + i_{N}(I_{C})/g_{m}^{2}) = 4KTr_B + 2KT/g_m \]
**$g_m$-Cell Noise Contributions**

- **phase-related noise power:**
  \[
  i_{PM}^2(g_{m-IN}) = \frac{1}{2d}(g_{2i-2} + g_{2i})^2 v_N^2(g_{m-IN}) = \frac{2g_{2i}^2}{d} v_N^2(g_{m-IN}) = 2dg^2 v_N^2(g_{m-IN})
  \]

- **$g_m$-cell noise transfer function:**
  \[
  g^2(g_{m-IN}) = 2dg^2 = 2d\left(\frac{g_m}{2}\right)^2 = 2\frac{1}{2k} (kG_{TK})^2 = kG_{TK}^2
  \]

- **$g_m$-cell noise factors:**

  \[
  F(2r_B) = \frac{2i_{PM}^2(2r_B)}{4KTG_{TK}} = \frac{2kG_{TK}^2 4KTr_B}{4KTG_{TK}} = 2kr_B G_{TK} = kc
  \]

  \[
  F(2I_C) = \frac{2i_{PM}^2(2I_C)}{4KTG_{TK}} = \frac{2kG_{TK}^2 2KT / g_m}{4KTG_{TK}} = kG_{TK} / g_m = \frac{1}{2}
  \]

- input $g_m$-cell noise around odd multiples of the oscillation frequency is folded to the LC-tank noise around the oscillation frequency.
Tail-Current Noise Transfer Function

- $g_m$-cell large-signal $V$-to-$I$ transfer function

- Fourier domain – (magnitude) complex harmonic components
TCS-Noise Contribution

• phase-related noise power:

\[ i_{PM}^2(I_{TCS}) = \frac{i_{PM,\,DIFF}^2(I_{TCS})}{4} = \frac{1}{4} i_N^2(I_{TCS}) \]

• TCS-noise transfer function:

\[ g^2(I_{TCS}) = \frac{1}{4} \]

• TCS-noise factor:

\[
F(I_{TCS}) = \frac{2i_{PM}^2(I_{TCS})}{4KTG_{TK}} = \frac{KTg_m(1 + 2r_Bg_m)}{4KTG_{TK}} = \frac{KT2kG_{TK}(1 + 2kc)}{4KTKG_{TK}} = \frac{1}{2} k(1 + 2kc) = k\left(\frac{1}{2} + kc\right) = k\left(F(2I_C + 2I_B) + F(2r_B)\right)
\]

• TCS noise around even multiples of the oscillation frequency is folded to the LC-tank noise around the oscillation frequency
Switching-Oscillator Phase Noise

• Noise factor:

\[
F = F(R_{TK}) + F(2I_{C}) + F(2I_{B}) + F(2r_{B}) + F(I_{TCS})
\]

\[
F = 1 + \frac{1}{2} + \frac{1}{2\beta} + kc + k\left(\frac{1}{2} + kc\right) \approx 1 + (1 + k)\left(\frac{1}{2} + kc\right)
\]

• Phase noise:

\[
\mathcal{L} = \frac{\mathcal{L}(R_{TK}) + \mathcal{L}(2I_{C}) + \mathcal{L}(2I_{B}) + \mathcal{L}(2r_{B}) + \mathcal{L}(I_{TCS})}{(4\pi C_{TOT} \Delta)^2} = \frac{4KTG_{TK}F}{v_s^2(4\pi C_{TOT} \Delta)^2}
\]

\[
\mathcal{L} = \frac{4KTG_{TK}}{(4\pi C_{TOT} \Delta)^2} \left(\frac{\pi}{8V_T}\right)^2 \frac{1 + (1 + k)\left(\frac{1}{2} + kc\right)}{k^2}
\]
Phase Noise Model of Bipolar Switching LC-Oscillators

\[ F = 1 + \frac{1}{2} + kc + k\left(\frac{1}{2} + kc\right) \]

- **Constant phase-noise contributions**
  - LC-tank noise contribution \( \sim 1 \)
  - \( g_m \)-cell current shot noise contribution \( \sim \frac{1}{2} \)

- **Loop-gain related contributions**
  - \( g_m \)-cell base-resistance noise contribution \( \sim ck \)
  - phase-noise contribution of the bias current source is \( k \)-times larger than the noise contribution of the \( g_m \)-cell \( \sim k(\frac{1}{2} + ck) \)
Low/High-Performance VCO Designs

• high loop-gain, high quality LC-tank, BCS noise eliminated:
  e.g., $k > > 1 (= 10)$, $c << 1 (~0.01)$
  
  $$\mathcal{L} \sim \frac{1 + \frac{1}{k} + kc}{k^2} = \frac{8}{5} \frac{1}{10} \frac{1}{10} \sim \frac{1}{k^2}$$

• high loop-gain, high quality LC-tank:
  e.g., $k > > 1 (= 10)$, $c << 1 (~0.01)$
  
  $$\mathcal{L} \sim \frac{k(\frac{1}{2} + kc)}{k^2} = \frac{1}{2} + kc = \frac{3}{5} \frac{1}{10} \sim \frac{1}{k}$$

• low loop-gain, low quality LC-tank:
  e.g., $k \sim 1 (= 2)$, $c \sim 1 (~0.5)$
  
  $$\mathcal{L} \sim \frac{1 + (1 + k)^{\frac{3}{2}}}{k^2} = \frac{11}{8} \sim 1$$
Single/Double-Switch CMOS LC-VCOs

\[ i_N^2(G_{TK}) = 2KTG_{TK} \]

\[ i_N^2(I_D) = 2KT\gamma g_m \]

\[ d = \frac{1}{2k} \]

\[ i_N^2(I_{BCS}) = 2KT\gamma g_{m,CS} \]

\[ i_N^2(I_{D,P}) = 2KT\gamma_P g_{m,P} \]

\[ i_N^2(I_{D,N}) = 2KT\gamma_N g_{m,N} \]

\[ d = \frac{1}{4k} \]
Phase Noise Model of CMOS LC-Oscillators

• Noise factor \( \gamma = \gamma_p, g_{m,N} = g_{m,P}, g_{m,N,CS} = 2g_{m,N} \):

\[
F_{SS} = F(R_{TK}) + F(2I_D) + F(I_{BCS}) \quad F_{DS} = F(R_{TK}) + F(4I_D) + F(I_{BCS}) \nn F_{SS} = 1 + \gamma + k\gamma = 1 + (1 + k)\gamma \quad F_{DS} = 1 + \gamma + 2k\gamma = 1 + (1 + 2k)\gamma
\]

• Phase noise:

\[
\mathcal{L} = \frac{\mathcal{L}(R_{TK}) + \mathcal{L}(2I_D) + \mathcal{L}(I_{BCS})}{(4\pi C_{TOT}\Delta)^2} = \frac{4KTG_{TK}F}{v_s^2(4\pi C_{TOT}\Delta)^2}
\]

\[
\mathcal{L}_{SS} = \frac{4KTG_{TK}}{(4\pi C_{TOT}\Delta)^2} \frac{1 + (1 + k)\gamma}{\left(\frac{2}{\pi} I_{TAIL}R_{TK}\right)^2} \quad \mathcal{L}_{DS} = \frac{4KTG_{TK}}{(4\pi C_{TOT}\Delta)^2} \frac{1 + (1 + 2k)\gamma}{\left(\frac{4}{\pi} I_{TAIL}R_{TK}\right)^2}
\]
Phase Noise Model of CMOS LC-Oscillators

\[ F = 1 + \gamma + \alpha k \gamma \]

- **Constant phase-noise contributions**
  - LC-tank noise contribution \( \sim 1 \)
  - \( g_m \)-cell drain-current thermal noise contribution \( \sim \gamma \)

- **Loop-gain related contributions**
  - Bias current source noise contribution \( \sim k\gamma \)
CMOS vs. Bipolar LC-Oscillators

- noise factors (for removed BCS noise):

\[ F_{BIP} = 1 + \frac{1}{2} + kc \]
\[ F_{CMOS} = 1 + \gamma \]

- bipolar VCO better for the same power consumption \((v_{s,BIP}=v_{s,CMOS})\)

\[ \frac{3}{2} + kc < \frac{5}{3} \quad \implies \quad kr_B < \frac{R_{TK}}{12} \]

- e.g., \(k=10, R_{TK}=1000\Omega\)

\[ r_B < \frac{1000}{120} = 8.4\Omega \]

- e.g., \(k=10, R_{TK}=100\Omega\)

\[ r_B < \frac{100}{120} = 0.84\Omega \]
CMOS vs. Bipolar LC-Oscillators

- power consumption figure of merit:

\[
FOM = 10 \log \left( L(\Delta) \left( \frac{\Delta}{\omega_0} \right)^2 V_{CC} I_{CC} \right)
\]

- \(V_{CC}=1.8\text{V}, v_{s,BIP}=0.4\text{V}, v_{s,SS-CMOS}=1.2\text{V}, (3I_{CC,BIP}=I_{CC,SS-CMOS})\)

\[
FOM_{BIP} = FOM_{SS-CMOS} + 4.8\text{dB}
\]

- \(V_{CC}=1.8\text{V}, v_{s,BIP}=0.4\text{V}, v_{s,DS-CMOS}=1.2\text{V}, (1.5I_{CC,BIP}=I_{CC,DS-CMOS})\)

\[
FOM_{BIP} = FOM_{DS-CMOS} + 7.8\text{dB}
\]
Phase-Noise Model Conclusions

- Parametric Phase-Noise Model
  - electrical circuit parameters (loop gain)
  - worst-case phase noise (bandwidth unlimited)
- Bipolar vs. CMOS LC-Oscillators
  - bipolar loop-gain related contributions
  - $v_{S,BIP} \sim k (\ll V_{CC}), \; v_{S,CMOS} \sim V_{CC}$
  - bipolar capacitive tapping for larger $v_{S,BIP}$, but also larger $k$-related noise contributions and power consumption
## So Far

### VCO design parameters

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<tr>
<th>Parameter</th>
<th>Design requirement</th>
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<tbody>
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<td>2.1GHz</td>
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<td>Tuning range</td>
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### Technology parameters

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Suppression of Noise in Oscillator’s Tail-Current Source
VCO Phase-Noise

\[ PN = \frac{\text{noise power}(\text{LC - tank}, g_m \text{ cell, current source})}{\text{signal power} \sim k^2} \]

- TCS noise >> LC-tank noise + \( g_m \)-cell noise
  - VCO noise power \sim 1 \text{ or } c \cdot k^2
  - phase noise \sim 1/k^2 \text{ or } \text{const}

- TCS noise suppressed
  - VCO noise power \sim 1 \text{ or } c \cdot k
  - phase noise \sim 1/k^2 \text{ or } 1/k
Bias Noise Reduction Techniques

- **Resistive Degeneration (RD)**
  - High supply required
  - Large area if integrated
  - Noise injection if discrete

- **Inductive Degeneration (RID)**
  - Transconductor noise always ON
  - Reduced output impedance

- **Filtering**
Capacitive Filtering

\[ d' = \frac{1}{2}(1 + d) \]

\[ F'(2I_C) \sim k \frac{n}{2} \]

\[ d = \frac{1}{2k} \]

\[ F(2I_C) \sim \frac{n}{2} \]

- for \( k = 10 \), \( d = 0.5\% \), \( d' = 50.25\% \), and \( F' > F \)
Resonant-Inductive Degeneration (RID)

- high TCS noise suppression
- no voltage headroom
- integration
- integrated degenerative inductor \((L_{RID})\) matched with base-emitter capacitance \((C_\Pi)\) at \(2f_0\)
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