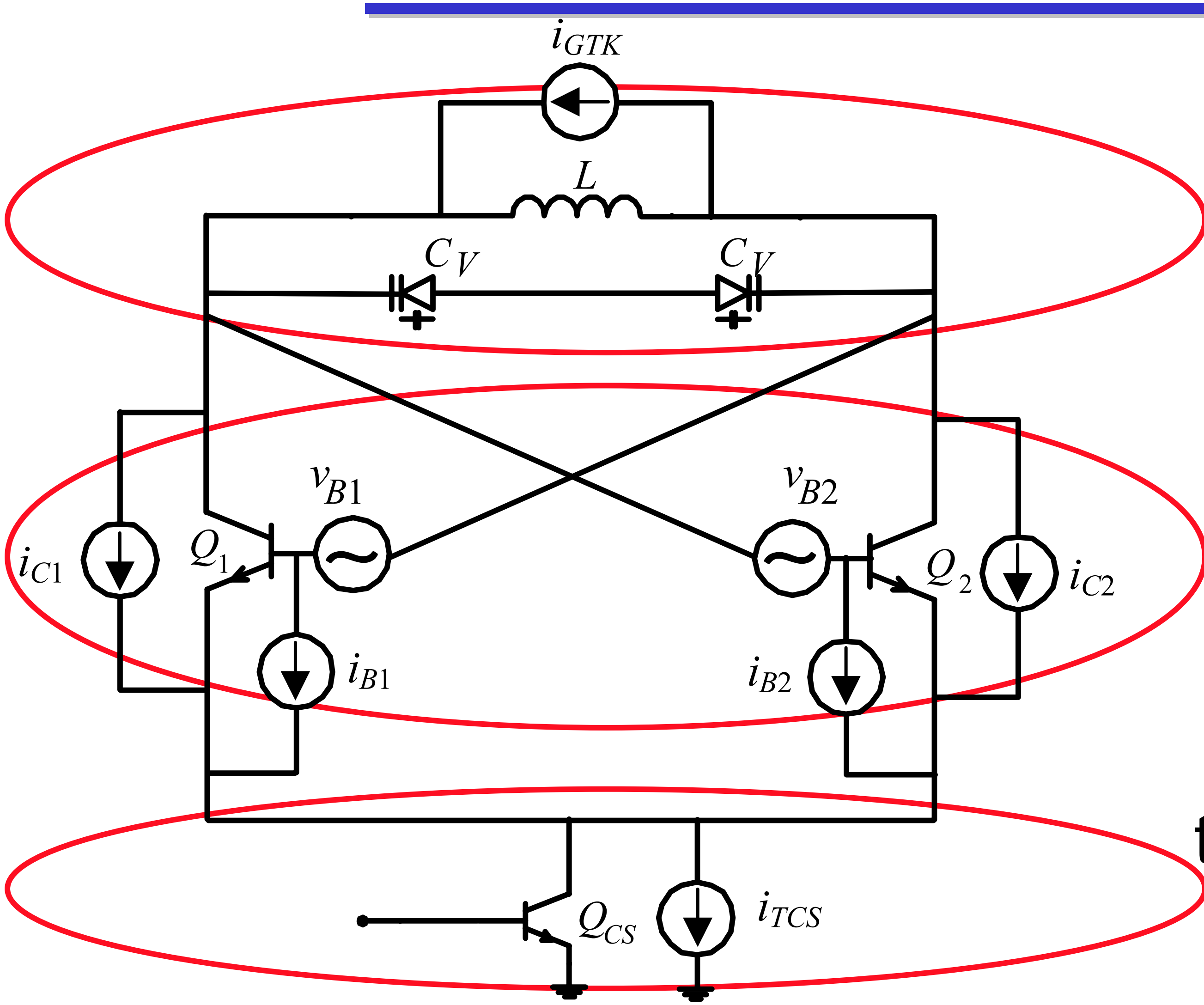
Spectral Analysis of Noise in Switching LC-Oscillators

Sub-Outline

- Duty Cycle of g_m-cell Small-Signal Gain
- Oscillation Condition
- LC-Tank Noise
- g_m-cell Noise
- Tail-Current Source Noise
- (Phase) Noise Factor – Bipolar VCO
- (Phase) Noise Factor – CMOS VCO
- Bipolar vs. CMOS VCO

Is it Indeed so Simple?



LC-tank noise

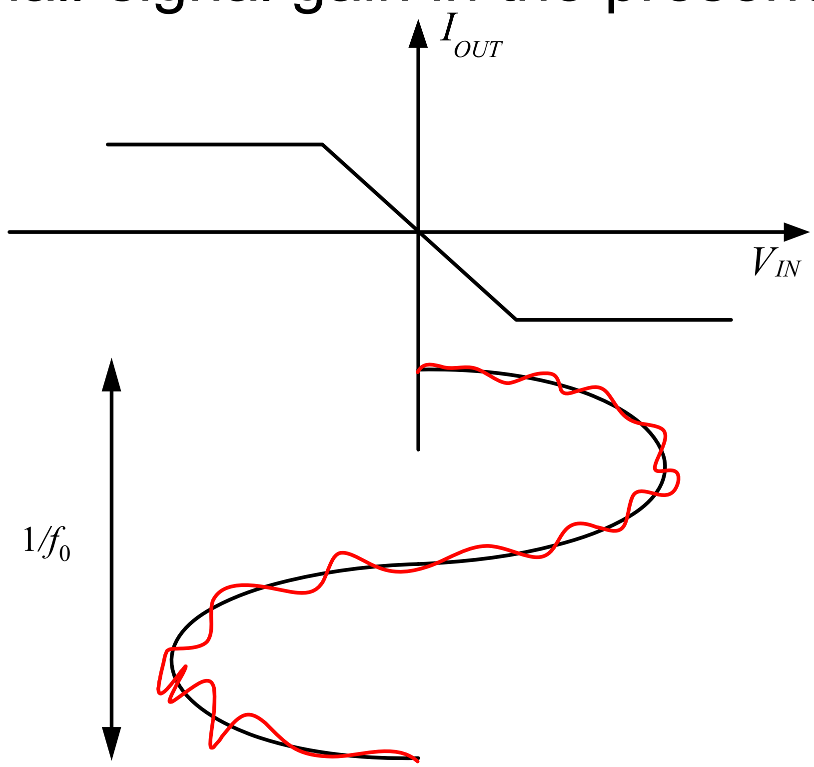
transconductor noise

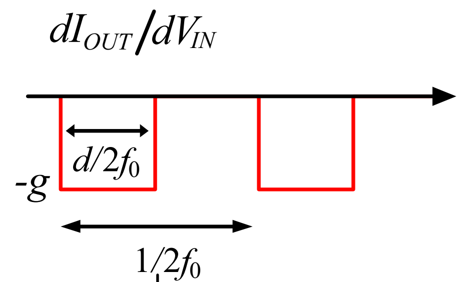
tail-current source noise

- noise from the transistors Q_1 and Q_2 is switched ON and OFF
- noise from current source Q_{CS} is modulated by oscillator switching

g_m-cell Transfer Function

- small-signal gain in the presence of a large signal



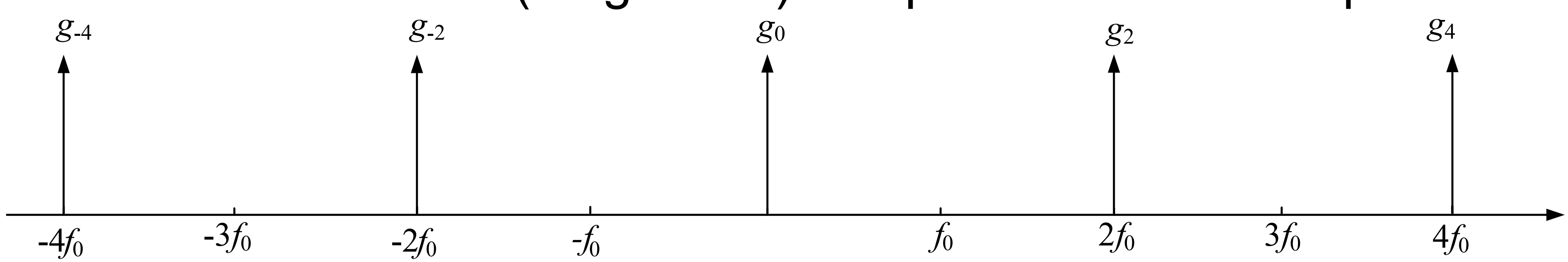


$$g_{2i} = \left| \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} g(t) e^{-j2i\omega_0 t} dt \right|$$

$$g_{2i} = \left| gd \frac{\sin(i\pi d)}{i\pi d} \right|$$

$$d = \frac{1}{2k}$$

- Fourier domain – (magnitude) complex harmonic components



LC-Tank Noise Contribution

- phase-modulating noise component:

$$i_{PM} = \frac{1}{2}\left[i_{N,O}(f_0 + \Delta) + i_{N,O}(-f_0 + \Delta) + i_{N,O}(-f_0 - \Delta) + i_{N,O}(f_0 - \Delta)\right]$$

$$i_{PM} = (g_0 + g_2)v_N(f_0 + \Delta) + (g_0 + g_2)v_N(f_0 - \Delta)$$

- phase-related noise power (k>>1, g=g_0=$g_{\pm 2}$):

$$i^2_{PM}(R_{TK}) = (g_0 + g_2)^2 v^2_N(R_{TK})$$

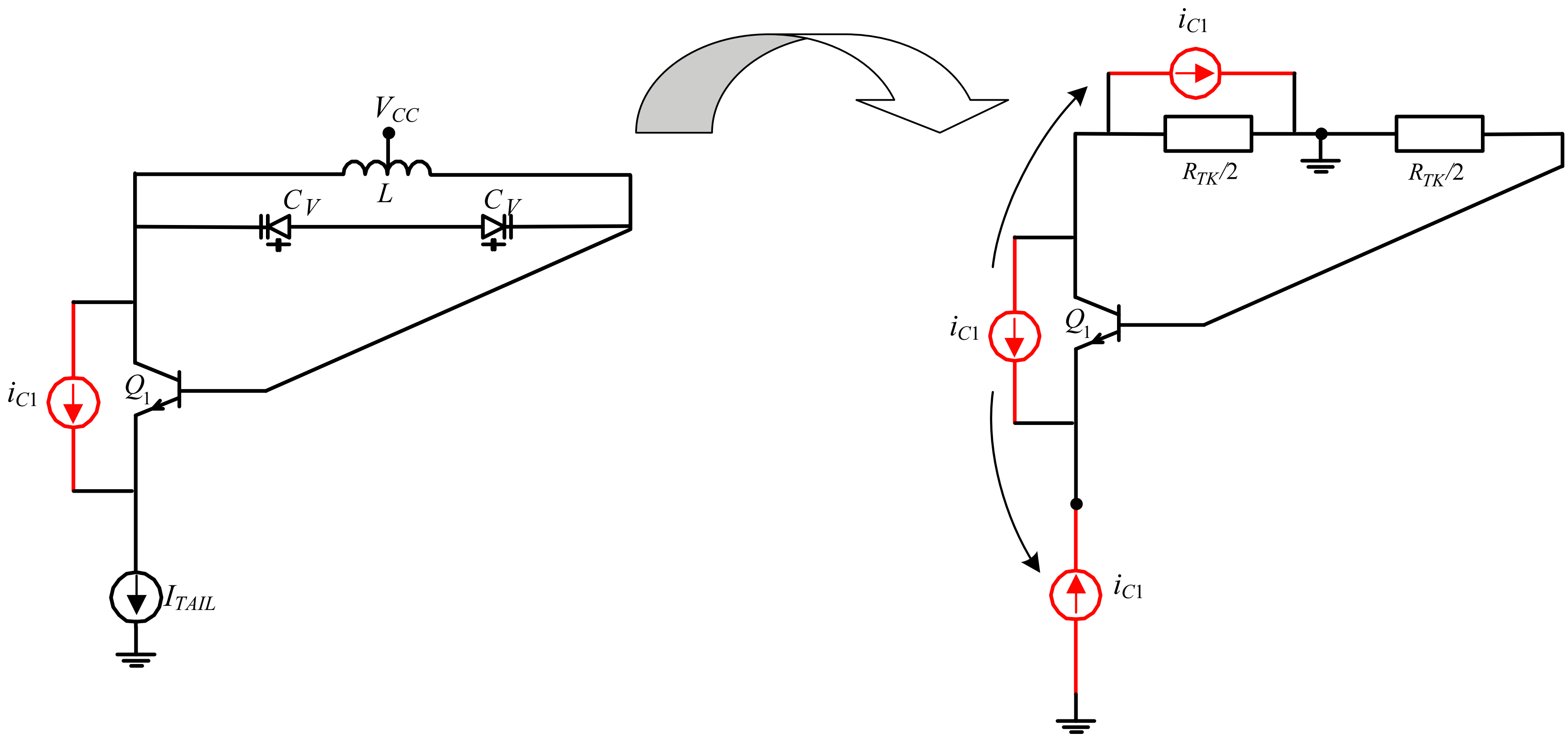
- LC-tank noise transfer function:

$$g^2(R_{TK}) = (g_0 + g_2)^2 = G^2_{TK}$$

- LC-tank noise factor:

$$F(R_{TK}) = \frac{2i^2_{PM}(R_{TK})}{4KTG_{TK}} = \frac{2(g_0 + g_2)^2 v^2_N(R_{TK})}{4KTG_{TK}} = \frac{2G^2_{TK} v^2_N(R_{TK})}{4KTG_{TK}} = \frac{4KTG_{TK}}{4KTG_{TK}} = 1$$

Collector-Current Noise Transfer Function in Limiting Region



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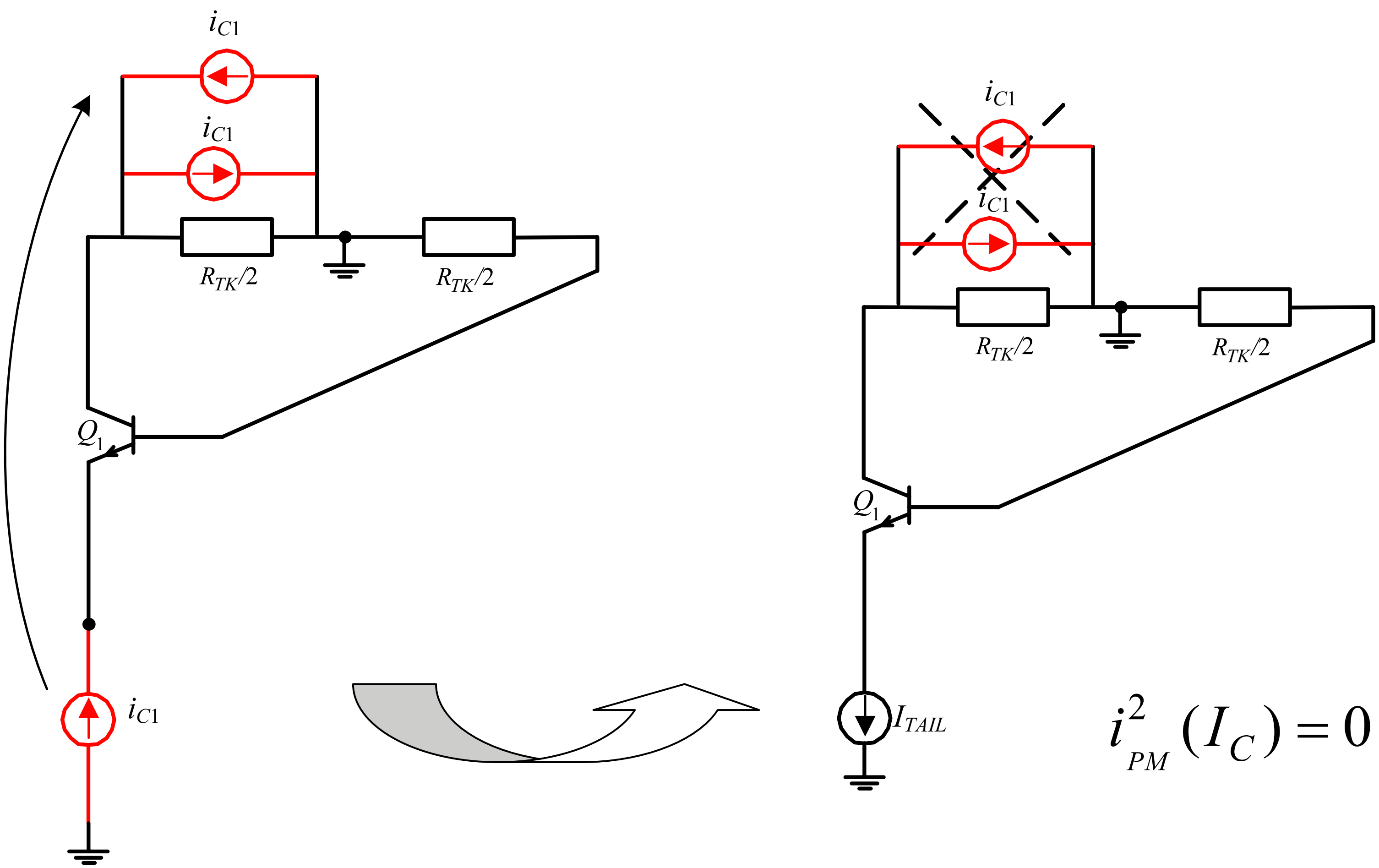
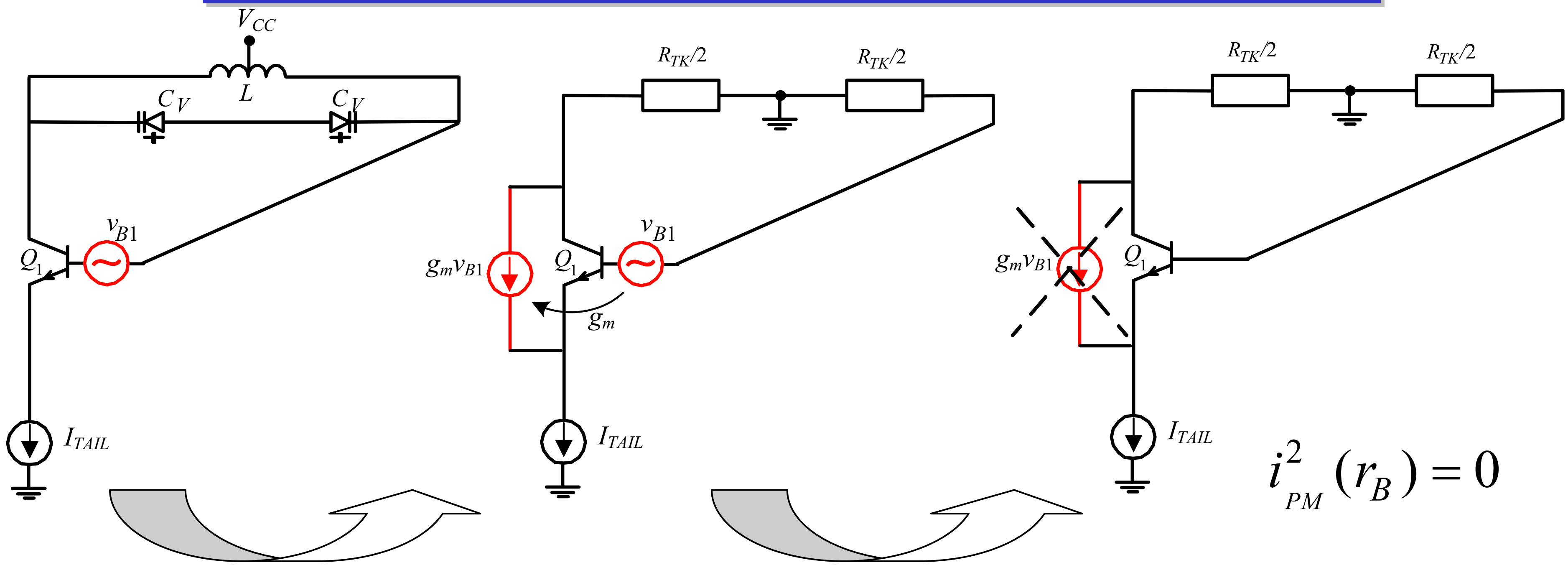


$$i_{PM}^2(I_C) = 0$$

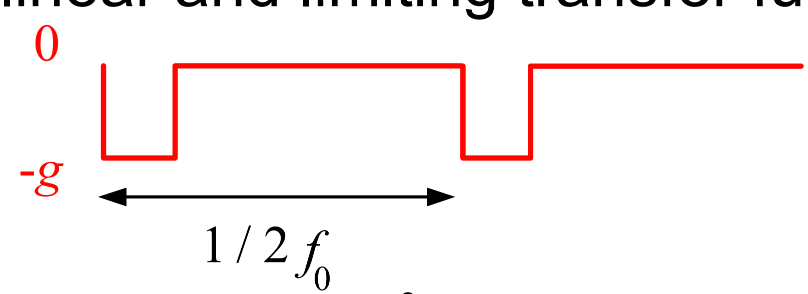
Base-Resistance Noise Transfer Function in Limiting Region



$$i^2_{PM}(r_B) = 0$$

- g_m-cell input-noise linear and limiting transfer function:

$$g(v_{N,g_{m-IN}}):$$



$$v_N(g_{m-IN}) = 2(v_N(r_B) + i_N(I_C)/g_m^2) = 4KTr_B + 2KT/g_m$$

g_m-Cell Noise Contributions

- phase-related noise power:

$$i_{PM}^2(g_{m-IN}) = \frac{1}{2d}(g_{2i-2} + g_{2i})^2 v_N^2(g_{m-IN}) = \frac{2g_{2i}^2}{d} v_N^2(g_{m-IN}) = 2dg^2 v_N^2(g_{m-IN})$$

- g_m-cell noise transfer function:

$$g^2(g_{m-IN}) = 2dg^2 = 2d(\frac{g_m}{2})^2 = 2\frac{1}{2k}(kG_{TK})^2 = kG_{TK}^2$$

- g_m-cell noise factors:

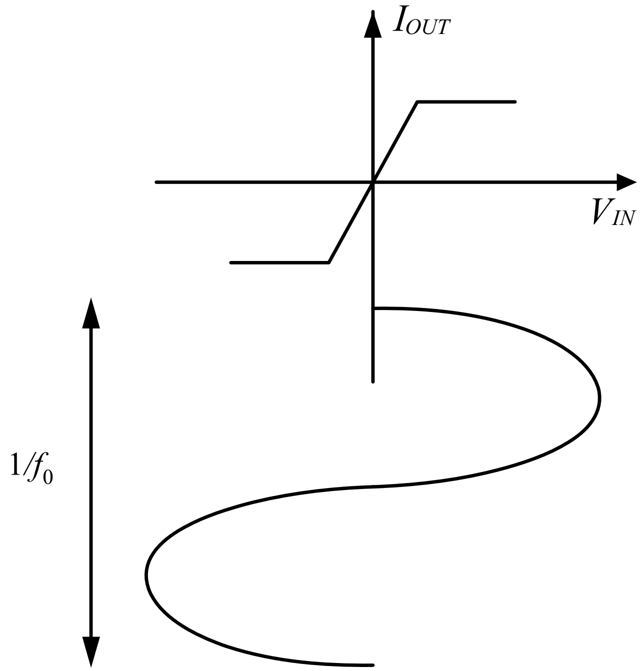
$$F(2r_B) = \frac{2i_{PM}^2(2r_B)}{4KTG_{TK}} = \frac{2kG_{TK}^2 4KTr_B}{4KTG_{TK}} = 2kr_B G_{TK} = kc$$

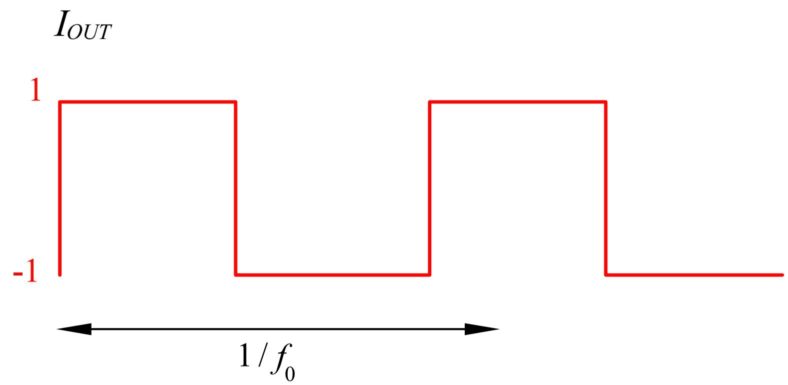
$$F(2I_C) = \frac{2i_{PM}^2(2I_C)}{4KTG_{TK}} = \frac{2kG_{TK}^2 2KT/g_m}{4KTG_{TK}} = kG_{TK}/g_m = \frac{1}{2}$$

- input g_m-cell noise around odd multiples of the oscillation frequency is folded to the LC-tank noise around the oscillation frequency

Tail-Current Noise Transfer Function

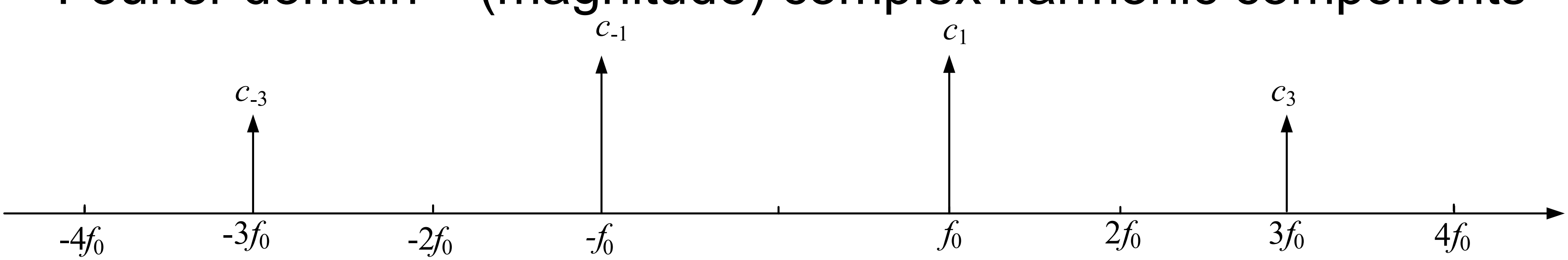
- g_m-cell large-signal *V*-to-*I* transfer function





$$c_{2i+1} = \left| \frac{\sin((2i+1)\pi/2)}{(2i+1)\pi/2} \right|$$

- Fourier domain – (magnitude) complex harmonic components



TCS-Noise Contribution

• phase-related noise power:

$$i_{PM}^2(I_{TCS}) = \frac{i_{PM,DIFF}^2(I_{TCS})}{4} = \frac{1}{4} i_N^2(I_{TCS})$$

• TCS-noise transfer function:

$$g^2(I_{TCS}) = \frac{1}{4}$$

• TCS-noise factor:

$$F(I_{TCS}) = \frac{2i_{PM}^2(I_{TCS})}{4KTG_{TK}} = \frac{KTg_m(1+2r_B g_m)}{4KTG_{TK}} = \frac{KT2kG_{TK}(1+2kc)}{4KTG_{TK}} =$$

$$= \frac{1}{2}k(1+2kc) = k(\frac{1}{2}+kc) = k\left(F(2I_C+2I_B)+F(2r_B)\right)$$

• TCS noise around even multiples of the oscillation frequency is folded to the LC-tank noise around the oscillation frequency

Switching-Oscillator Phase Noise

- Noise factor:

$$F = F(R_{TK}) + F(2I_C) + F(2I_B) + F(2r_B) + F(I_{TCS})$$

$$F = 1 + \frac{1}{2} + \frac{1}{2\beta} + kc + k(\frac{1}{2} + kc) \cong 1 + (1+k)(\frac{1}{2} + kc)$$

- Phase noise:

$$\mathcal{L} = \frac{\mathcal{L}(R_{TK}) + \mathcal{L}(2I_C) + \mathcal{L}(2I_B) + \mathcal{L}(2r_B) + \mathcal{L}(I_{TCS})}{(4\pi C_{TOT}\Delta)^2} = \frac{4KTG_{TK}F}{v_s^2(4\pi C_{TOT}\Delta)^2}$$

$$\mathcal{L} = \frac{4KTG_{TK}}{(4\pi C_{TOT}\Delta)^2}(\frac{\pi}{8V_T})^2 \frac{1 + (1+k)(\frac{1}{2} + kc)}{k^2}$$

Phase Noise Model of Bipolar Switching LC-Oscillators

$$F = 1 + \frac{1}{2} + kc + k(\frac{1}{2} + kc)$$

- **Constant phase-noise contributions**
  - LC-tank noise contribution ~ 1
  - g_m-cell current shot noise contribution ~ ½
- **Loop-gain related contributions**
  - g_m-cell base-resistance noise contribution ~ *ck*
  - phase-noise contribution of the bias current source is *k*-times larger then the noise contribution of the g_m-cell ~ *k*(½+*ck*)!

Low/High-Performance VCO Designs

- high loop-gain, high quality LC-tank, BCS noise eliminated:

  e.g., *k*>>1(=10), *c*<<1(~0.01)

$$\mathcal{L} \sim \frac{1+\frac{1}{2}+kc}{k^2} = \frac{8}{5}\frac{1}{10}\frac{1}{10} \sim \frac{1}{k^2}$$

- high loop-gain, high quality LC-tank:

  e.g., *k*>>1(=10), *c*<<1(~0.01)

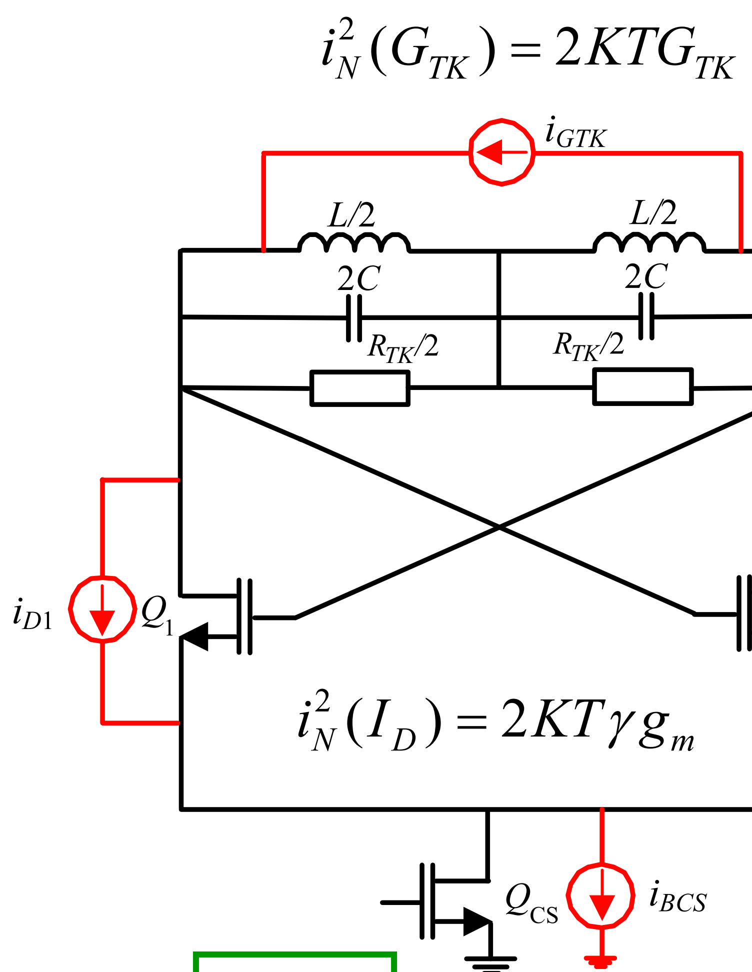
$$\mathcal{L} \sim \frac{k(\frac{1}{2}+kc)}{k^2} = \frac{\frac{1}{2}+kc}{k} = \frac{3}{5}\frac{1}{10} \sim \frac{1}{k}$$

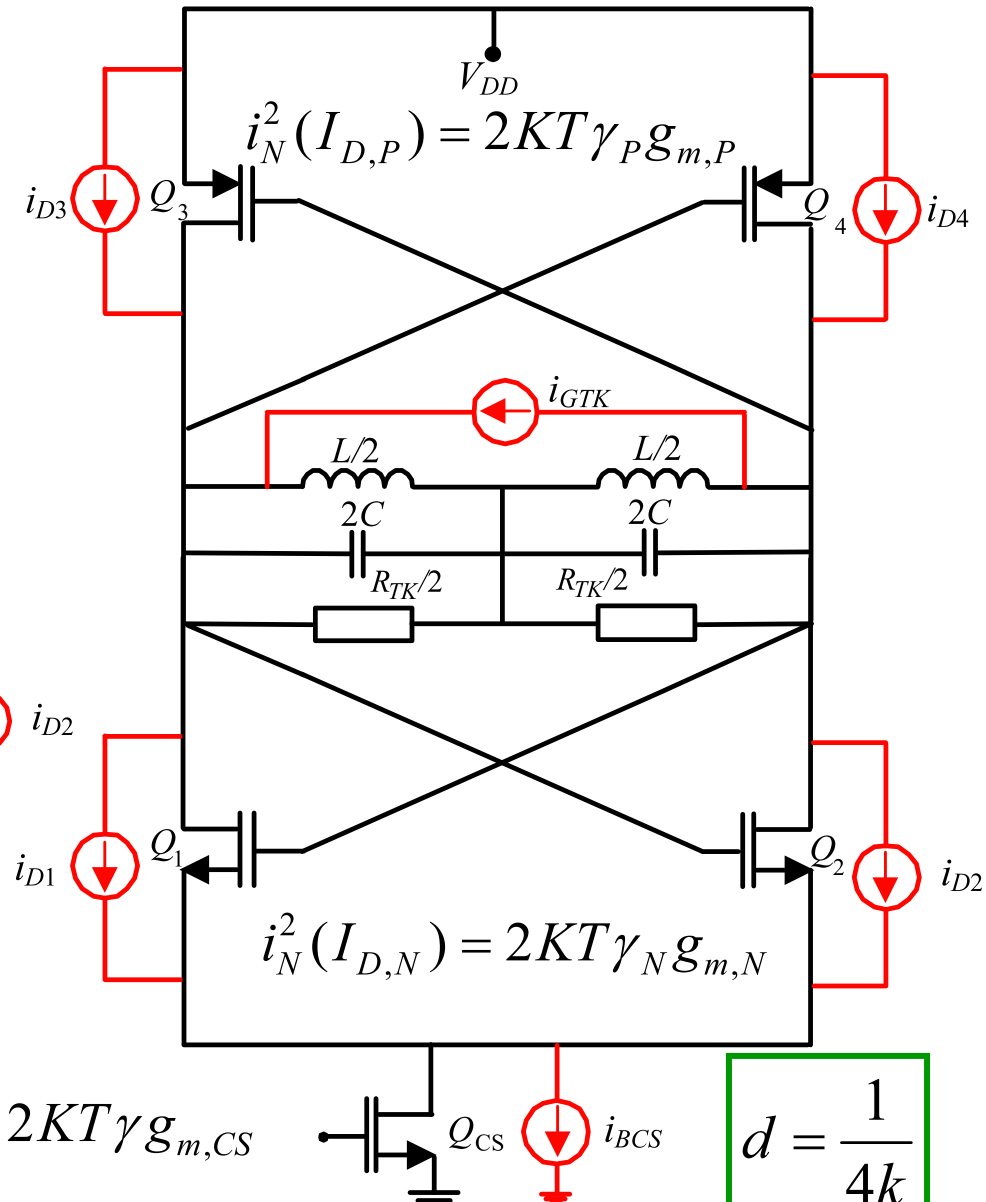
- low loop-gain, low quality LC-tank:

  e.g., *k*~1(=2), *c*~1(~0.5)

$$\mathcal{L} \sim \frac{1+(1+k)\frac{3}{2}}{k^2} = \frac{11}{8} \sim 1$$

Single/Double-Switch CMOS LC-VCOs





$$d = \frac{1}{2k}$$

$$i_N^2(I_{BCS}) = 2KT\gamma g_{m,CS}$$

$$d = \frac{1}{4k}$$

Phase Noise Model of CMOS LC-Oscillators

- Noise factor ($\gamma_N = \gamma_P$, $g_{m,N} = g_{m,P}$, $g_{m,N,CS} = 2g_{m,N}$):

$$F_{SS} = F(R_{TK}) + F(2I_D) + F(I_{BCS})$$
$$F_{SS} = 1 + \gamma + k\gamma = 1 + (1+k)\gamma$$

$$F_{DS} = F(R_{TK}) + F(4I_D) + F(I_{BCS})$$
$$F_{DS} = 1 + \gamma + 2k\gamma = 1 + (1+2k)\gamma$$

- Phase noise:

$$\mathcal{L} = \frac{\mathcal{L}(R_{TK}) + \mathcal{L}(2I_D) + \mathcal{L}(I_{BCS})}{(4\pi C_{TOT}\Delta)^2} = \frac{4KTG_{TK}F}{v_s^2(4\pi C_{TOT}\Delta)^2}$$

$$\mathcal{L}_{SS} = \frac{4KTG_{TK}}{(4\pi C_{TOT}\Delta)^2}\frac{1+(1+k)\gamma}{(\frac{2}{\pi}I_{TAIL}R_{TK})^2}$$

$$\mathcal{L}_{DS} = \frac{4KTG_{TK}}{(4\pi C_{TOT}\Delta)^2}\frac{1+(1+2k)\gamma}{(\frac{4}{\pi}I_{TAIL}R_{TK})^2}$$

Phase Noise Model of CMOS LC-Oscillators

$$F = 1 + \gamma + \alpha k \gamma$$

- **Constant phase-noise contributions**
  - LC-tank noise contribution ~ 1
  - g_m-cell drain-current thermal noise contribution ~ γ
- **Loop-gain related contributions**
  - bias current source noise contribution ~ $k\gamma$

CMOS vs. Bipolar LC-Oscillators

- noise factors (for removed BCS noise):

$$F_{BIP} = 1 + \frac{1}{2} + kc$$

$$F_{CMOS} = 1 + \gamma$$

- bipolar VCO better for the same power consumption ($v_{s,BIP}=v_{s,CMOS}$)

$$\frac{3}{2} + kc < \frac{5}{3} \quad \Rightarrow \quad \boxed{kr_B < \frac{R_{TK}}{12}}$$

- e.g., k=10, R_{TK}=1000Ω

$$r_B < \frac{1000}{120} = 8.4\Omega$$

- e.g., k=10, R_{TK}=100Ω

$$r_B < \frac{100}{120} = 0.84\,\Omega$$

CMOS vs. Bipolar LC-Oscillators

- power consumption figure of merit:

$$FOM = 10\log\left(\mathcal{L}(\Delta)\left(\frac{\Delta}{\omega_0}\right)^2 V_{CC} I_{CC}\right)$$

- V_{CC}=1.8V, $v_{s,BIP}$=0.4V, $v_{s,SS\text{-}CMOS}$=1.2V, ($3I_{CC,BIP}=I_{CC,SS\text{-}CMOS}$)

$$FOM_{BIP} = FOM_{SS-CMOS} + 4.8dB$$

- V_{CC}=1.8V, $v_{s,BIP}$=0.4V, $v_{s,DS\text{-}CMOS}$=1.2V, ($1.5I_{CC,BIP}=I_{CC,DS\text{-}CMOS}$)

$$FOM_{BIP} = FOM_{DS-CMOS} + 7.8dB$$

Phase-Noise Model Conclusions

- Parametric Phase-Noise Model
  - electrical circuit parameters (loop gain)
  - worst-case phase noise (bandwidth unlimited)
- Bipolar vs. CMOS LC-Oscillators
  - bipolar loop-gain related contributions
  - $v_{S,\mathrm{BIP}} \sim k$ ($\ll V_{CC}$), $v_{S,CMOS} \sim V_{CC}$
  - bipolar capacitive tapping for larger $v_{S,\mathrm{BIP}}$, but also larger *k*-related noise contributions and power consumption

So Far

VCO design parameters	*Design requirement*
Oscillating frequency	2.1GHz
Tuning range	400MHz
Voltage swing	0.7V
Phase noise	-110dBc@1MHz
Supply voltage	3V
Power consumption	10mW

Technology parameters	*Values*
Technology	BiCMOS
Number of metals	4
Transit frequency	50GHz
MIM capacitors	available
Varactors	available

Suppression of Noise in Oscillator’s Tail-Current Source

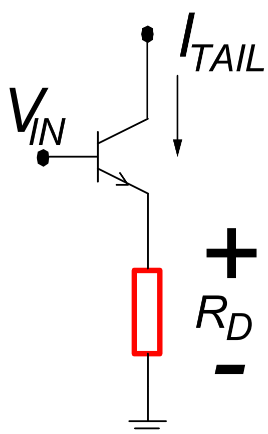
VCO Phase-Noise

$$PN = \frac{\text{noise power(LC - tank,- } g_m \text{ cell, current source)}}{\text{signal power } (\sim k^2)}$$

- TCS noise >> LC-tank noise + g_m-cell noise
  - VCO noise power ~ 1 or $c \bullet k^2$
  - phase noise ~ $1/k^2$ or *const*
- TCS noise suppressed
  - VCO noise power ~ 1 or $c \bullet k$
  - phase noise ~ $1/k^2$ or $1/k$

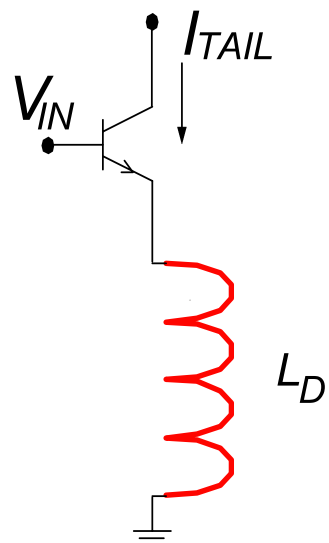
Bias Noise Reduction Techniques

resistive degeneration



☹ high supply required

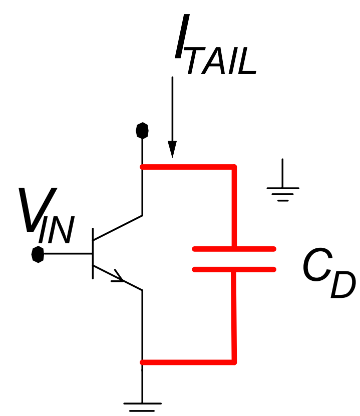
inductive degeneration



☹ large area if integrated

☹ noise injection if discrete

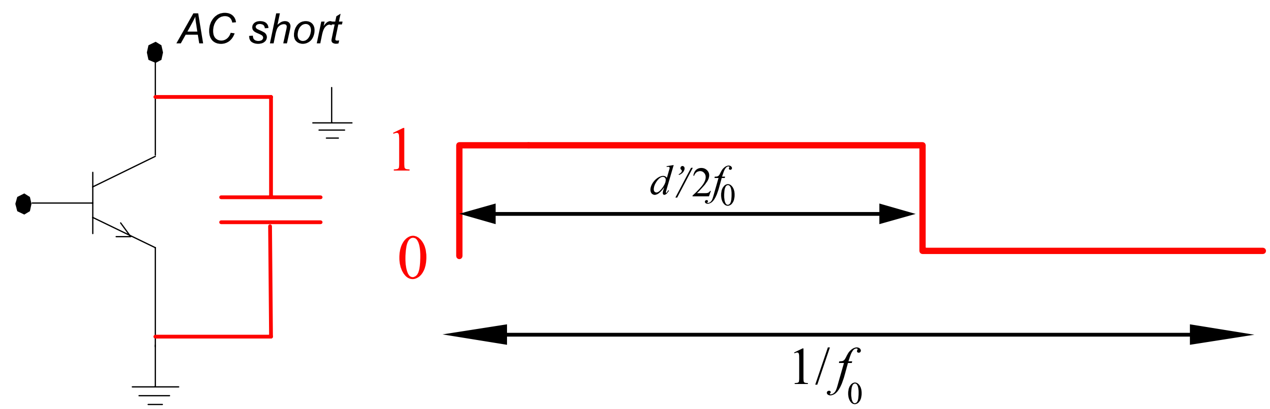
filtering?



☹ transconductor noise always ON

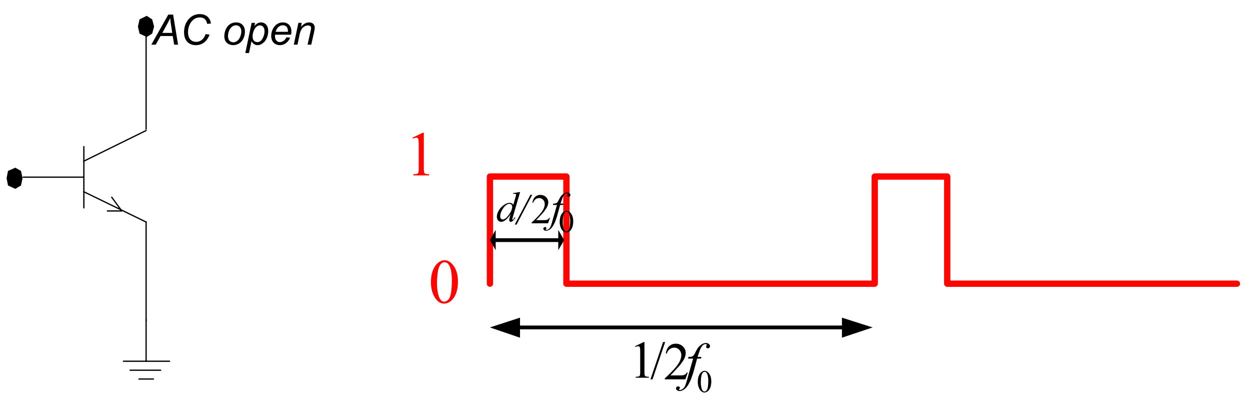
☹ reduced output impedance

Capacitive Filtering



$$d' = \frac{1}{2}(1+d)$$

$$F'(2I_C) \sim k\frac{n}{2}$$



$$d = \frac{1}{2k}$$

$$F(2I_C) \sim \frac{n}{2}$$

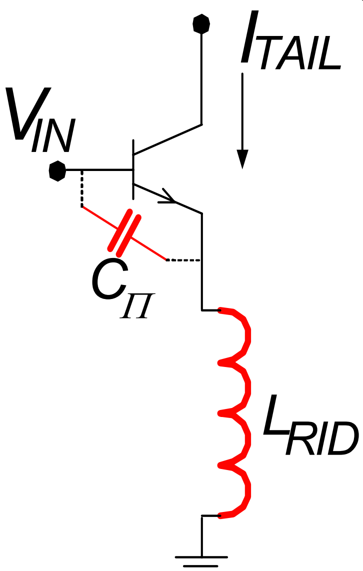
- for k=10, d=0.5%, d'=50.25%, and F'>F

Resonant-Inductive Degeneration (RID)

high TCS noise suppression

no voltage headroom

integration



integrated degenerative inductor (L_{RID}) matched with base-emitter capacitance (C_{Π}) at $2f_0$

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