AE4520: Advanced Structural Analysis

Stress in 2D and 3D

Mostafa Abdalla

Learning Objectives

- Understand the nature of internal forces
- Understand the dependence of tractions on orientation
- Define the stress tensor
- Derive the equilibrium equations
- Understand the symmetry of the stress tensor

Summary

- Traction is force per unit area. Traction is a vector.
- At a point in the body tractions depend on surface orientation
- Equilibrium necessitates that tractions are linear in orientation.
- Stress is the tensor that maps orientations to tractions
- Equilibrium of moments requires the stress tensor to be symmetric

Equilibrium equations

- Cartesian components

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

- Equilibrium equations

Axis	*Force*	*Moment*
x	$\sigma_{x,x} + \tau_{yx,y} + \tau_{zx,z} + b_x = 0,$	$\tau_{yz} - \tau_{zy} = 0$
y	$\tau_{xy,x} + \sigma_{y,y} + \tau_{zy,z} + b_y = 0,$	$\tau_{zx} - \tau_{xz} = 0$
z	$\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} + b_z = 0,$	$\tau_{xy} - \tau_{yx} = 0$