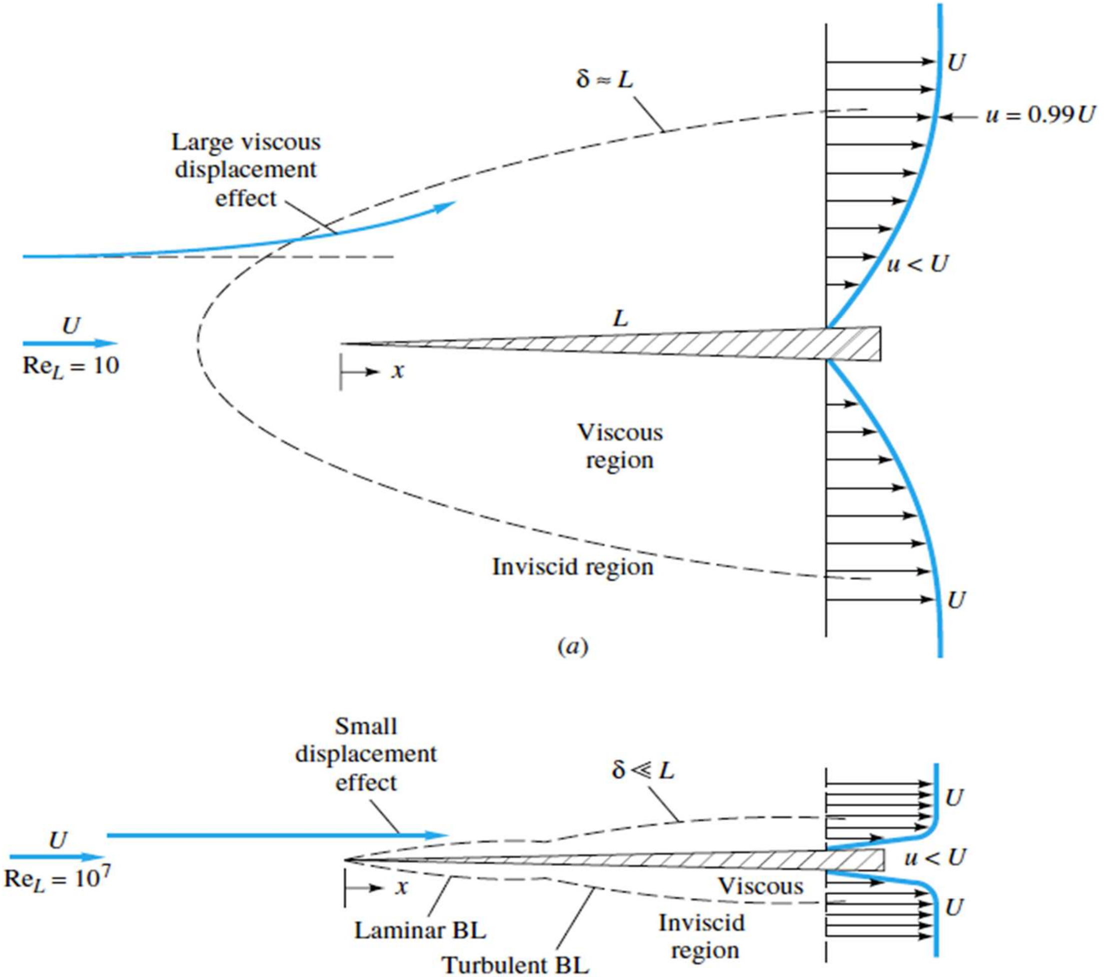
Fluid mechanics (wb1225)

Lecture 10: boundary layers

Flow past a thin plate



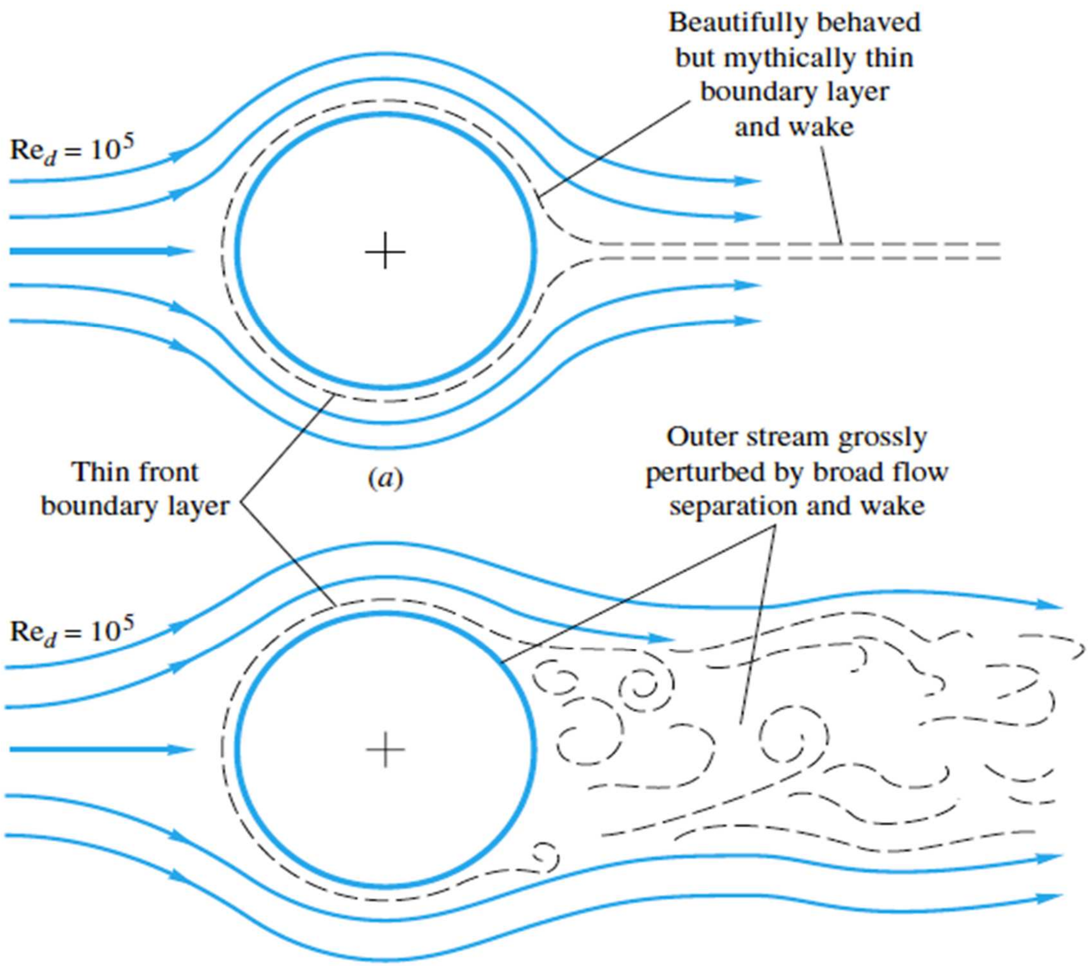
laminar flow:

$$\frac{\delta}{x} \approx 5.0\,\mathrm{Re}_x^{-1/2}$$

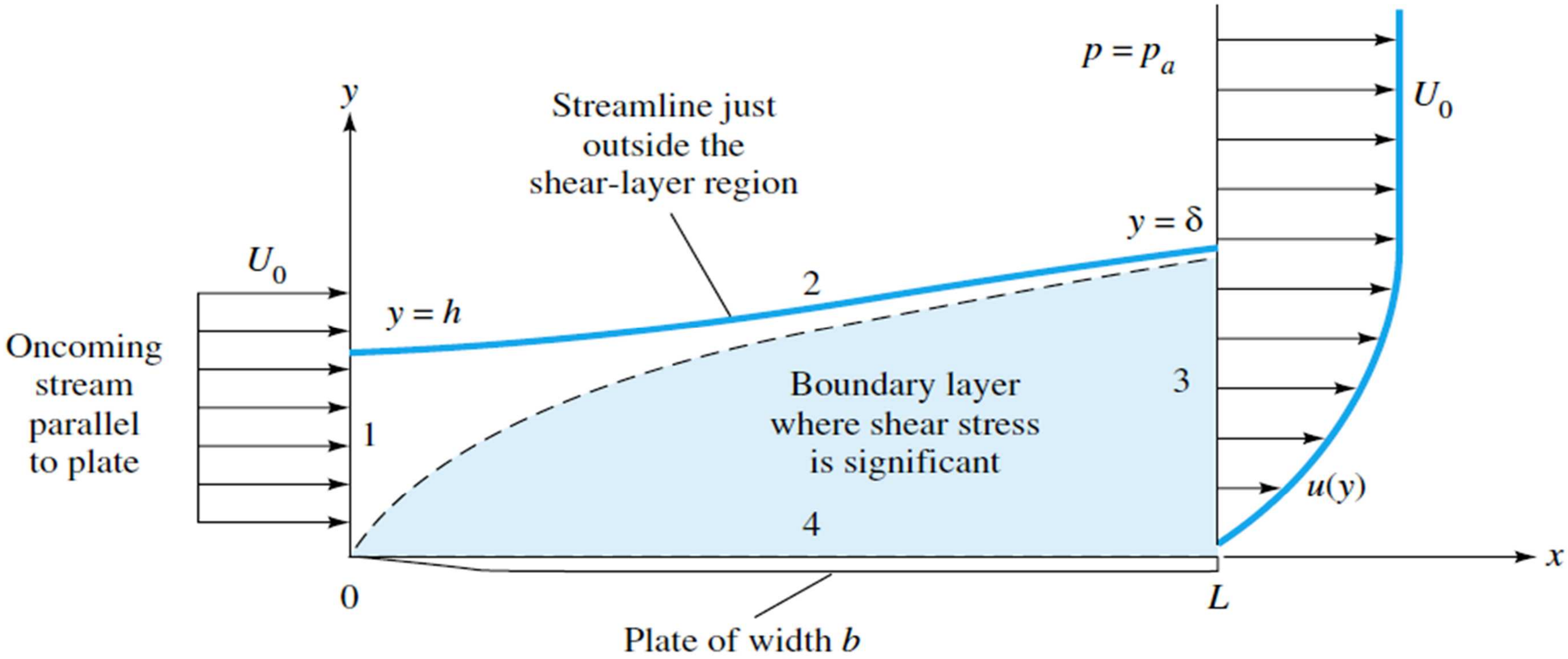
turbulent flow:

$$\frac{\delta}{x} \approx 0.16\,\mathrm{Re}_x^{-1/7}$$

Flow around an object



Boundary layer



Example 3.11

$$\sum F_x = -D = \rho\int_1 u\,V_n\,dA + \rho\int_3 u\,V_n\,dA$$

$$= \rho\int_0^h U_0\left(-U_0\right)b\,dy + \rho\int_0^\delta u(y)\left(+u(y)\right)b\,dy$$

$$\Rightarrow \quad D = \rho U_0^2 bh - \rho b\int_0^\delta u^2(y)\,dy$$

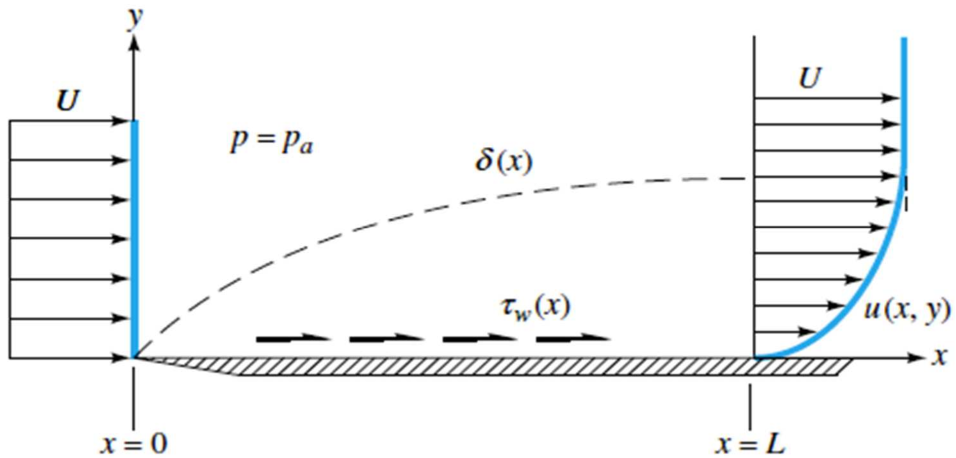
$h \rightarrow \delta$? $\Rightarrow$ conservation of mass $\quad U_0 h = \int_0^\delta u(y)\,dy$

$$D = \rho b\int_0^\delta \underbrace{u\left(U_0 - u\right)}dy$$

momentum defect

Theodore von Kármán (1921)

Growth of a boundary layer



momentum integral:

$$D(x) = \rho b \int_0^{\delta(x)} u(U-u)\,dy$$

$$D(x) = \rho b U^2 \theta, \quad \theta = \int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right) dy$$

momentum thickness

drag equals integrated wall shear stress:

$$D(x) = b\int_0^x \tau_w(x)\,dx \quad \Rightarrow \quad \frac{dD}{dx} = b\tau_w$$

also:

$$\frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

momentum integral relation:

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

$$\text{assume: } u(x,y) \approx U\left(\frac{2y}{\delta(x)} - \frac{y^2}{\delta(x)^2}\right), \quad 0 \le y \le \delta(x) \quad \Rightarrow \quad \frac{\delta}{x} \approx 5.5\left(\frac{\nu}{Ux}\right)^{1/2} = 5.5\,\mathrm{Re}_x^{-1/2}$$

$$\tau_w = \rho U^2 \frac{d\theta}{dx} \qquad \theta = \frac{2}{15}\delta \qquad \tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = \frac{2\mu U}{\delta} \quad \Rightarrow \quad \frac{2\mu U}{\delta} = \frac{2}{15}\rho U^2 \frac{d\delta}{dx}$$

Turbulent boundary layer

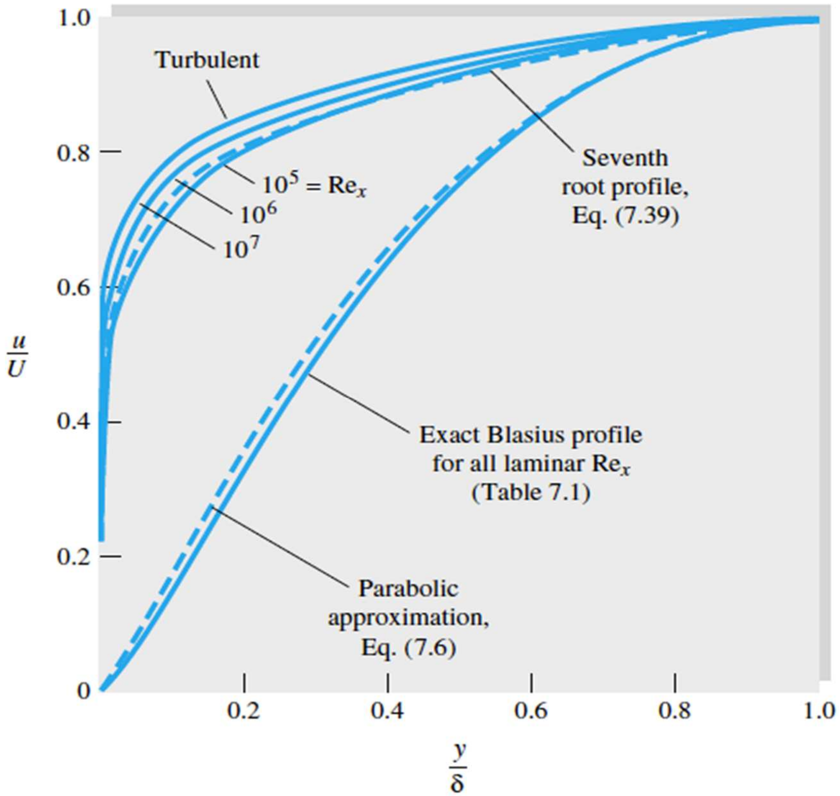
$$\tau_w = \rho U^2 \frac{d\theta}{dx} \qquad \frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} \qquad \theta = \frac{7}{72}\delta$$

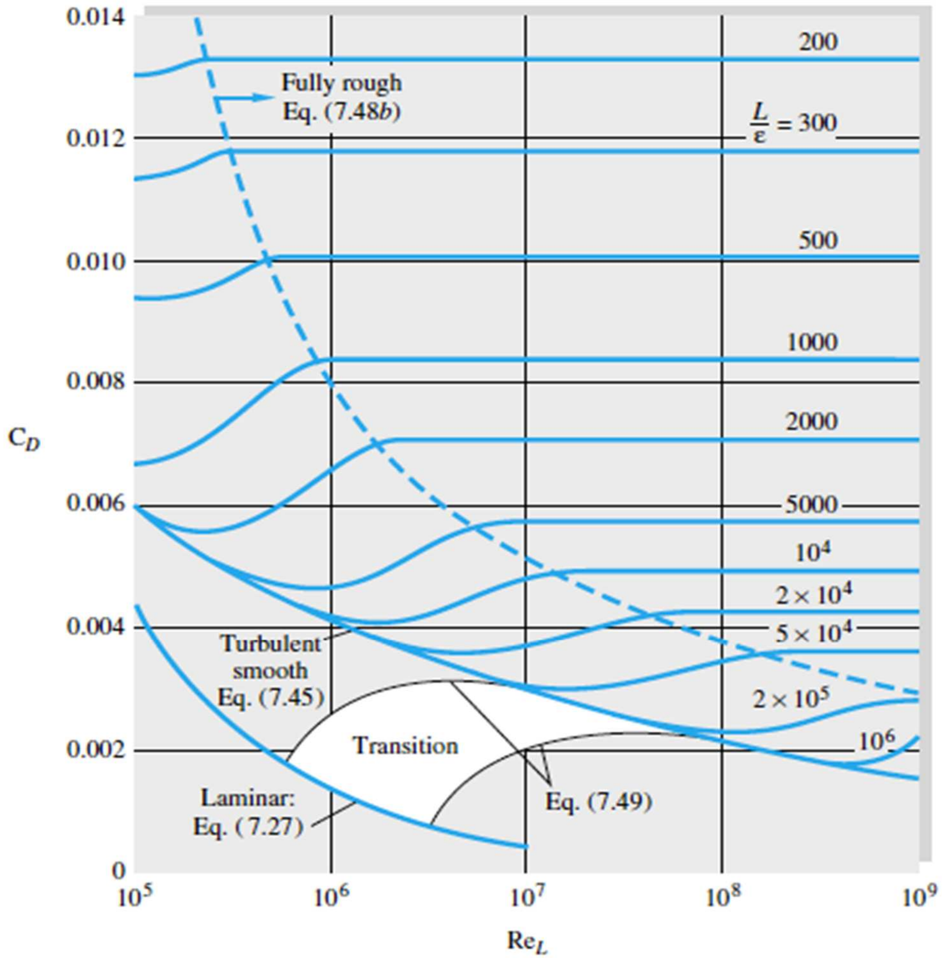
$$\text{empirical:} \quad \tau_w \cong 0.01 \rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/6}$$

$$\Rightarrow \quad \frac{\delta}{x} \cong \frac{0.16}{\mathrm{Re}_x^{1/7}}$$

Flat-plate boundary layer

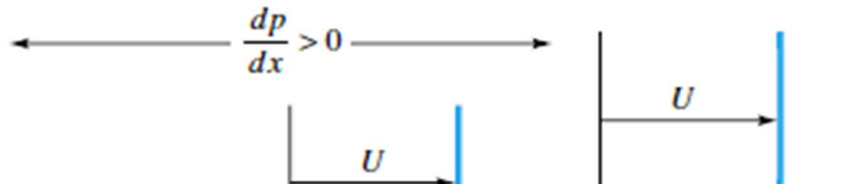
$$C_D = \frac{D(L)}{\frac{1}{2}\rho U^2 bL}$$

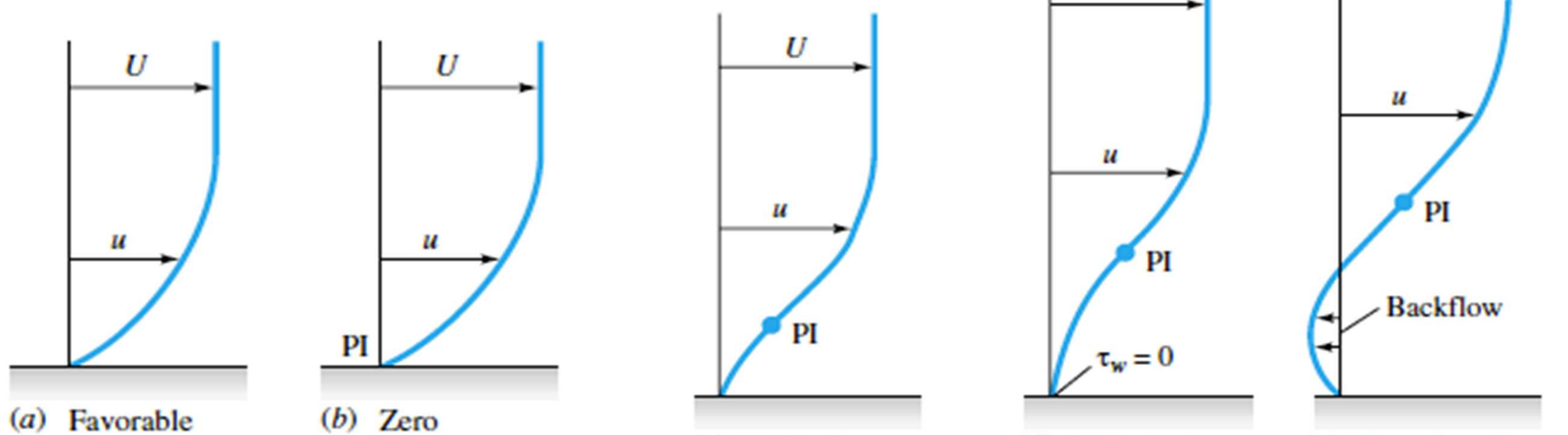




Effect of pressure gradient

$$\left.\frac{\partial \tau}{\partial y}\right|_{\text{wall}} = \mu \left.\frac{\partial^2 u}{\partial y^2}\right|_{\text{wall}} = \frac{dp}{dx} = -\rho U \frac{dU}{dx}$$





(a) Favorable gradient: $\frac{dU}{dx} > 0$, $\frac{dp}{dx} < 0$. No separation, PI inside wall

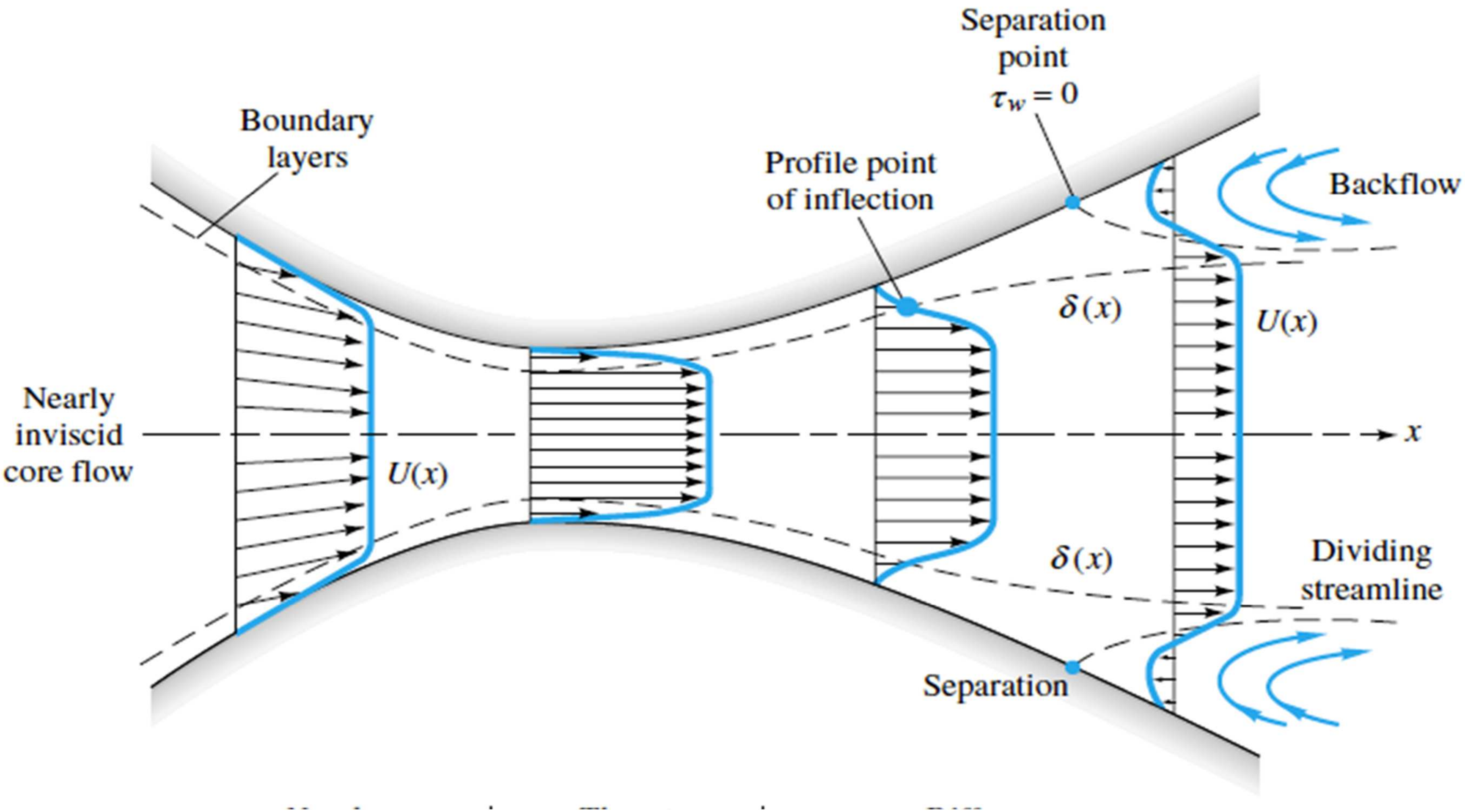
(b) Zero gradient: $\frac{dU}{dx} = 0$, $\frac{dp}{dx} = 0$. No separation, PI at wall

(c) Weak adverse gradient: $\frac{dU}{dx} < 0$, $\frac{dp}{dx} > 0$. No separation, PI in the flow

(d) Critical adverse gradient: Zero slope at the wall: *Separation*

(e) Excessive adverse gradient: Backflow at the wall: Separated flow region

Nozzle-diffuser



Boundary layer integral theorem

$$\frac{\tau_w}{\rho U^2} = \tfrac{1}{2} c_f = \frac{d\theta}{dx} + (2 + H)\frac{\theta}{U}\frac{dU}{dx}$$

shape factor:

$$H = \frac{\delta^*}{\theta}$$

$$H = \begin{cases} 3.5 & \text{laminar flow} \\ 2.4 & \text{turbulent flow} \end{cases}$$

momentum thickness:

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

displacement thickness:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

NB: $\delta = \delta_{0.99}$

Summary

- Chapter 7:
- Examples:
- Problems:

Source

All images are from the book of Frank M. White, *Fluid Mechanics*, McGraw-Hill Series in Mechanical Engineering.