Fluid mechanics (wb1225)

Lecture 13: turbomachinery

Classification of pumps

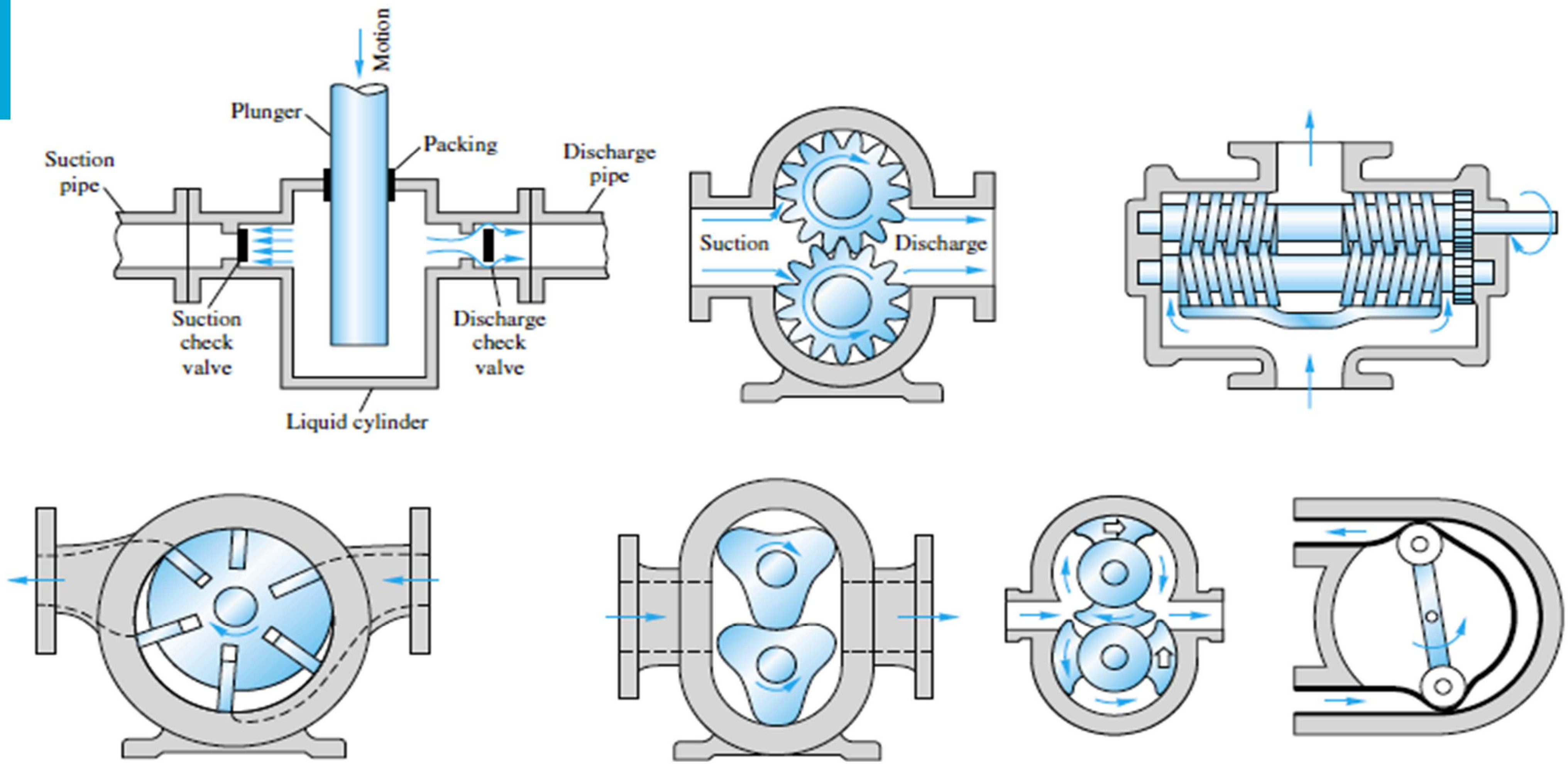
Positive displacement

- Reciprocating
  (piston pump)
- Rotary
  - Single rotor
    (screw pump,
    peristaltic pump)
  - Multiple rotor
    (gear pump)

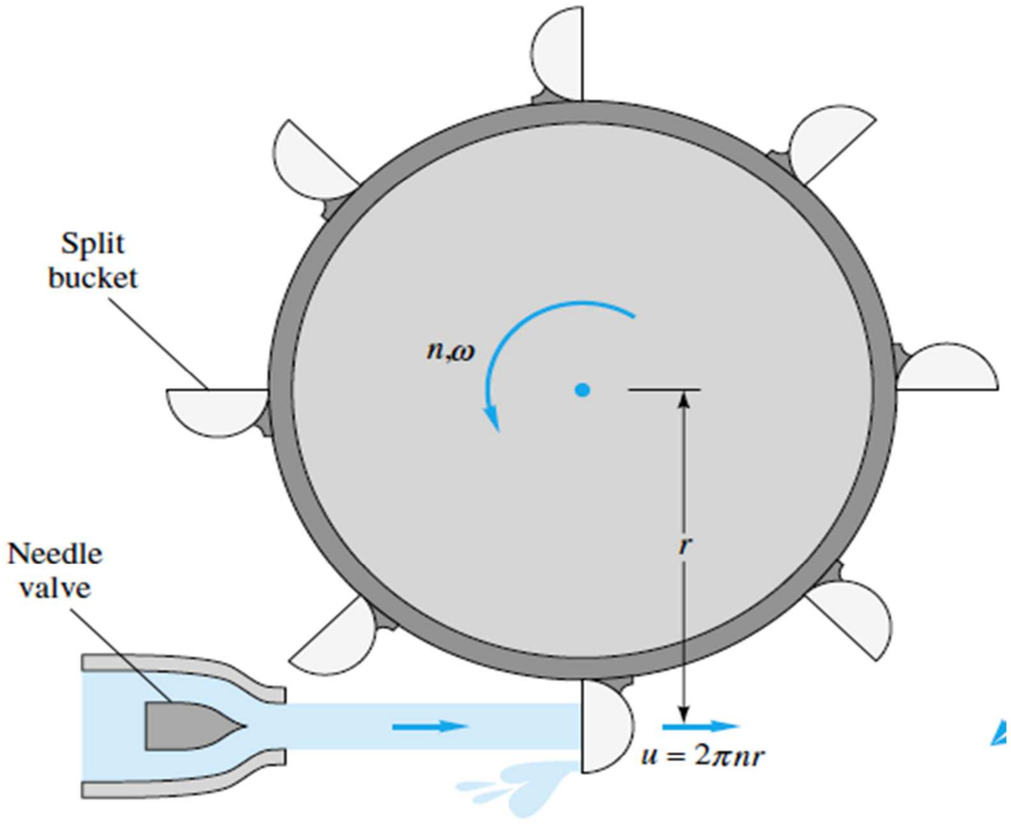
Dynamic pumps

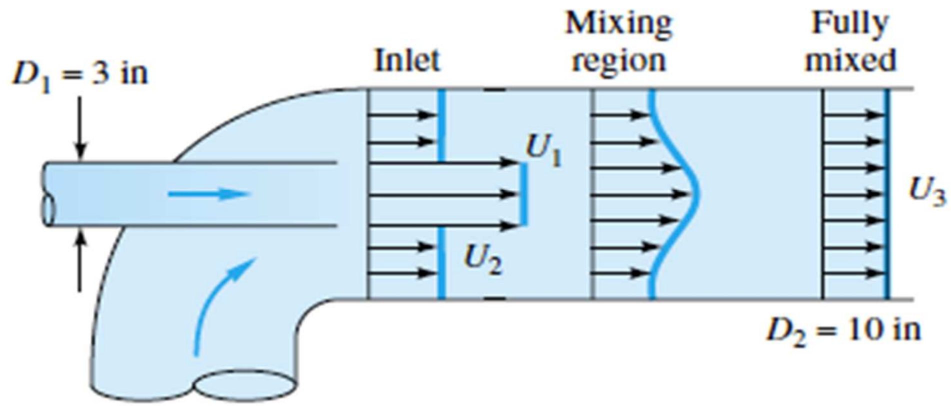
- Rotary
  - Centrifugal pump
  - Axial flow
  - Mixed flow
- Special
  - Jet pump or ejector
  - Electromagnetic pumps (liquid metals)
  - Fluid-actuated
    (gas-lift, hydraulic ram)

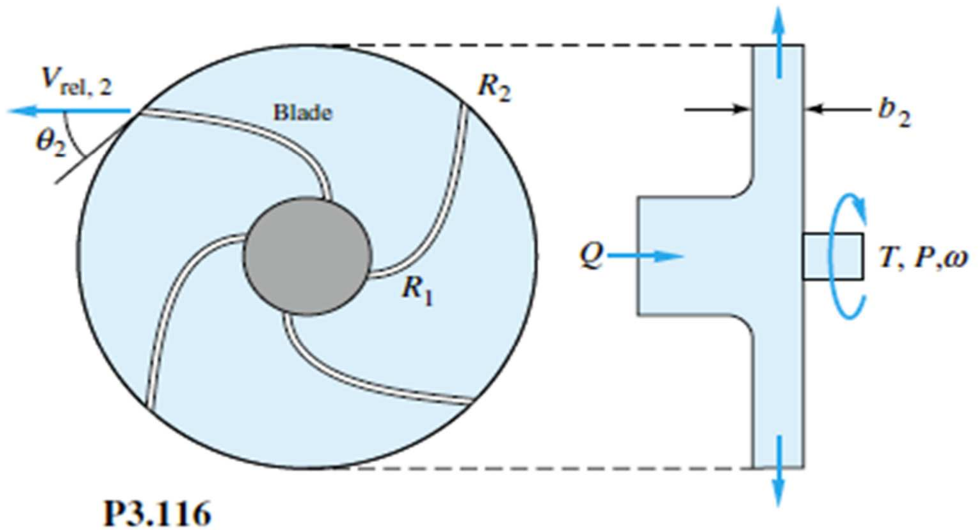
Positive-displacement pumps



Dynamic pumps

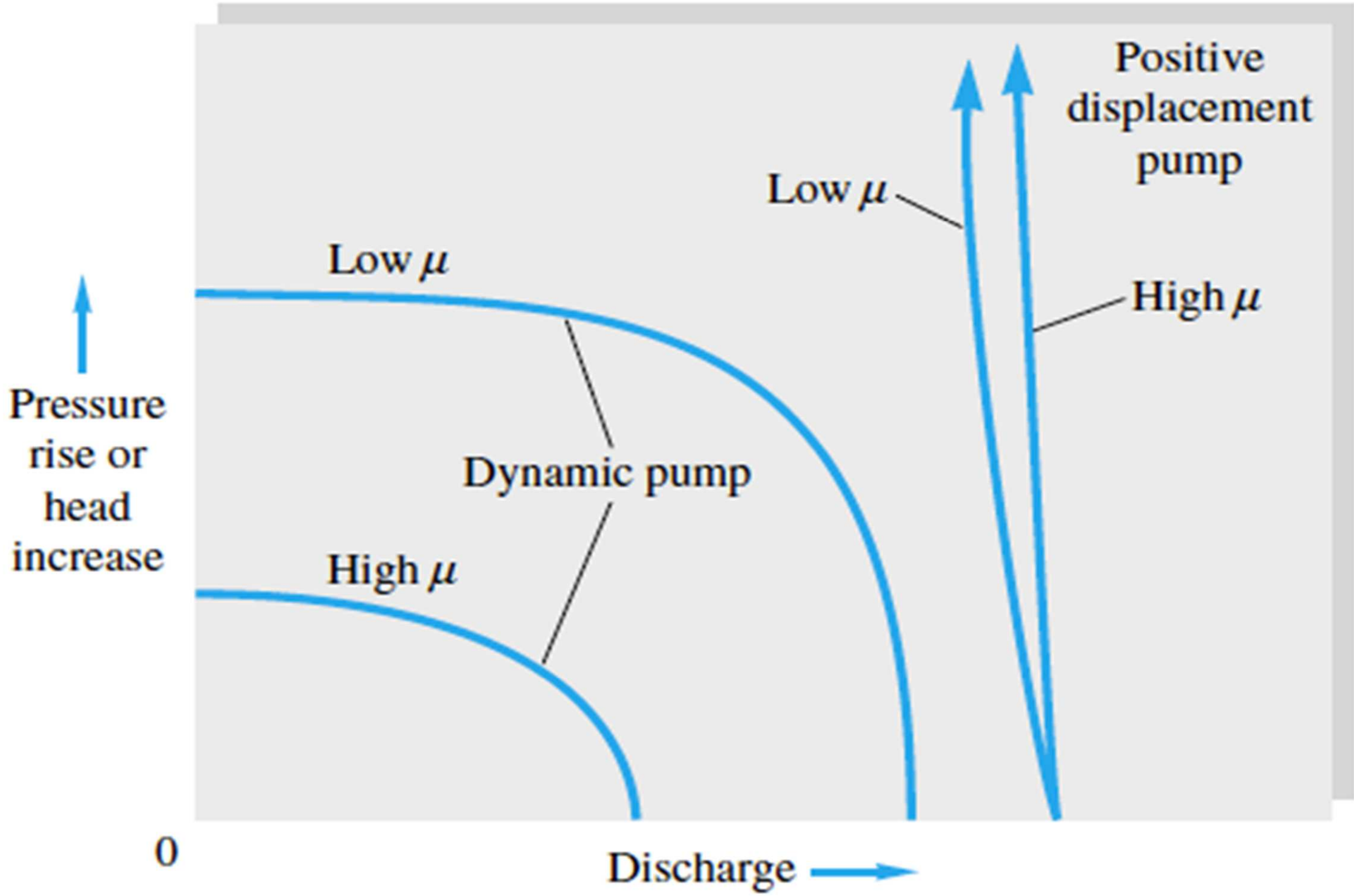




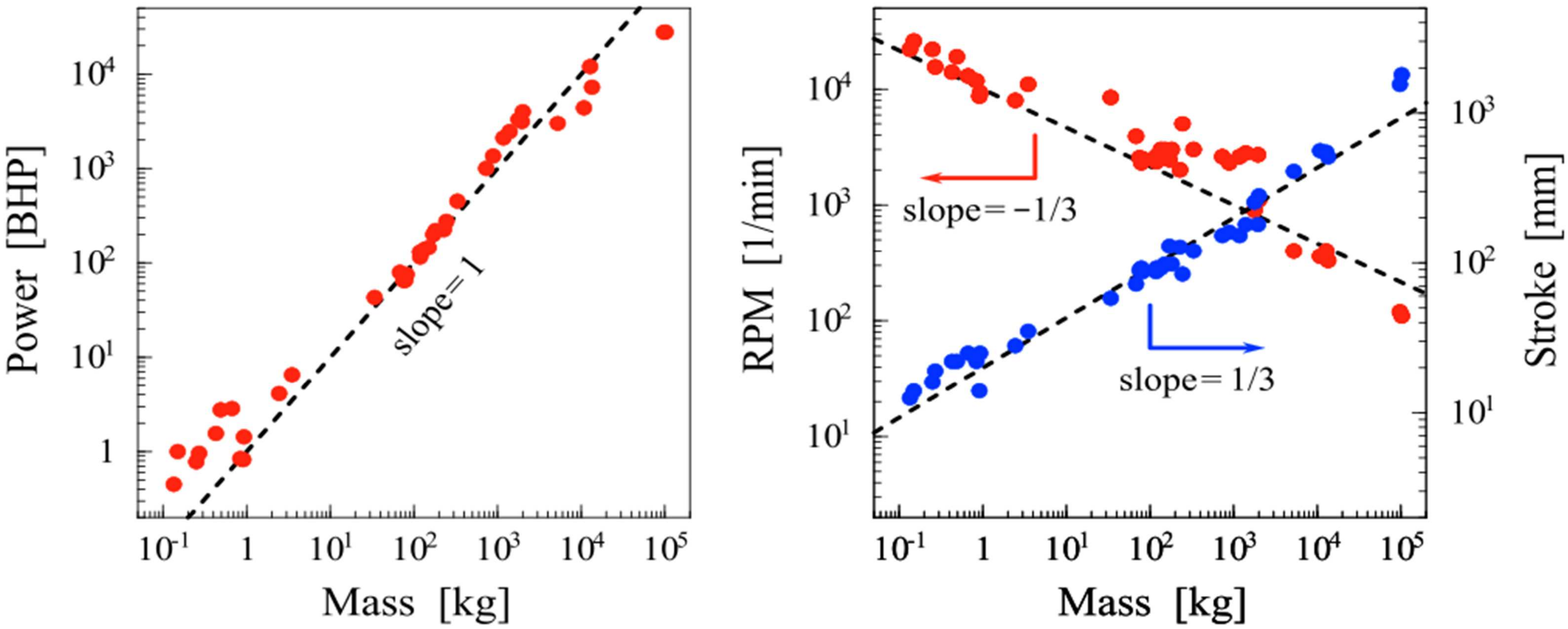


P3.116

Performance

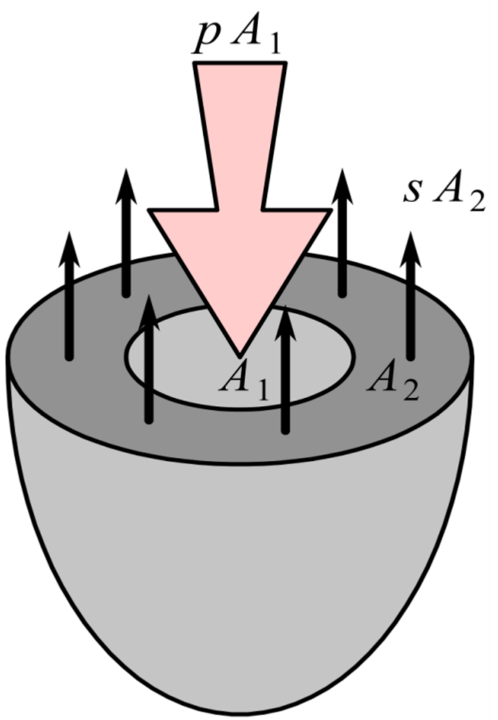


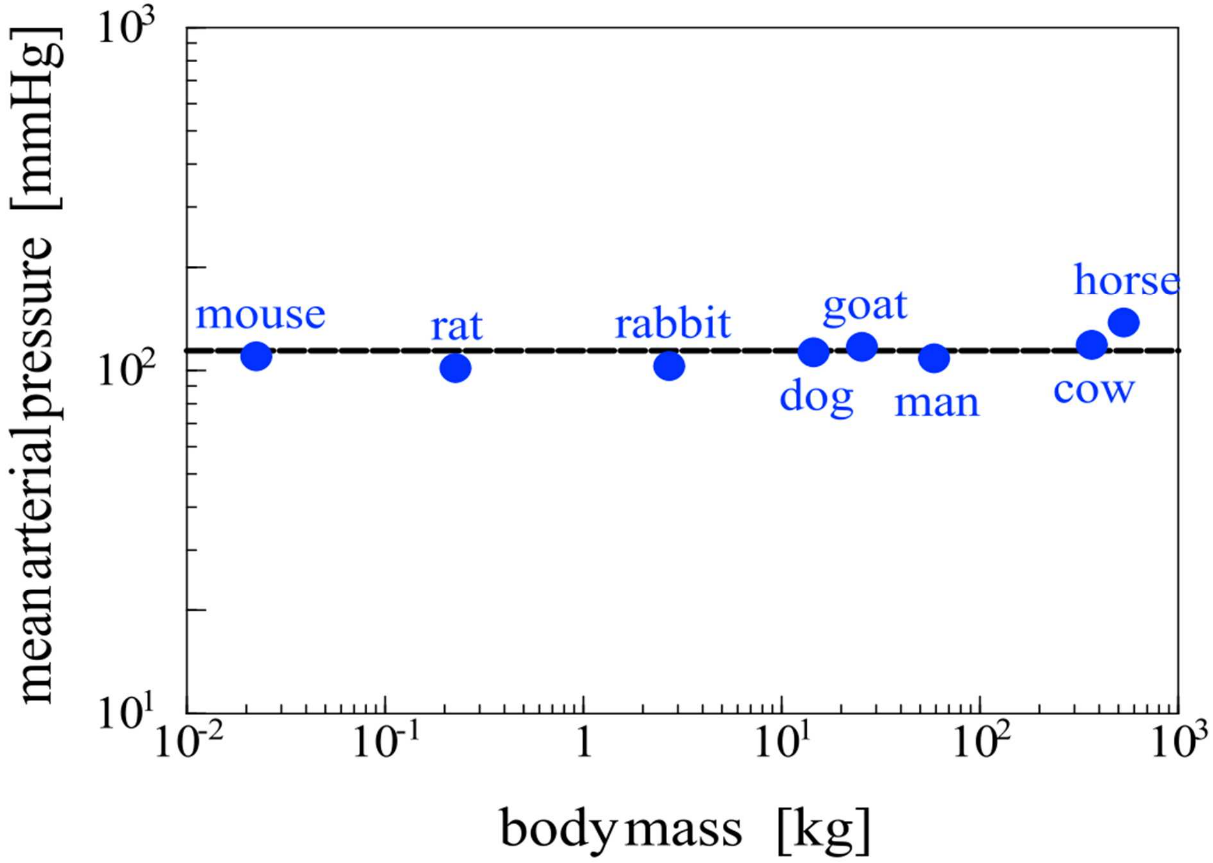
Similarity scaling example: internal combustion piston



Source: McMahon & Bonner, On Size and Life, 1983

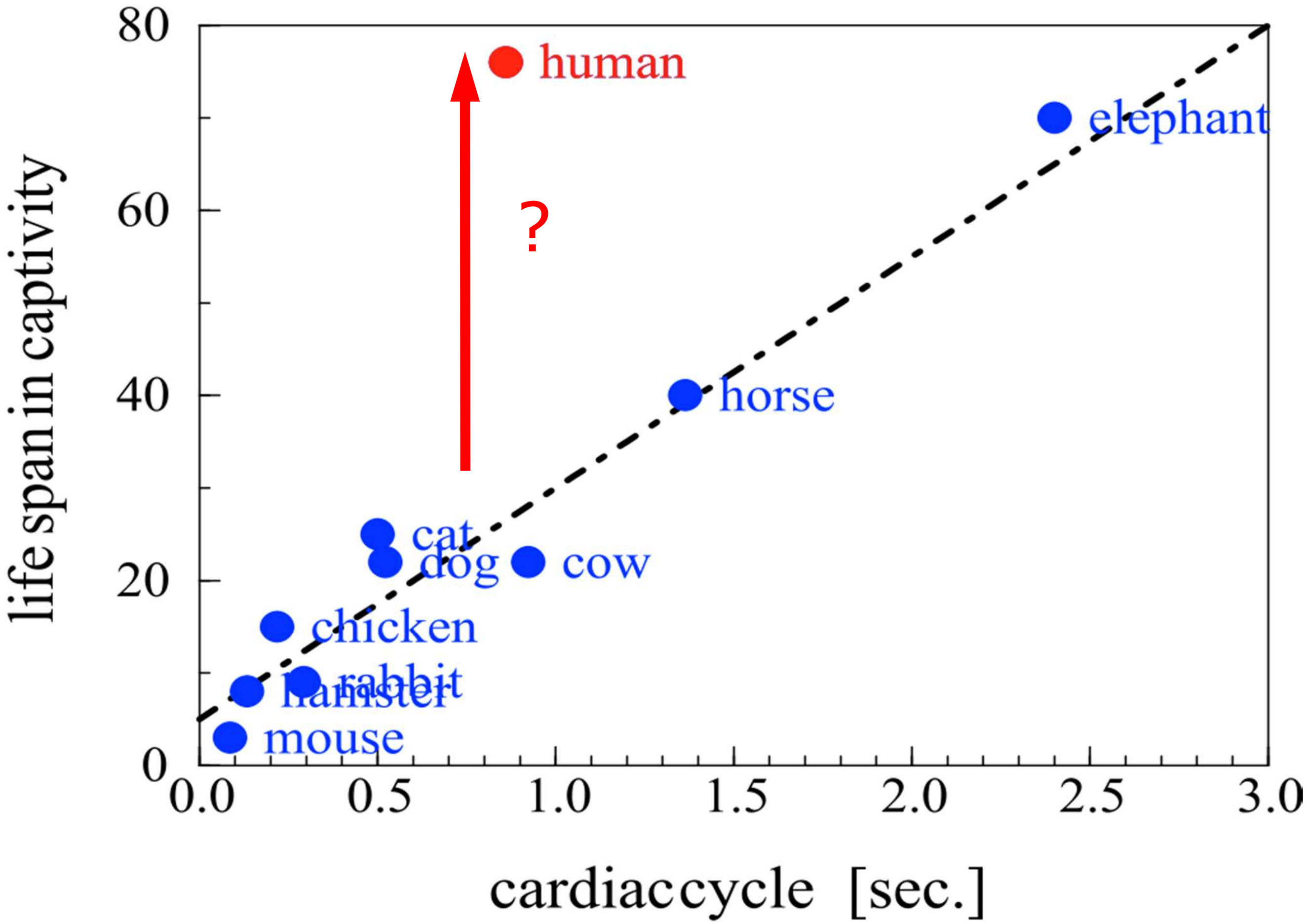
Another engine … the heart





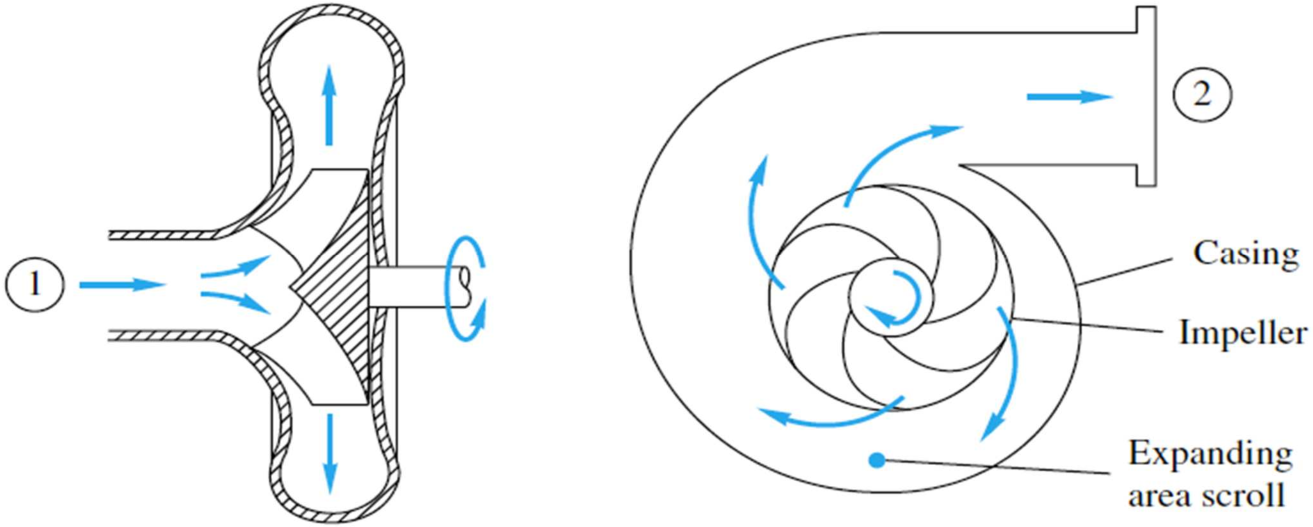
Source: McMahon & Bonner, On Size and Life, 1983

Heart rate vs. life span



Source: McMahon & Bonner, On Size and Life, 1983

The centrifugal pump



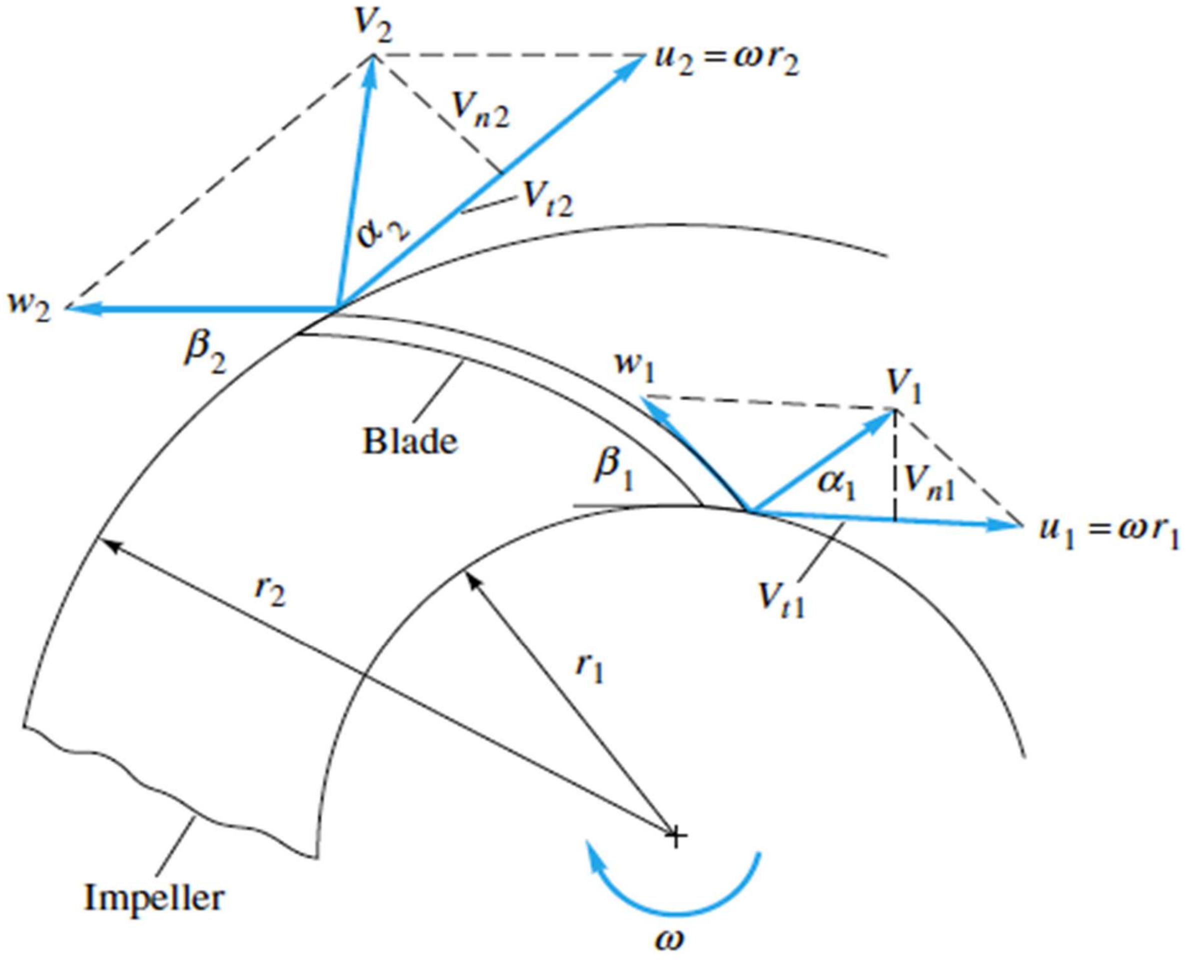
$$V_2 \approx V_1$$

Diffusor for pressure recovery

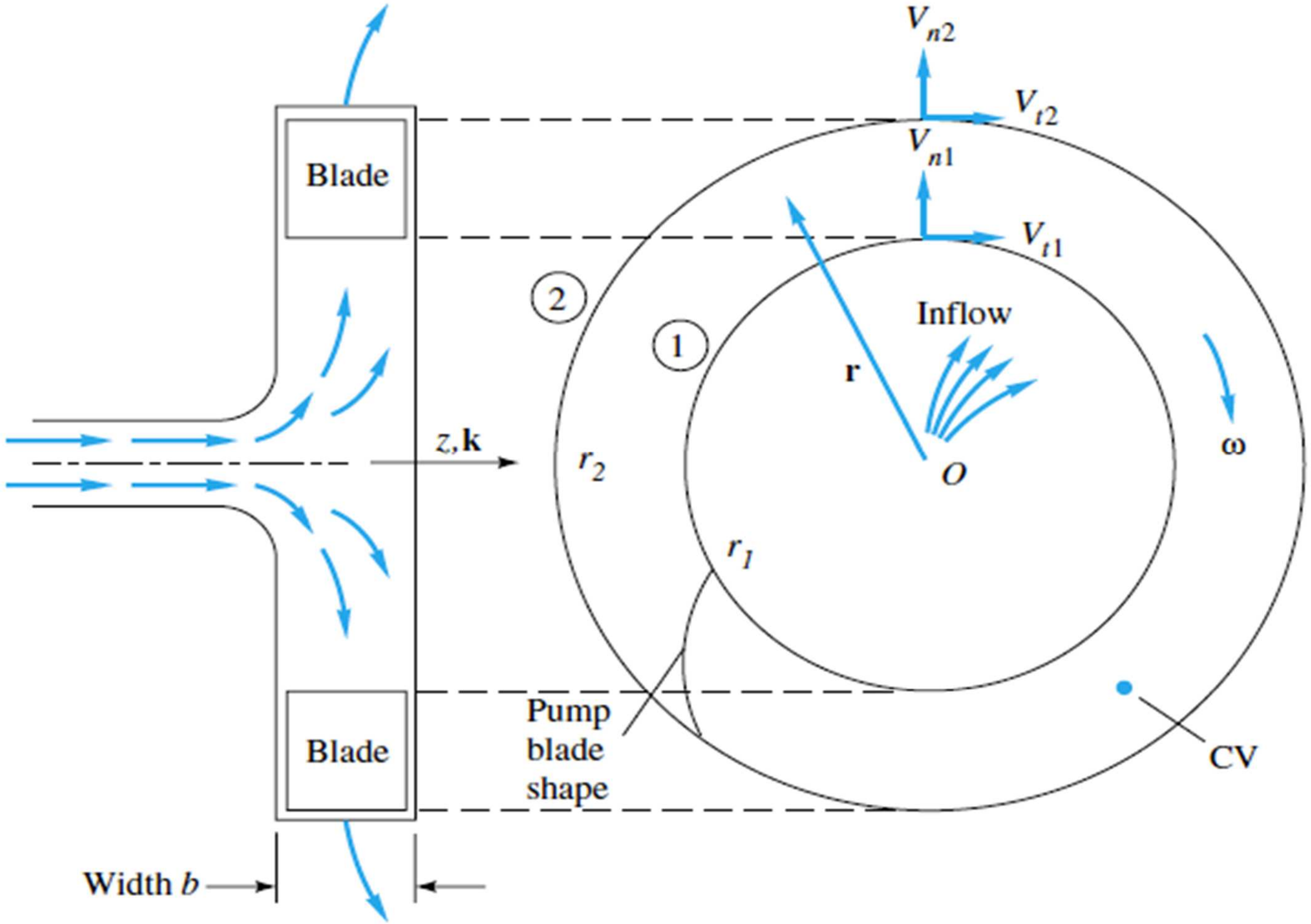
$$H = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z \right)_2 - \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z \right)_1 \approx \frac{\Delta p}{\rho g}$$

$$\left. \begin{array}{l} P_w = \rho g Q H \\ P_b = \omega T \end{array} \right\} \Rightarrow \eta = \frac{\rho g Q H}{\omega T} \Rightarrow \left\{ \begin{array}{c} \text{volumetric efficiency} \\ \text{hydraulic efficiency} \\ \text{mechanical efficiency} \end{array} \right. \Rightarrow \left\{ \begin{array}{c} \text{shock loss} \\ \text{friction loss} \\ \text{circulation loss} \end{array} \right.$$

Idealized pump impeller



Control volume analysis



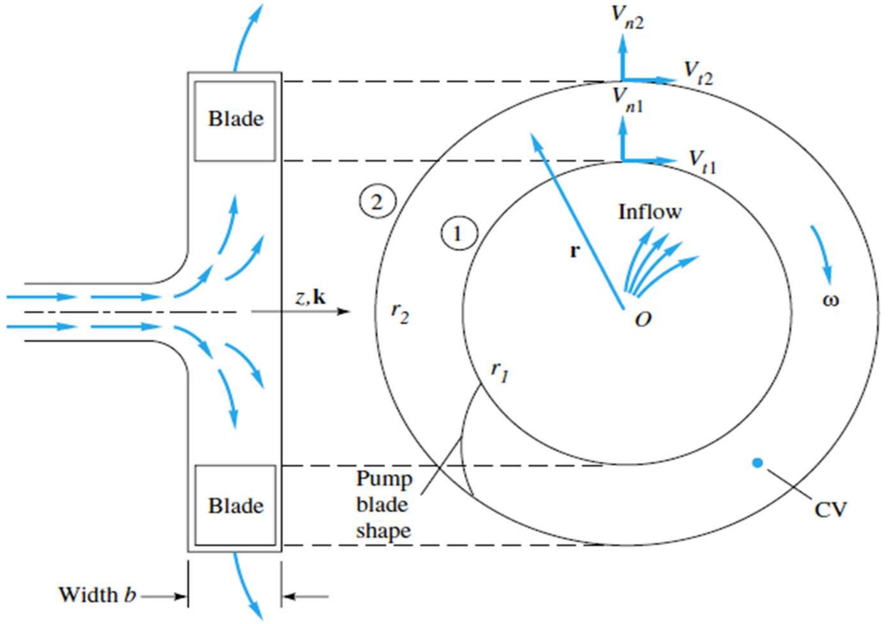
$$\sum M_O = T_O = (\mathbf{r}_2 \times \mathbf{V}_2)\dot{m}_{\text{out}} - (\mathbf{r}_1 \times \mathbf{V}_1)\dot{m}_{\text{in}}$$

$$\dot{m}_{\text{in}} = \rho V_{n1} 2\pi r_1 b = \dot{m}_{\text{out}} = \rho V_{n2} 2\pi r_2 b = \rho Q$$

$$\mathbf{r}_2 \times \mathbf{V}_2 = r_2 V_{t2} \sin 90^{\text{o}} \mathbf{k} = r_2 V_{t2} \mathbf{k}$$

$$\mathbf{r}_1 \times \mathbf{V}_1 = r_1 V_{t1} \mathbf{k}$$

$$T_O = \rho Q (r_2 V_{t2} - r_1 V_{t1})\mathbf{k}$$



with $V_{t1} = \omega r_1, V_{t2} = \omega r_2 : T_O = \rho Q \omega \left(r_2^2 - r_1^2\right)$

assume tip velocities ($u_{1,2} = \omega r_{1,2}$) and fluid velocities are not equal:

Power: $P_w = \omega T = \rho Q \left(u_2 V_{t2} - u_1 V_{t1}\right)$

use continuity:

$$V_{n1} = \frac{Q}{2\pi r_1 b_1}, \quad V_{n1} = V_{t1} \tan \alpha_1, \quad V_{n2} = \frac{Q}{2\pi r_2 b_2}, \quad V_{n2} = V_{t2} \tan \alpha_2$$

$$P_w = \rho Q \left(u_2 V_{n2} \cot \alpha_2 - u_1 V_{n1} \cot \alpha_1\right), \quad \alpha_1 = 90^{\text{o}}, V_{n1} = V_1$$

Example 11.1

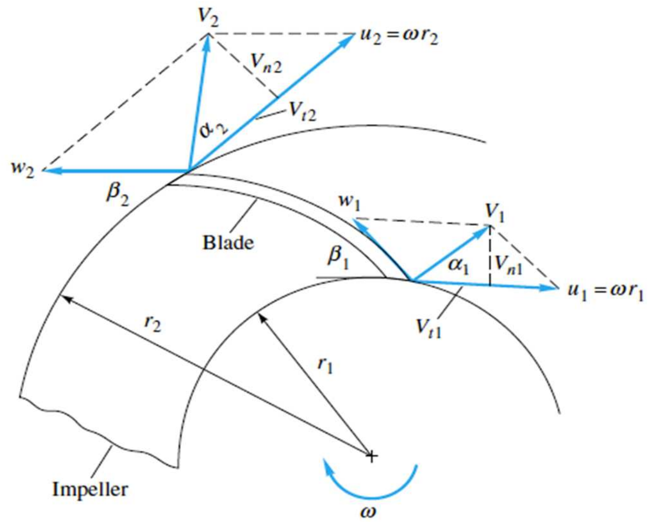
$$r_1 = 100\ \text{mm},\ r_2 = 175\ \text{mm},\ b_1 = b_2 = 45\ \text{mm}$$

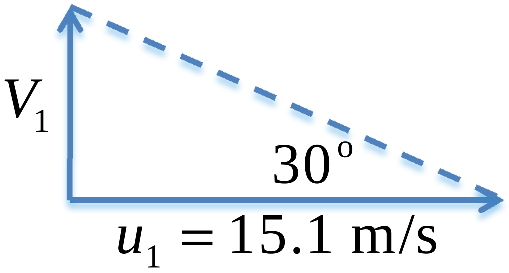
$$\beta_1 = 30^\circ,\ \beta_2 = 20^\circ$$

$$\text{rpm} = 1440 \quad (\omega = 2\pi \cdot 24\ \text{rad/s} = 150.8\ \text{rad/s})$$

$$u_1 = \omega r_1 = 15.1\ \text{m/s}$$

$$u_2 = \omega r_2 = 26.4\ \text{m/s}$$

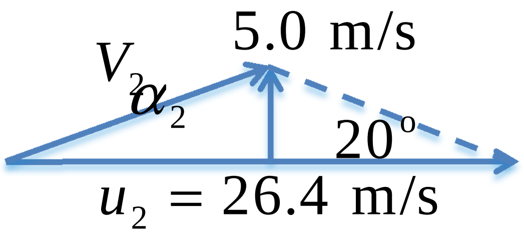




$$V_{n1} = u_1 \tan 30^\circ = 8.7\ \text{m/s}$$

$$Q = 2\pi r_1 b_1 V_{n1} = 0.246\ \text{m}^3/\text{s} = 14.8\ \text{m}^3/\text{min}$$

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = 5.0\ \text{m/s}$$



$$V_{t2} = u_2 - V_{n2} \cot \beta_2 = 12.7\ \text{m/s}$$

$$\alpha_2 = \tan^{-1} \frac{V_{n2}}{V_{t2}} = 21.5^\circ$$

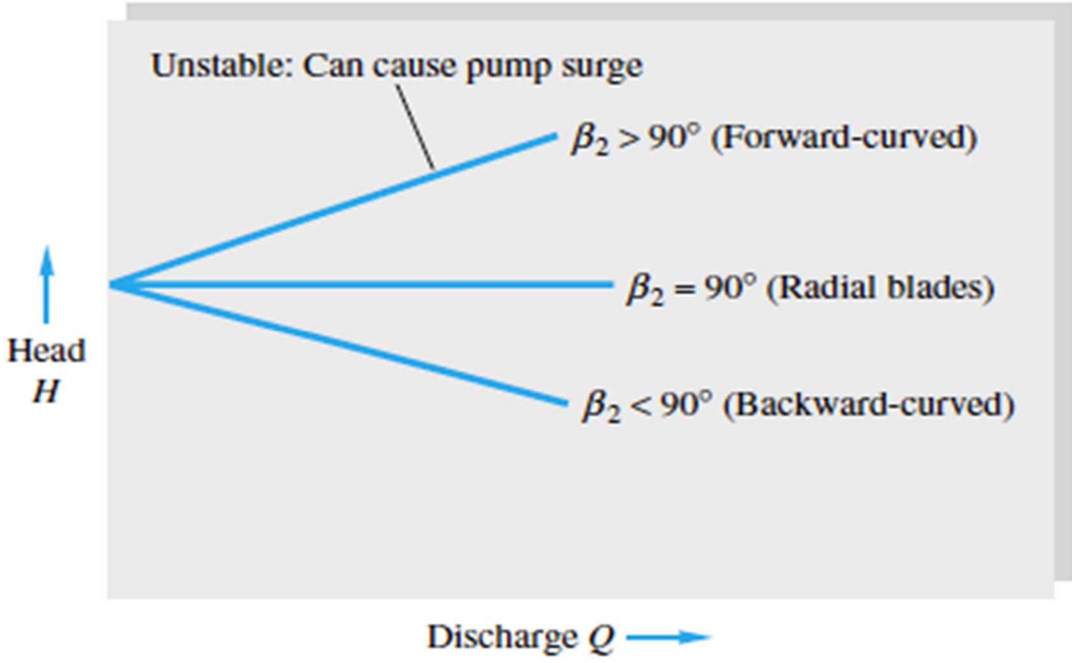
$$P_w = \rho Q u_2 V_{t2} = 82.5\ \text{kW}$$

Blade angle

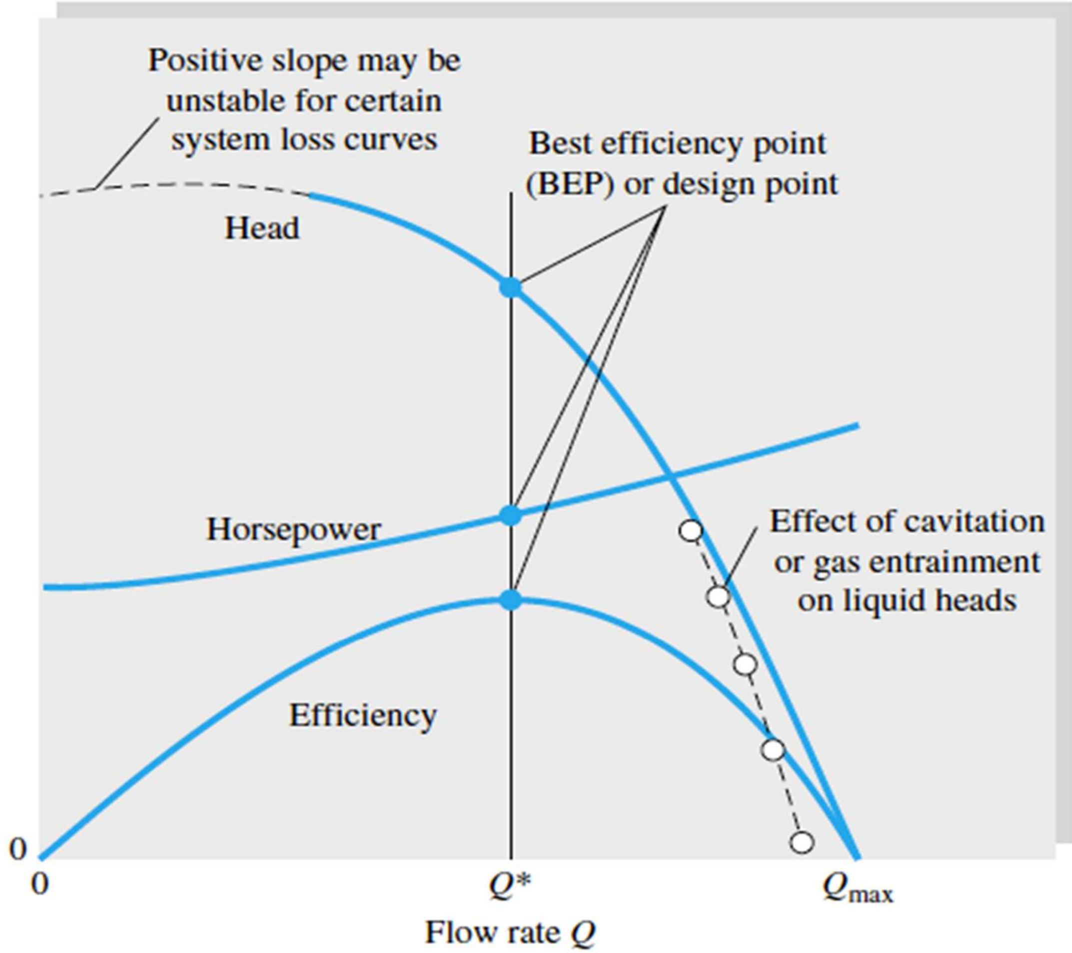
$$P_w = \rho Q u_2 V_{t2}$$

$$V_{t2} = u_2 - V_{n2} \cot \beta_2 \qquad V_{n2} = \frac{Q}{2\pi r_2 b_2}$$

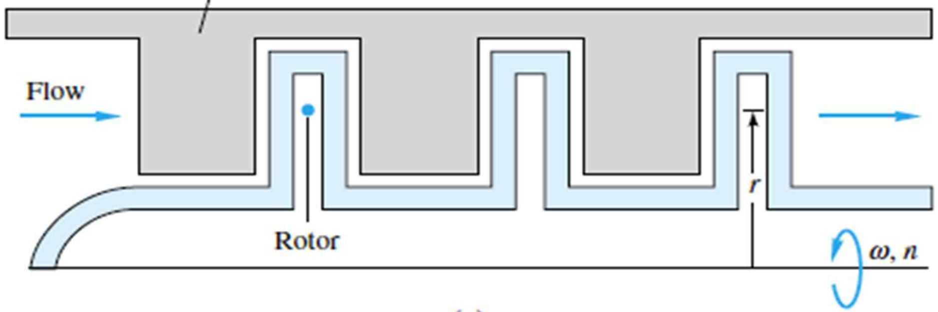
$$\Delta p \approx \rho u_2^2 - \rho \frac{u_2 \cot \beta_2}{2\pi r_2 b_2} Q$$



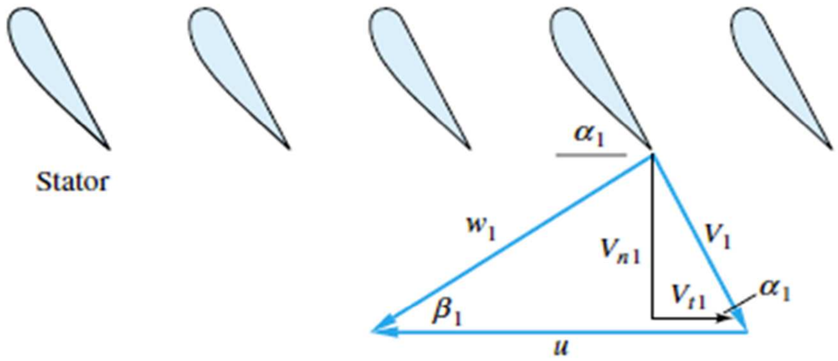
Efficiency


Positive slope may be unstable for certain system loss curves
Best efficiency point (BEP) or design point
Head
Horsepower
Effect of cavitation or gas entrainment on liquid heads
Efficiency
0
0
Q*
Q max
Flow rate Q

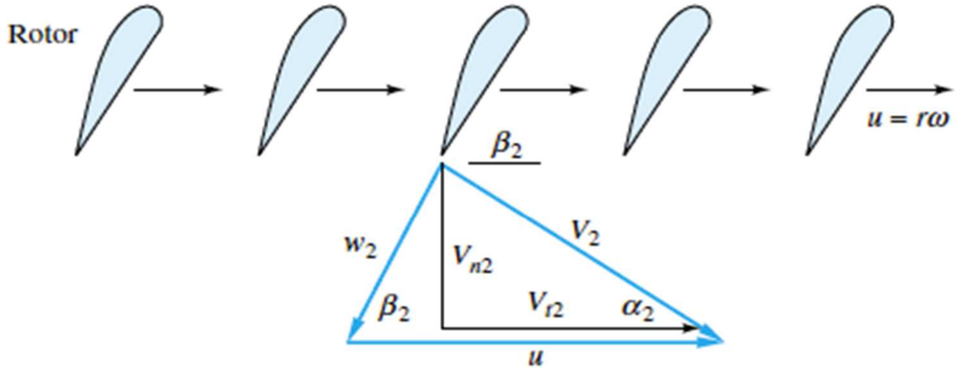
Axial pump



(a)



(b)



$$V_{n1} = V_{n2} = V_n = \frac{Q}{A} = \text{const.}$$

$$u = \omega r_{\text{av}} = V_{n1}(\tan\alpha_1 + \tan\beta_1)$$
$$= V_{n2}(\tan\alpha_2 + \tan\beta_2)$$

$$V_{t1} = \cot\alpha_1 \qquad V_{t2} = u - V_{n2}\cot\beta_2$$

$$\Delta p = \rho u^2 - \rho u V_n(\cot\alpha_1 + \cot\beta_2)$$

Wind turbine

available power:

$$P_{avail} = \tfrac{1}{2}\dot{m}V_1^2 = \tfrac{1}{2}\rho A V_1^3, \quad P_{max} = \tfrac{8}{27}\rho A V_1^3$$

$$C_P = \frac{P}{\tfrac{1}{2}\rho A V_1^3}$$