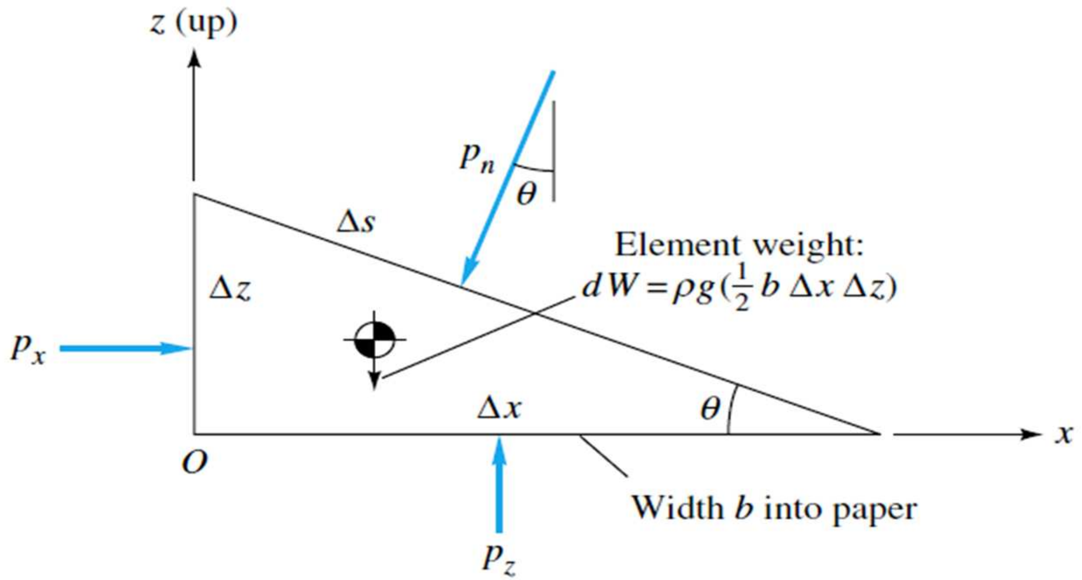
Fluid mechanics (wb1225)

Lecture 2: pressure distribution in a fluid

Fluid at rest

No fluid motion → no shear stresses → pressure forces only



Fluid at rest;
Equilibrium of forces

$$\left.\begin{aligned} p_x \Delta z - p_n \Delta s \sin\theta = 0 \\ p_x \Delta z - p_n \Delta z = 0 \end{aligned}\right\} \lim_{\Delta z \to 0} \Rightarrow p_x = p_n = p$$

Pressure is isotropic (does not depend on direction)

Variation of pressure

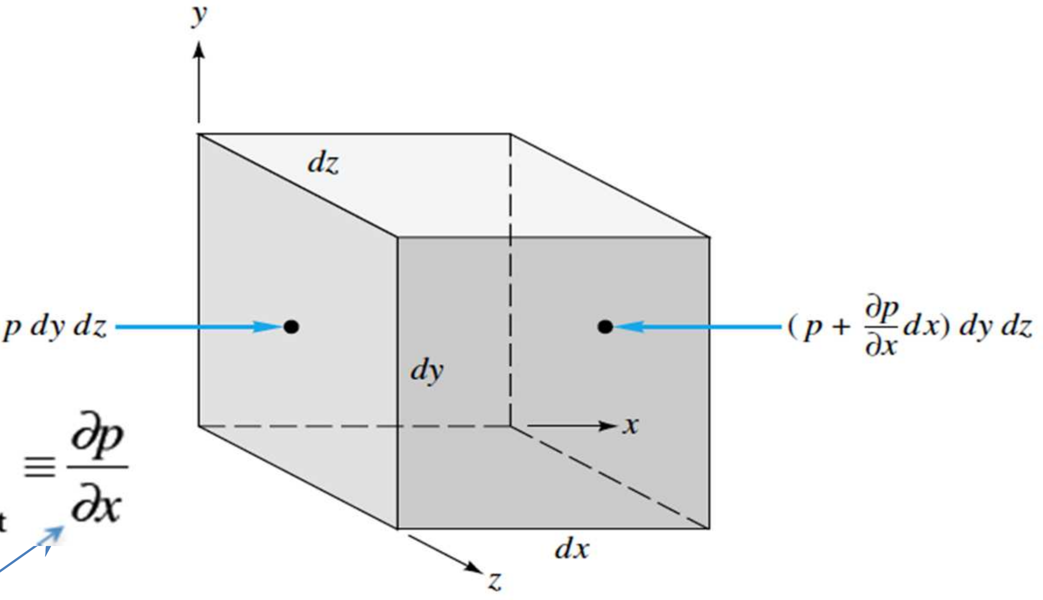
Force in x-direction:

$$F_x = (p+\Delta p)\Delta y \Delta z - p\,\Delta y \Delta z$$

Force density in x-direction:

$$f_x = \lim_{\Delta x \to 0} \frac{(p+\Delta p)\Delta y \Delta z - p\,\Delta y \Delta z}{\Delta x \Delta y \Delta z} = \left.\frac{dp}{dx}\right|_{y,z=\text{const}} \equiv \frac{\partial p}{\partial x}$$

partial derivative



$$\Delta p = \frac{\partial p}{\partial x} \Delta x$$

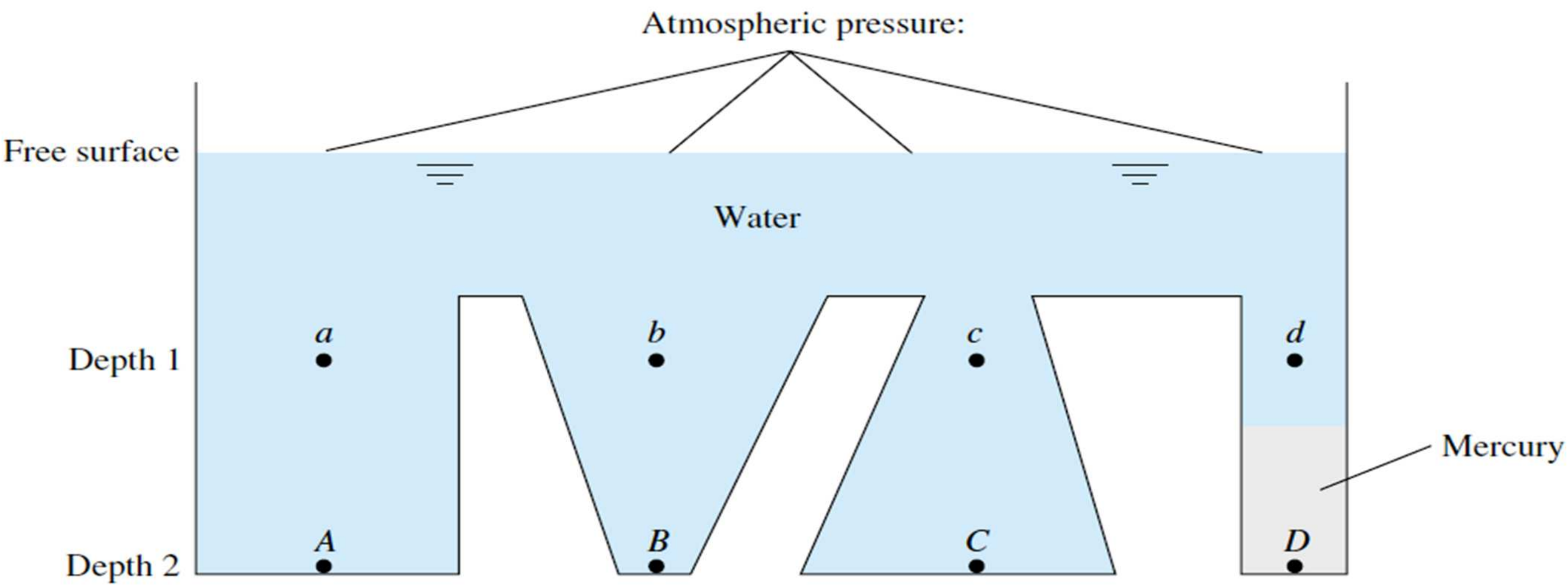
$$\mathbf{f} \equiv (f_x, f_y, f_z) = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right) \equiv \text{grad}\, p = \nabla p$$

Hydrostatic equilibrium (Pascal relation):

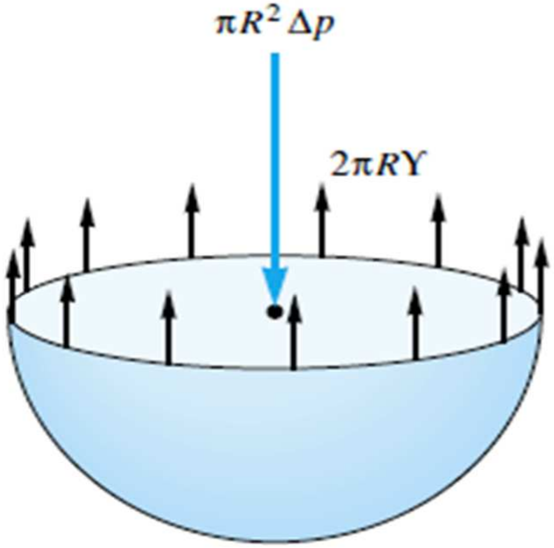
$$\frac{dp}{dz} = -\rho g \quad \text{or} \quad p_2 - p_1 = -\int_{z_1}^{z_2} \rho g \, dz$$

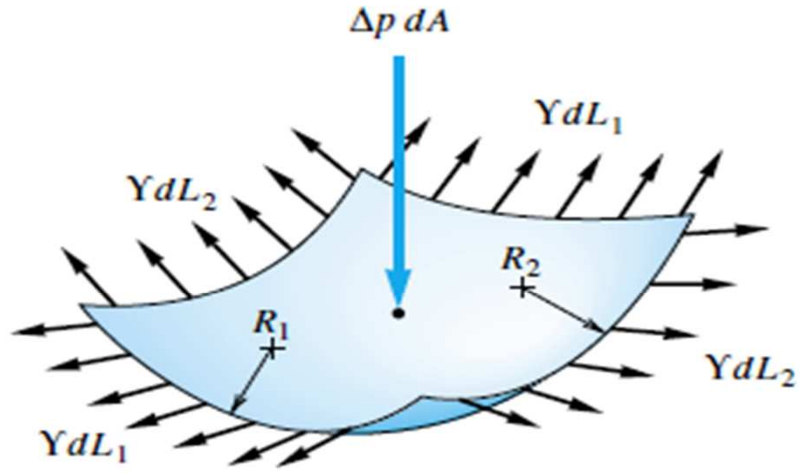
Fluid statics

- Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container.
- The pressure is the same at all points on a given horizontal plane in the fluid.
- The pressure increases with depth in the fluid.



Surface tension





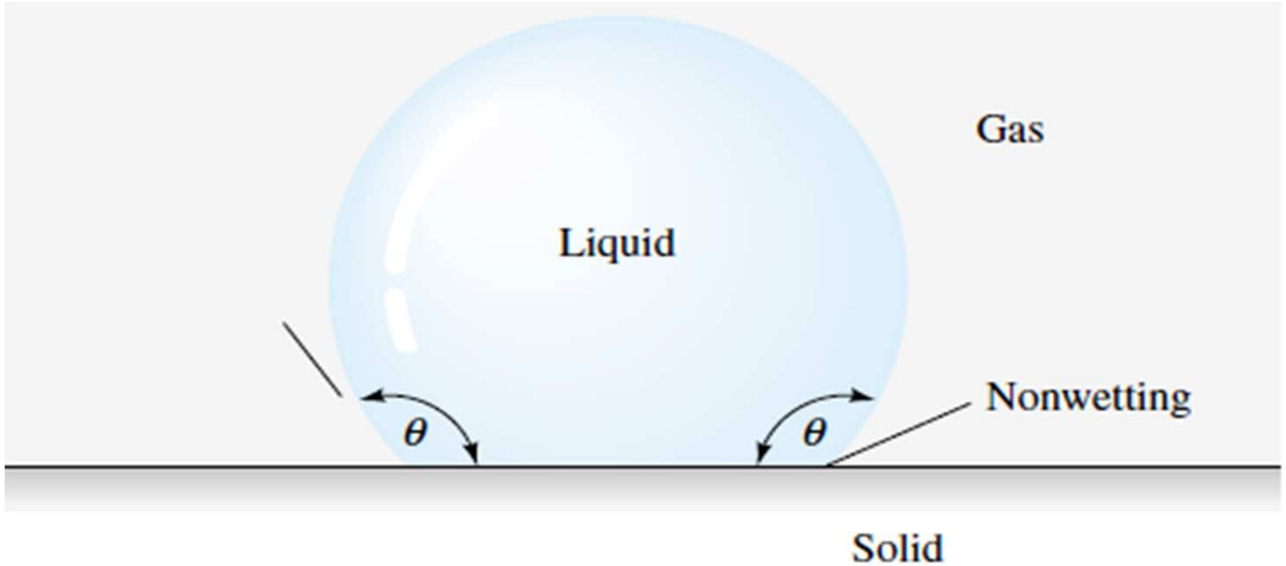
$$\Delta p = \frac{2\gamma}{R}$$

$$\Delta p = \gamma \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$\gamma =$ surface tension [N/m]

water: γ = 0.068 N/m

Contact angle

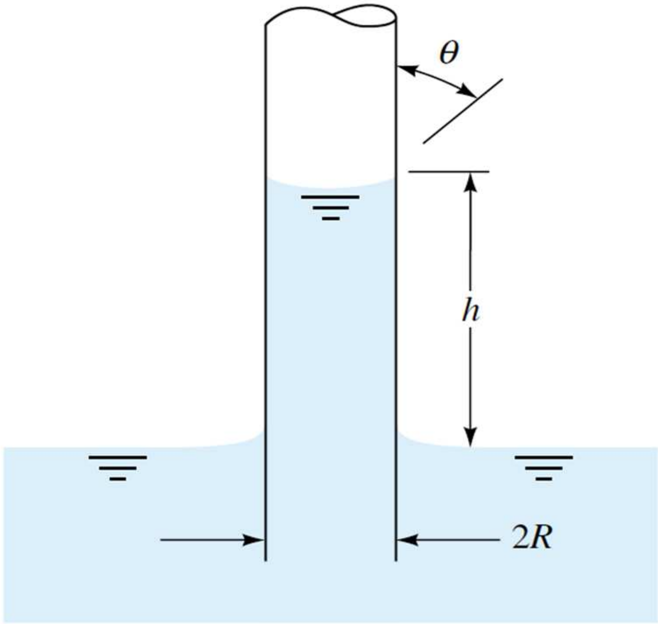


$$\text{contact angle [deg]}: \ \theta = \begin{cases} < 90^\circ & \text{wetting (hydrophilic)} \\ > 90^\circ & \text{non-wetting (hydrophobic)} \end{cases}$$

contact angle of water ~ 0° - 30° for most surfaces

θ ~ 180° ➞ super-hydrophobic surface (‘Lotus effect’)

Example 1.9



Derive an expression for the height h in a circular tube of a liquid with surface tension γ and contact angle θ.

Compressibility

Compressibility effects become important when velocity speed approaches the speed of sound (a):

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_S = k\left(\frac{\partial p}{\partial \rho}\right)_T \qquad k = \frac{c_p}{c_v}$$

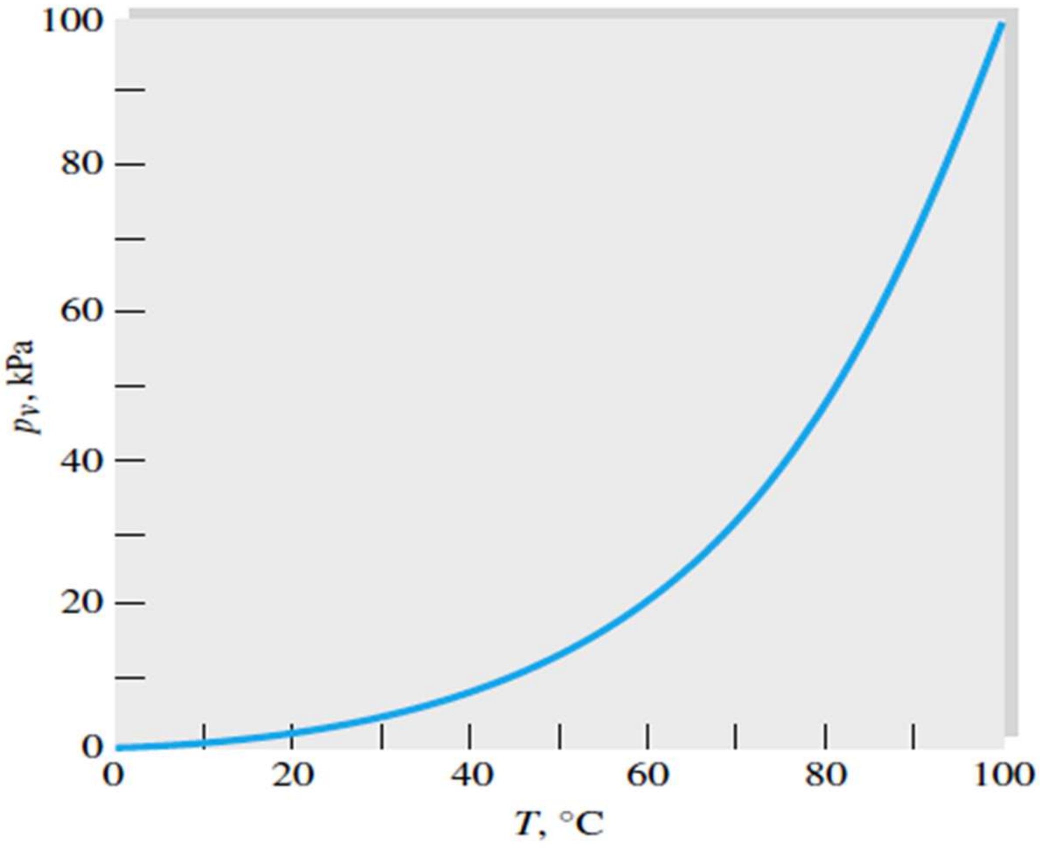
[1]

Ideal gas: $$a = (kRT)^{1/2}$$

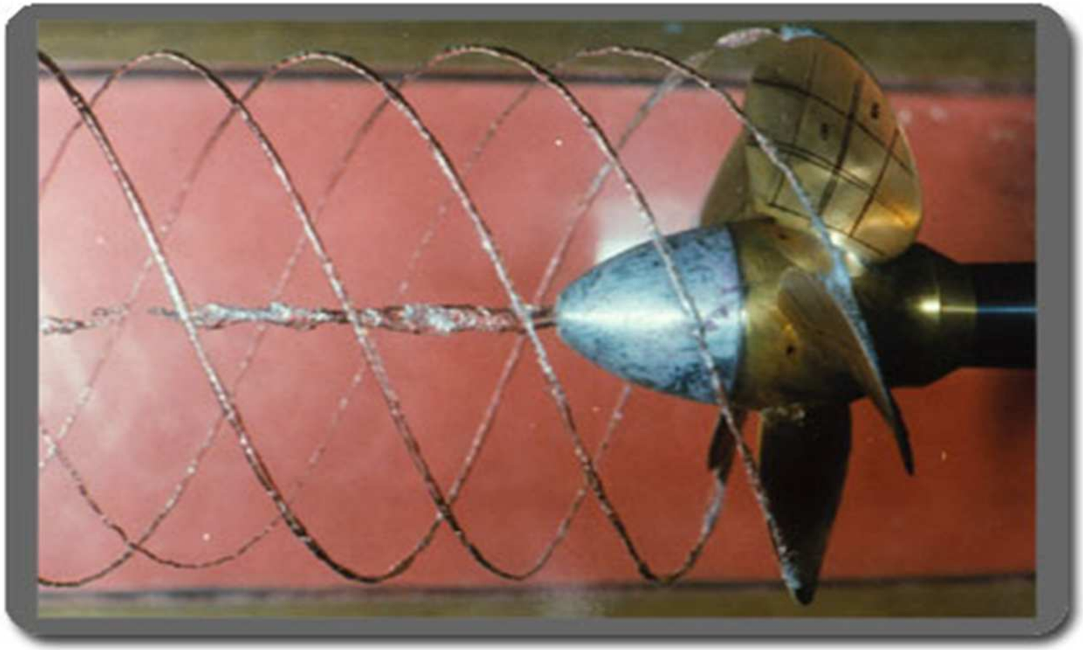
$$k = 1.4$$
$$R = (8314[\text{J/kmol/K}]/29[\text{kg/kmol}]) = 287[\text{J/kg/K}]$$
$$T = 293[K]$$
$$a = 343 \text{ m/s}$$

Mach number: $$\text{Ma} = \frac{V}{a}$$

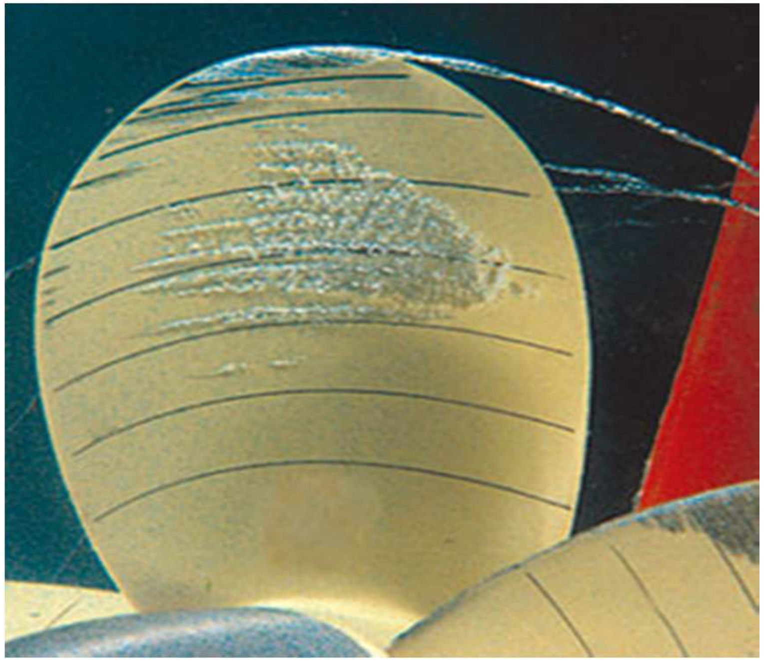
Vapor pressure



Cavitation



[2]



[3]

$$p_0 + \frac{1}{2}\rho V^2 = \text{constant}$$

$$\left.\begin{array}{r} p_0 = 10^5 \text{ Pa}, \\ \rho = 10^3 \text{ kg/m}^3 \end{array}\right\} \Rightarrow V \simeq 14 \text{ m/s (50 km/h)}$$

Cavitation

[4]

the amazing snapping shrimp

Atmospheric scale height

force balance: $dp = -\rho g dz$

ideal gas law: $pv = RT \Rightarrow \rho = \dfrac{p}{RT}$

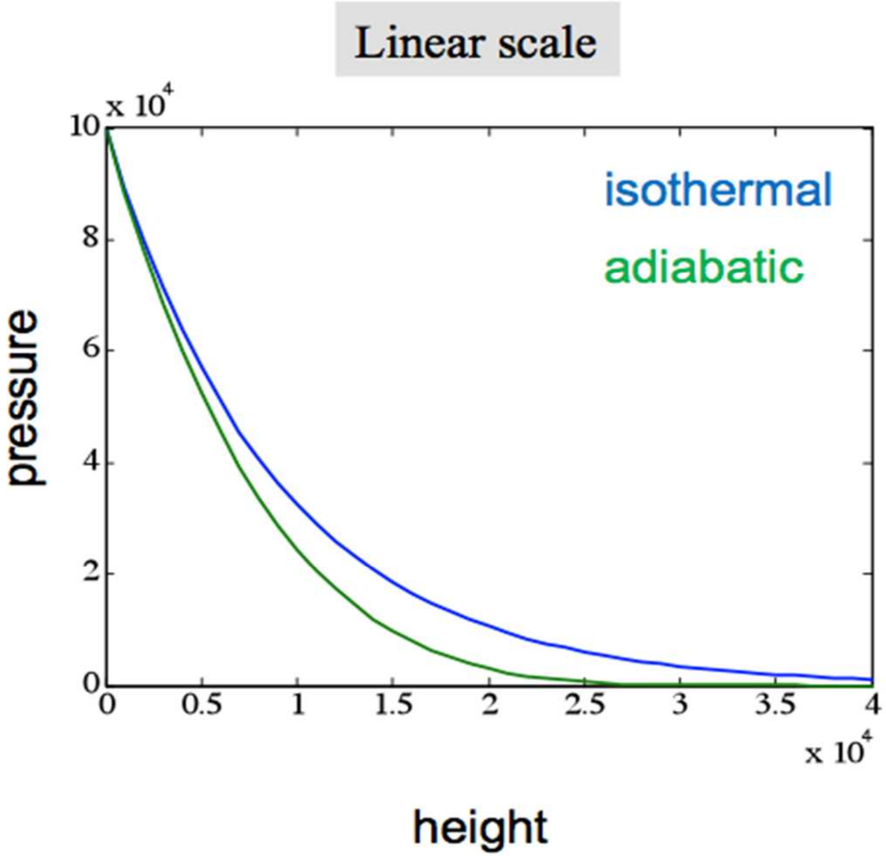
isothermal atmosphere: $p = p_0 \exp(-gz/RT_0)$ $\Rightarrow L = RT_0/g$

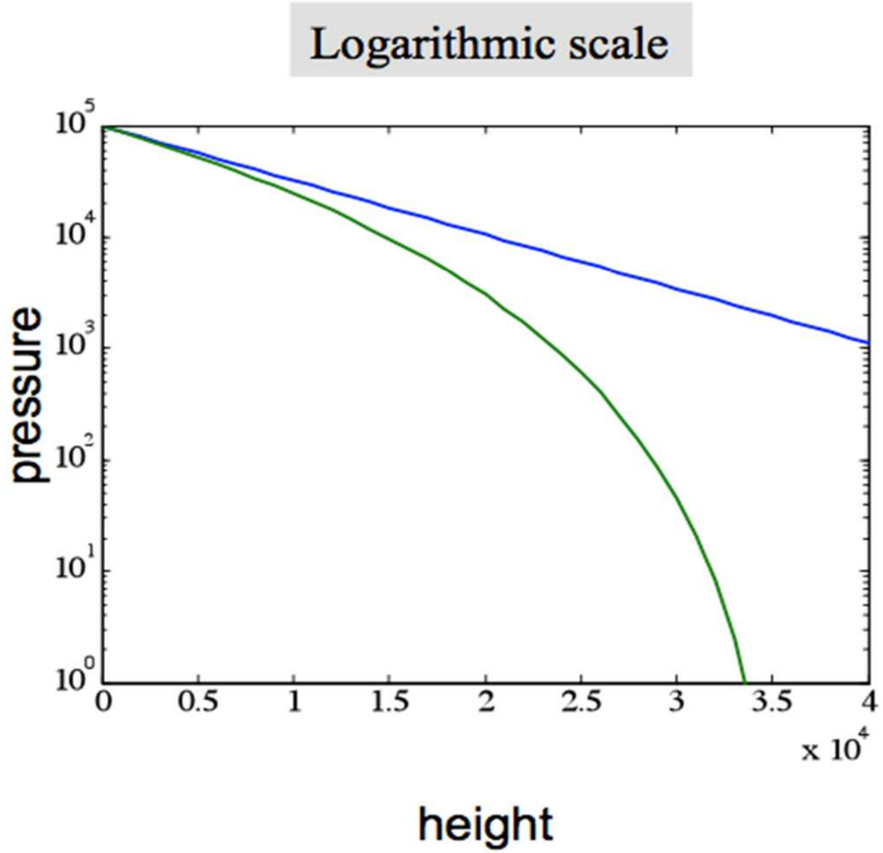
adiabatic atmosphere: Poisson relation: $p\rho^{-\kappa} = \text{const}$ $\kappa = c_p / c_v$

$$p = p_0\left(1 - \frac{\kappa-1}{\kappa}\frac{\rho_0 g}{p_0}z\right)^{\frac{\kappa}{\kappa-1}} \Rightarrow L_0 = \frac{\kappa}{\kappa-1}\frac{p_0}{\rho_0 g}$$

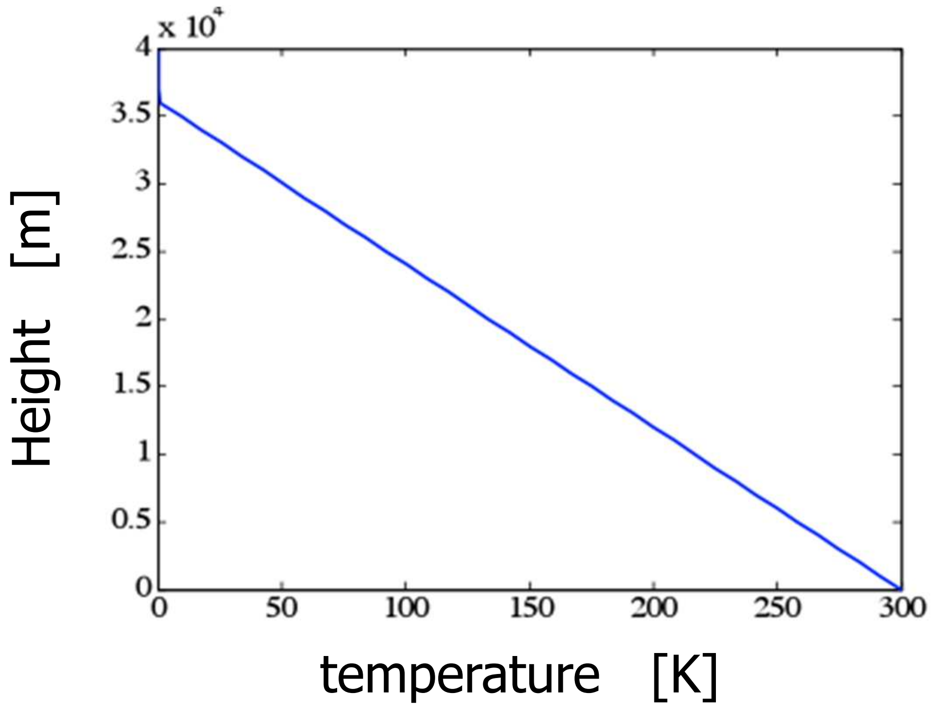
$$T = T_0\left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} \Rightarrow T = T_0\left(1 - \frac{z}{L_0}\right)$$

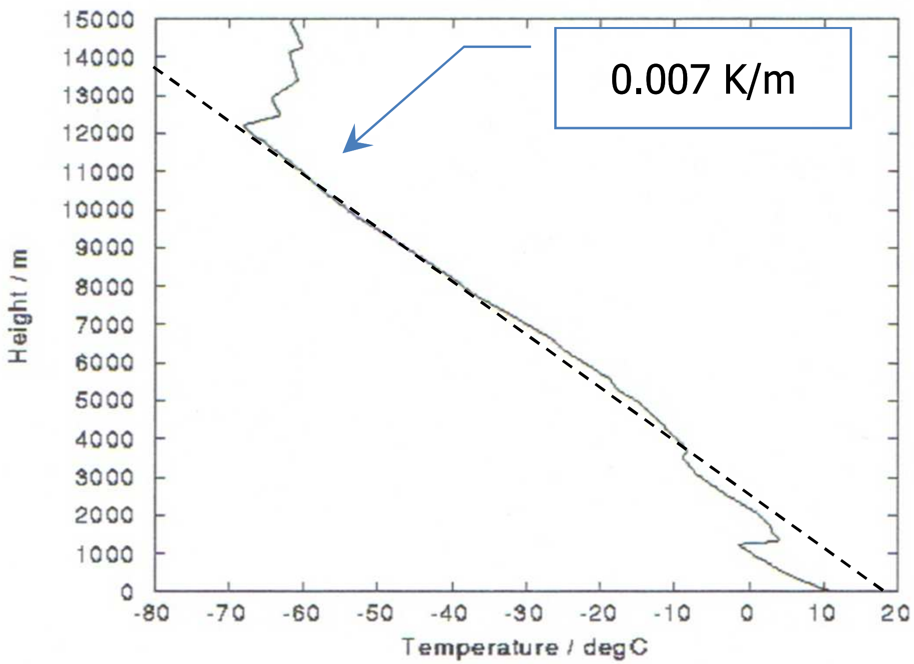
Isothermal vs. adiabatic





Temperature vs. height





$$\frac{T_0}{L_0} \cong \frac{270 \text{ K}}{37 \text{ km}} = 0.007 \text{ K/m}$$

Our atmosphere

Our atmosphere is ‘thin’ with respect to the diameter of Earth;

For example, for a 30-cm diameter globe, the equivalent thickness of the atmosphere is 0.18 mm (about the thickness of a sheet of paper)

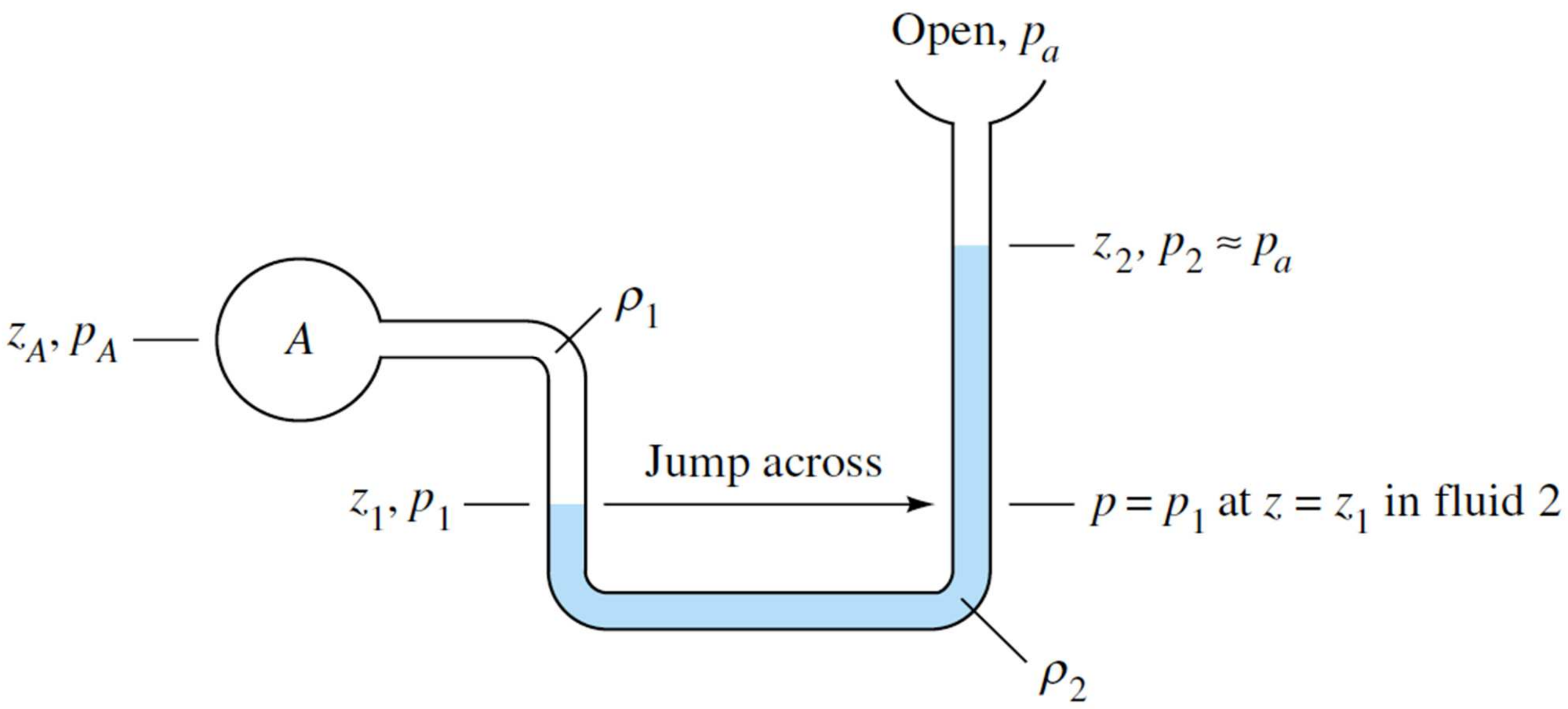
The atmosphere has a finite volume of $4\pi\times(6370\text{ km})^2\times(8\text{ km}) = 4\times10^{18}\text{ m}^3$, and has a weight of 5×10^{18} kg. This corresponds to 10^{44} molecules.

Suppose Julius Caesar’s last breath (1 liter, containing 2.7×10^{22} molecules) has mixed with the atmosphere, then your next intake of air contains about 4-5 molecules from Julius’ last breath!

You will also breathe molecules from the last breath of Ramses II, Dzjengis Khan, or any other favorite historical person
(with the exception of Thomas Edison, whose final breath is held at a museum!)

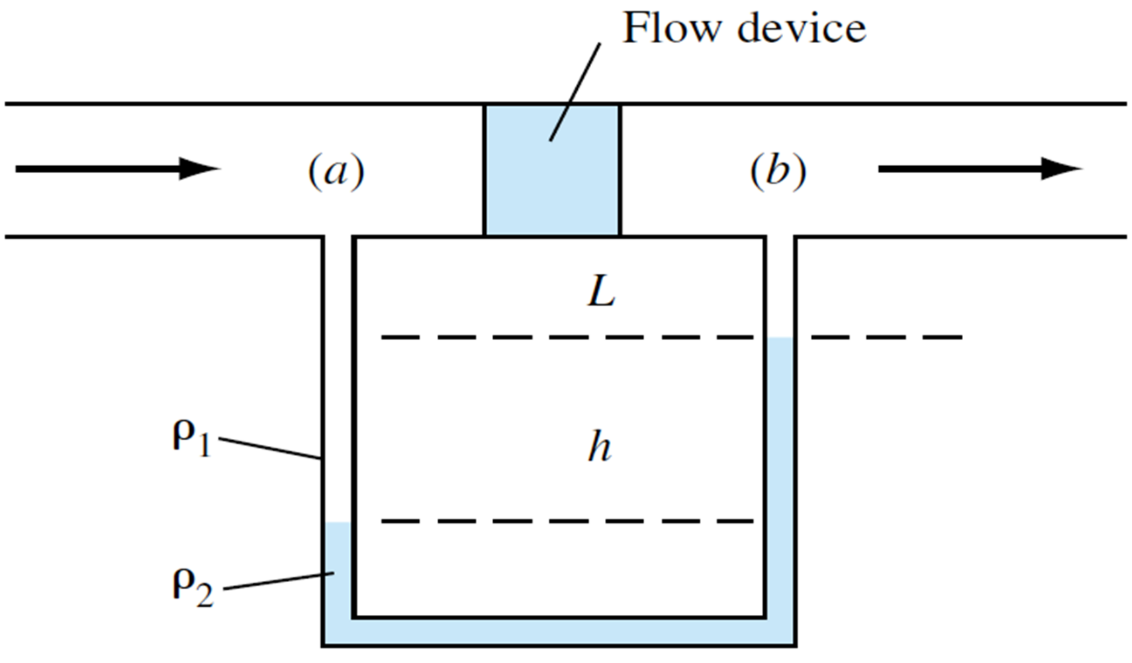
The world population (7×10^9 people) breathes a total of $1.5\times10^{15}\text{ m}^3$ over 67 years (human average life span).

Manometer



Open, p_a
$z_2, p_2 \approx p_a$
ρ_1
z_A, p_A
A
Jump across
z_1, p_1
$p = p_1$ at $z = z_1$ in fluid 2
ρ_2

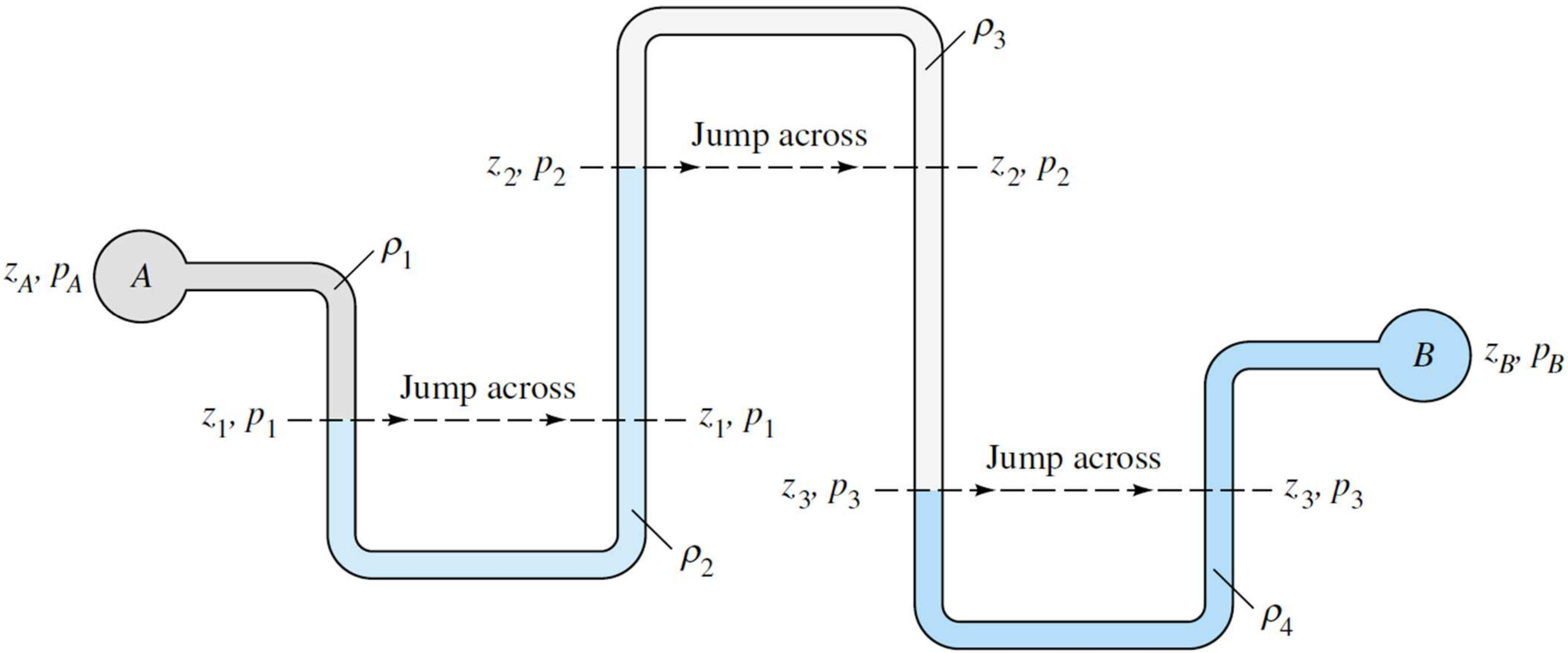
Example 2.3



$$p_a + \rho_1 gL + \rho_1 gh - \rho_2 gh - \rho_1 gL = p_b$$

$$p_a - p_b = (\rho_2 - \rho_1) gh$$

Complex tube systems



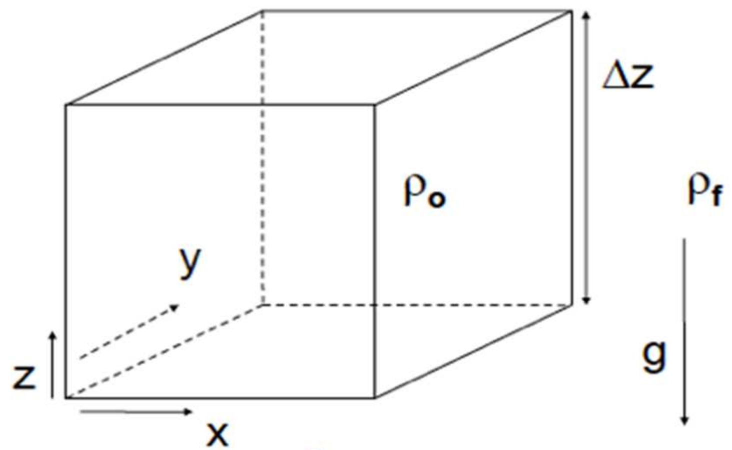
Archimedes' Law

Pressure force on bottom surface:

$$p_b \Delta x \Delta y = \left[p_0 - \rho_f g z\right] \Delta x \Delta y$$

Pressure force on top surface:

$$p_t \Delta x \Delta y = \left[p_0 - \rho_f g (z + \Delta z)\right] \Delta x \Delta y$$



Net pressure force:

$$F_p = (p_b - p_t) \Delta x \Delta y = \left[p_0 - \rho_f g z - \left(p_0 - \rho_f g (z + \Delta z)\right)\right] \Delta x \Delta y$$

$$= \rho_f g \Delta x \Delta y \Delta z$$

Weight of object:

$$F_o = -\rho_o g \Delta x \Delta y \Delta z$$

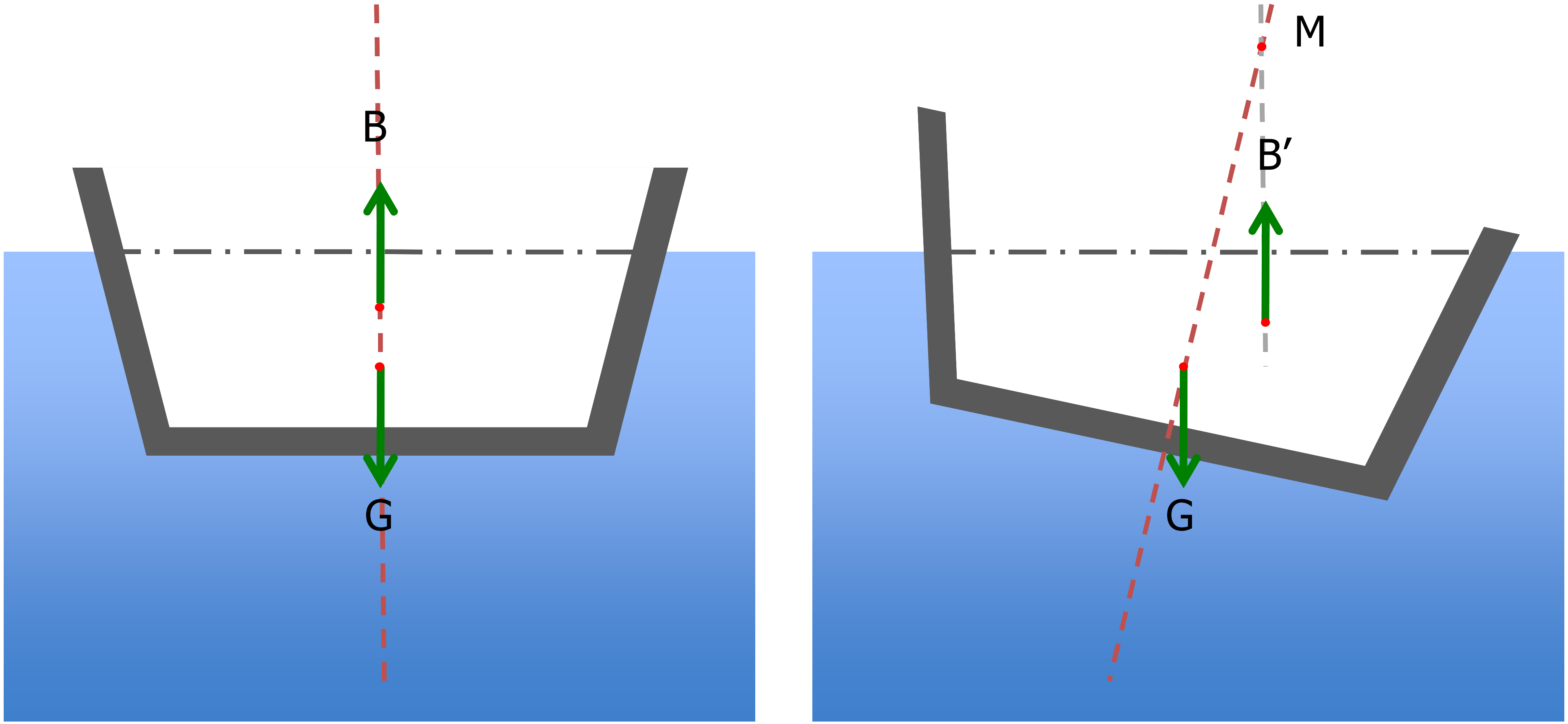
Net force on object:

$$F = F_o + F_p = -\rho_o g \Delta x \Delta y \Delta z + \rho_f g \Delta x \Delta y \Delta z$$

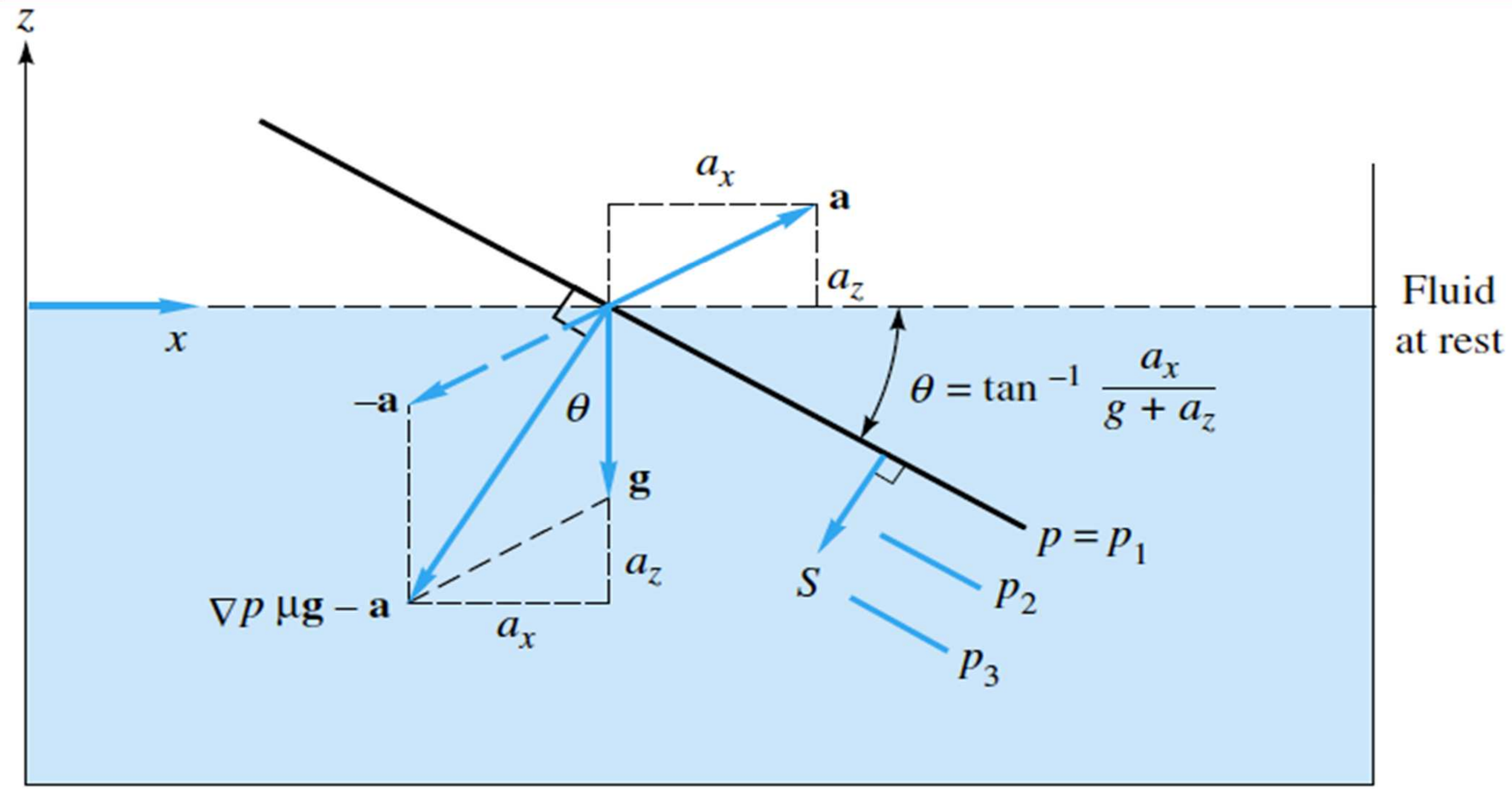
$$= -(\rho_o - \rho_f) g \Delta V$$

Weight of displaced volume

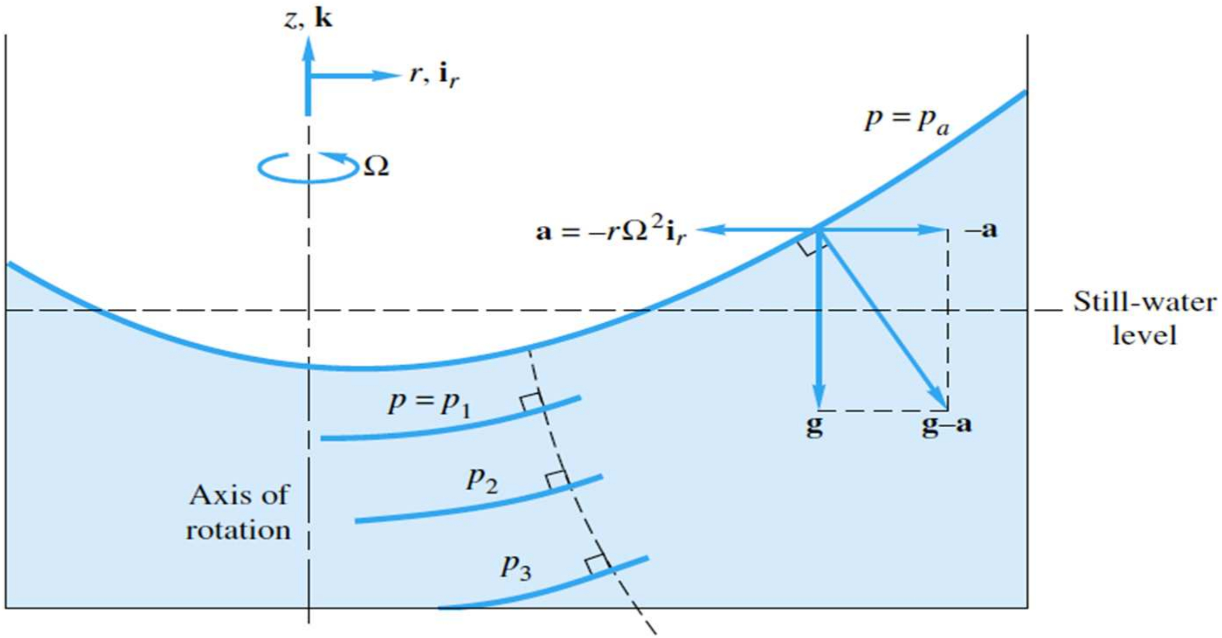
Stability of a floating body



Uniform linear acceleration



Rotating frame



$$\frac{\partial p}{\partial z} = -\rho g$$

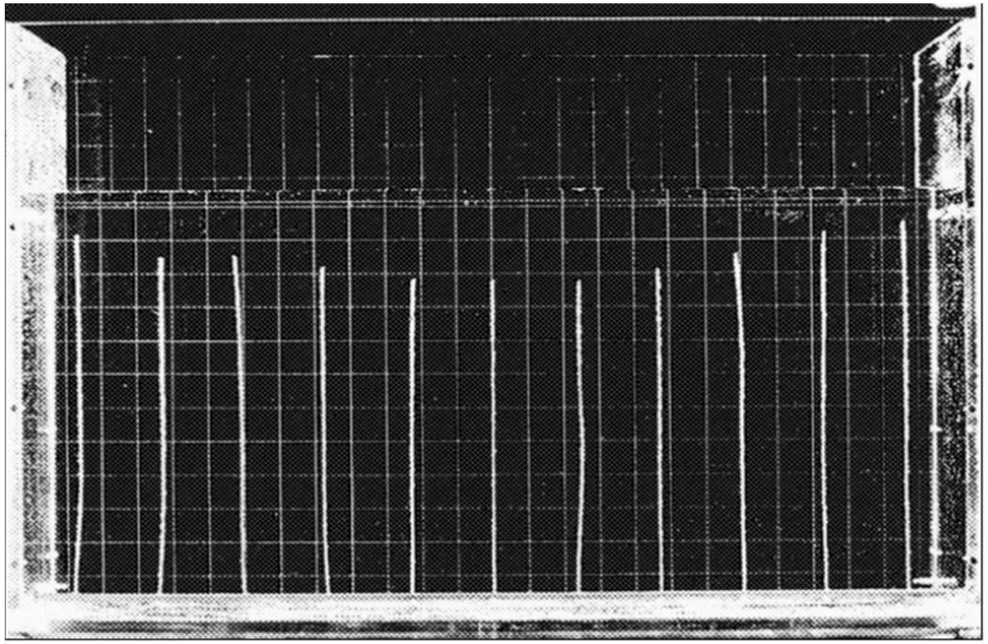
$$\frac{\partial p}{\partial r} = \rho r \Omega^2$$

$$p = \tfrac{1}{2}\rho r^2 \Omega^2 + f(z)$$

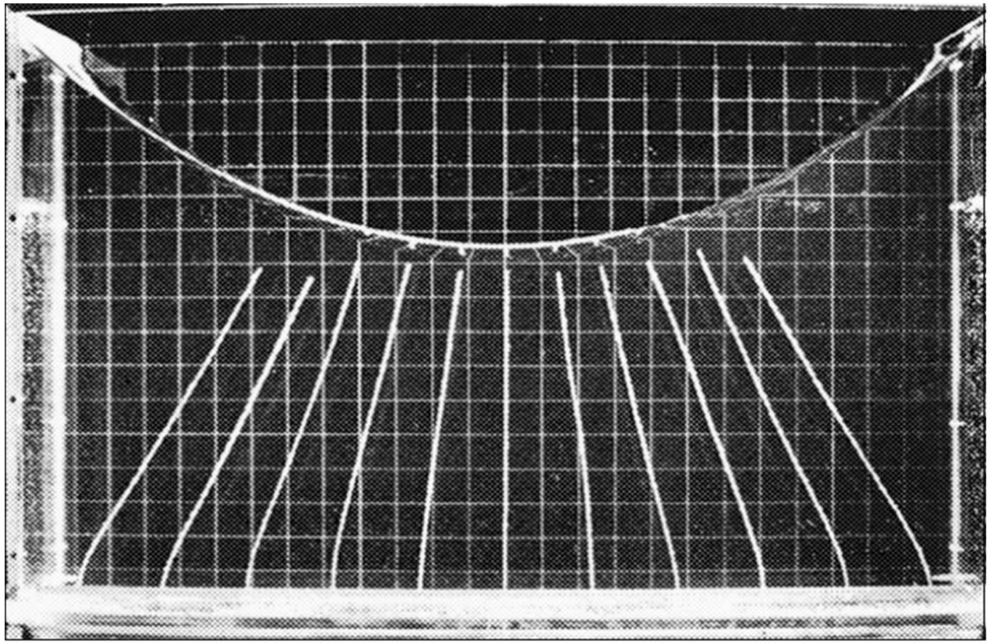
$$\frac{\partial p}{\partial z} = \quad 0 \quad + f'(z) \equiv -\rho g$$

$$\boxed{p = p_0 - \rho g z + \tfrac{1}{2}\rho r^2 \Omega^2}$$

Rigid body rotation



non-rotating



rotating

Casting of a telescope mirror

[5]

[6]

Example

[7]

Summary

- Chapter 2: 2.1, 2.3-2.9, (2.10)
- Examples: 2.3, 2.4
- Problems: 2.37, 2.44,

Sources

1. F/A-18 Hornet breaking sound barrier, *http://www.news.navy.mil,* Photo courtesy of Ensign John Gay, U.S. Navy
2. Cavitation Tunnel, *http://www.nrc-cnrc.gc.ca,* Photo courtesy of NRC-CNRC
3. Cavitation, *http://stilton.tnw.utwente.nl,* Photo courtesy of G. Kuiper
4. Cavitation Propeller Damage, *http://en.wikipedia.org,* Photo courtesy of Erik Axdahl
5. Mirror Cast, *http://www.eso.org,* Photo courtesy of European Southern Observatory
6. Parabolic moondust mirror, *http://science.nasa.gov,* Photo courtesy of Peter C. Chen, NASA/GSFC
7. Driving a Car with a Helium Balloon: Physics; *http://youtu.be/XXpURFYgR2E*; courtesy of Tessa Ricci and Austin Jaspers