Fluid mechanics
(wb1225)

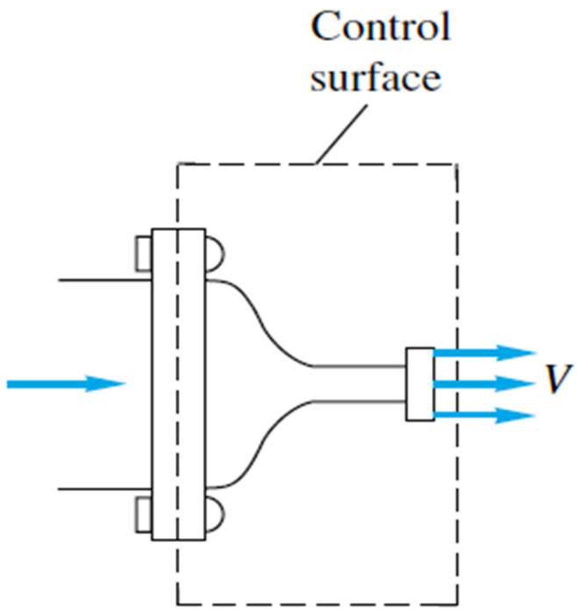
Lecture 3:
control volume analysis

Control volumes

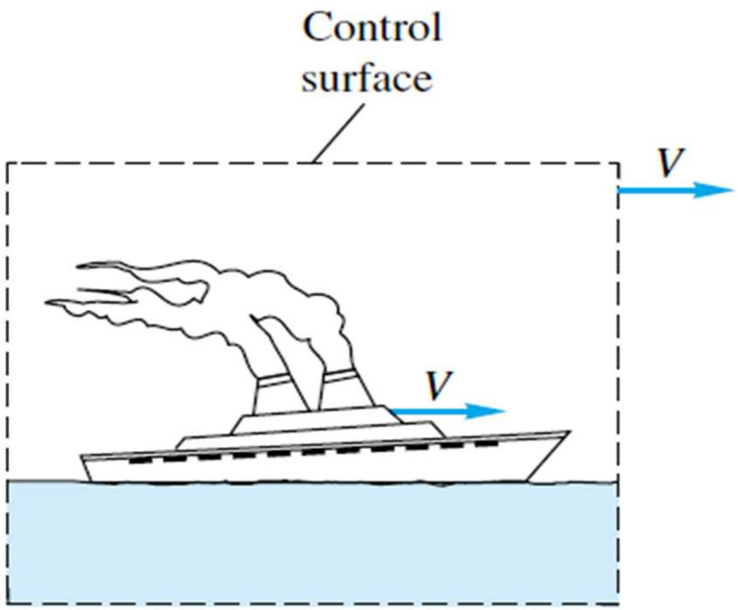
[1]

Fire hose rodeo [2]

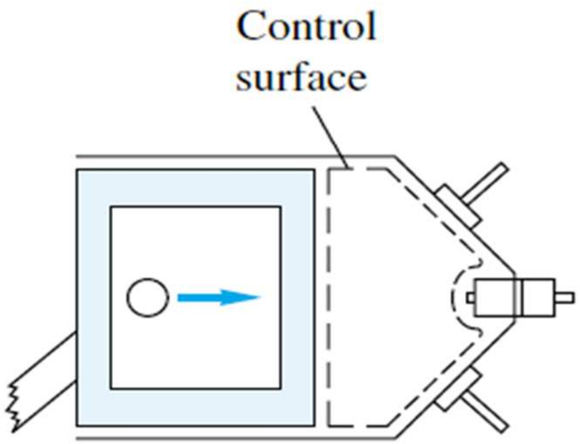
Control volume (CV)



Fixed CV



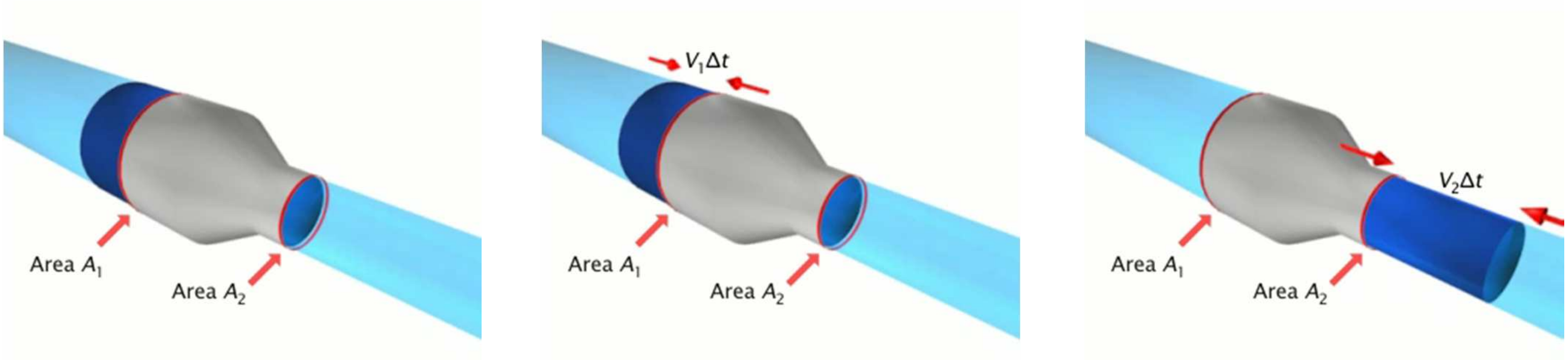
moving CV

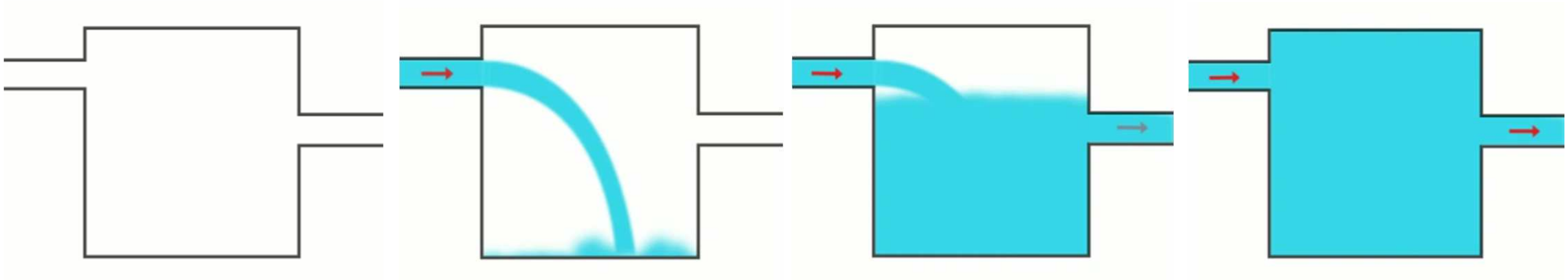


Deformable CV

[3]

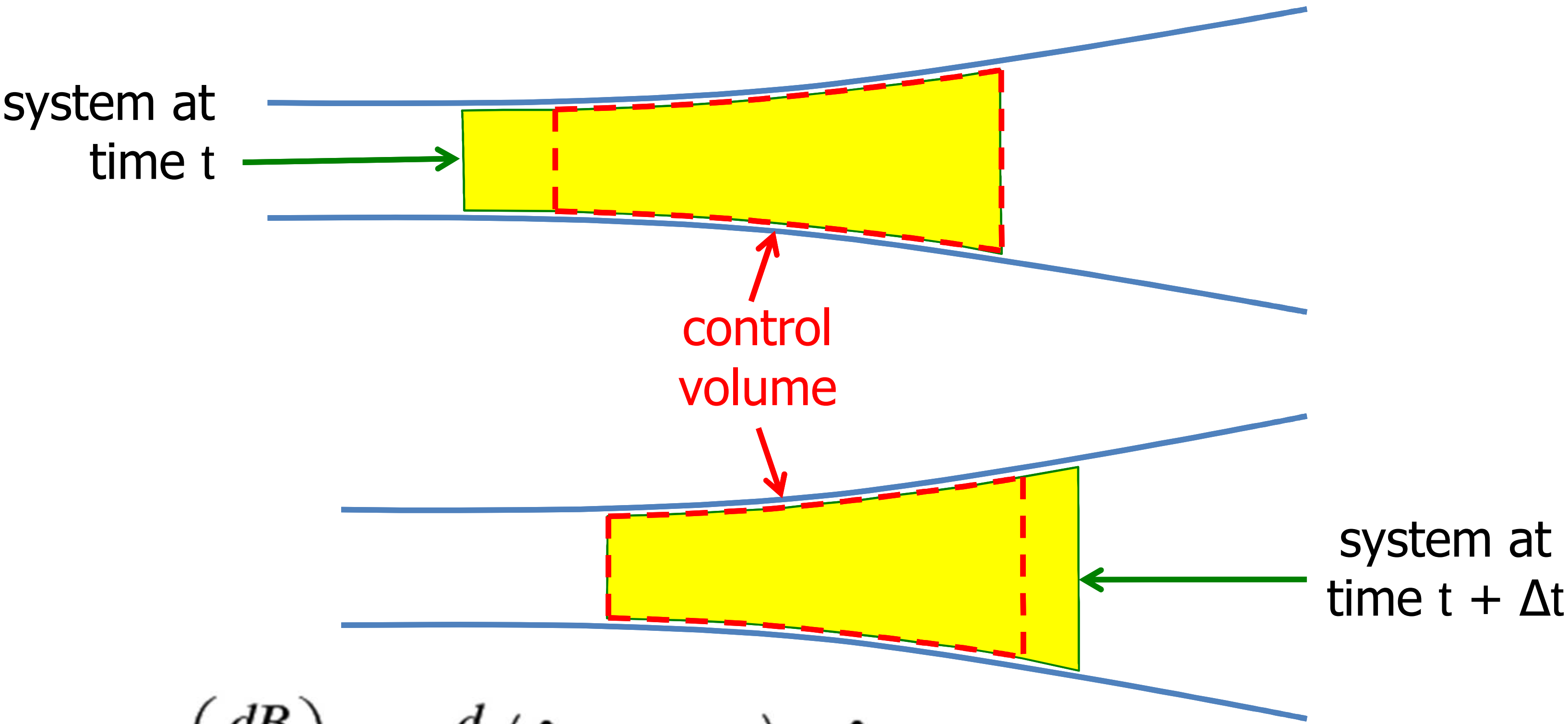
System vs. control volume





[4]

System vs. control volume

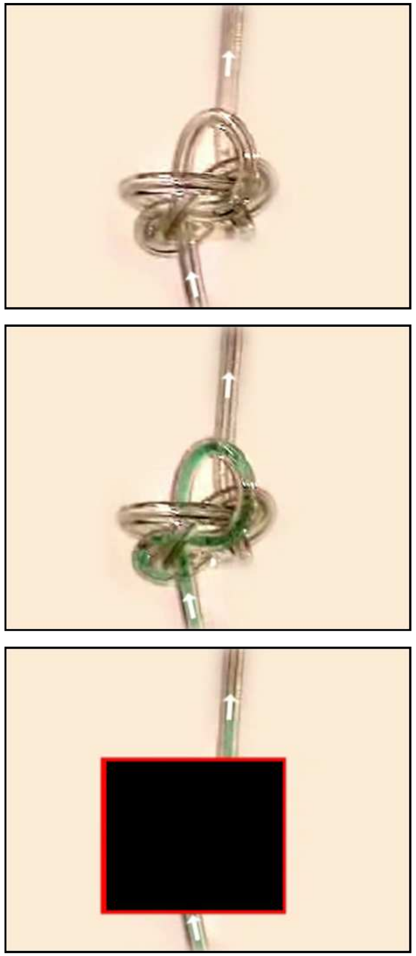


$$\underbrace{\left(\frac{dB}{dt}\right)_{syst}}_{\substack{\text{change of}\\ \text{system mass}}} = \underbrace{\frac{d}{dt}\left(\int_{CS}\rho\beta\, d\mathcal{V}\right)}_{\substack{\text{change of}\\ \text{mass in CV}}} + \underbrace{\int_{CS}\rho\beta\left(\mathbf{V}_r\cdot\mathbf{n}\right)dA}_{\substack{\text{flux through}\\ \text{surface of CV}}}$$

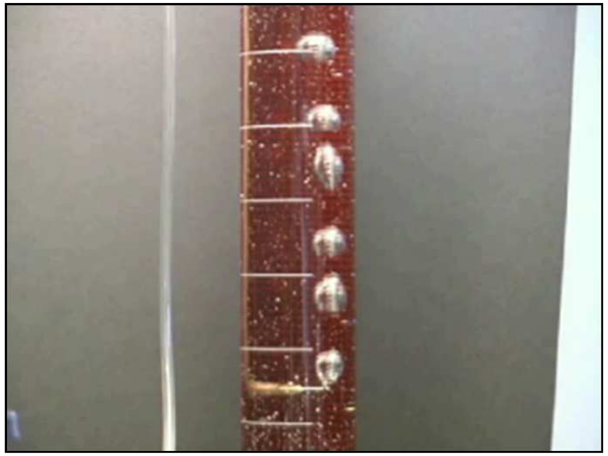
B : system property
(mass, momentum, energy)

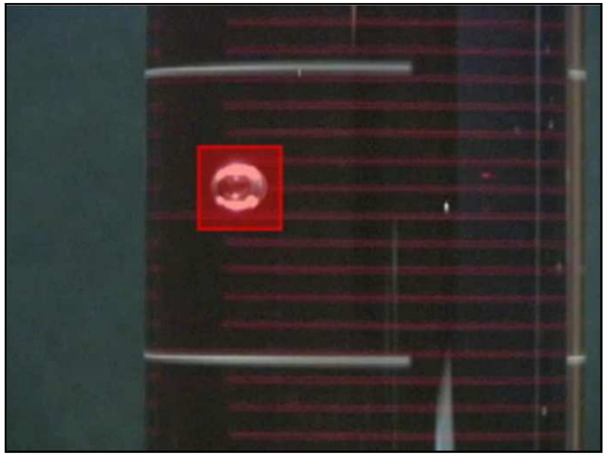
β : property / unit mass

Control volume

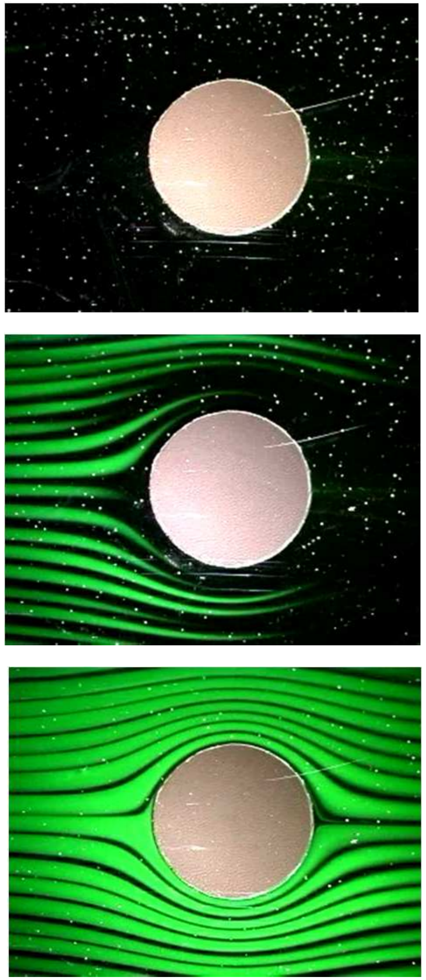


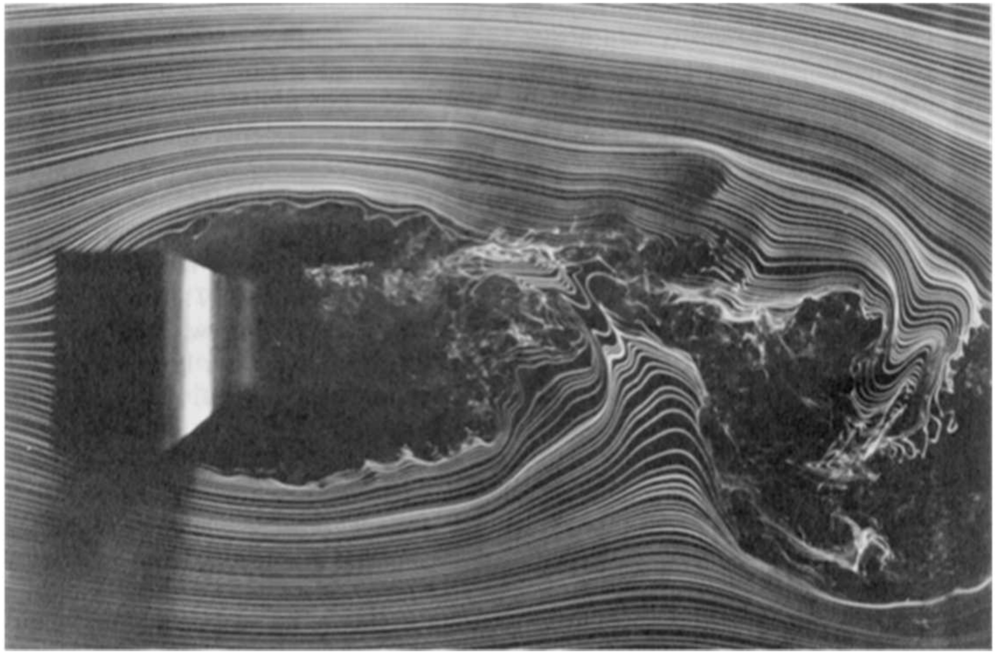
[4]





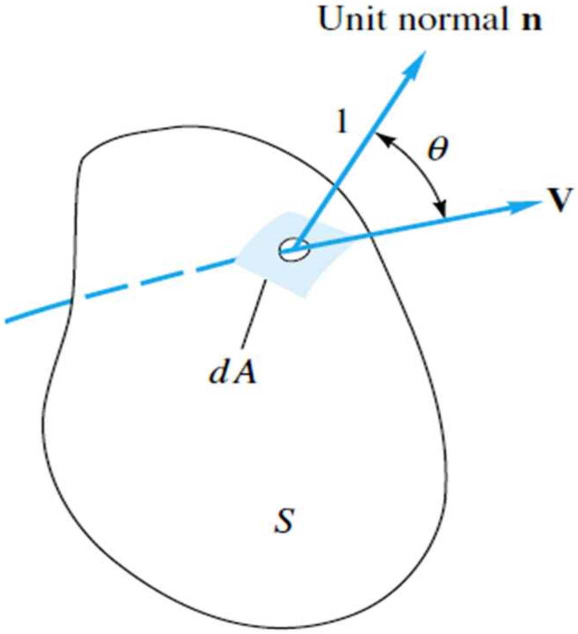
Control volume (CV)

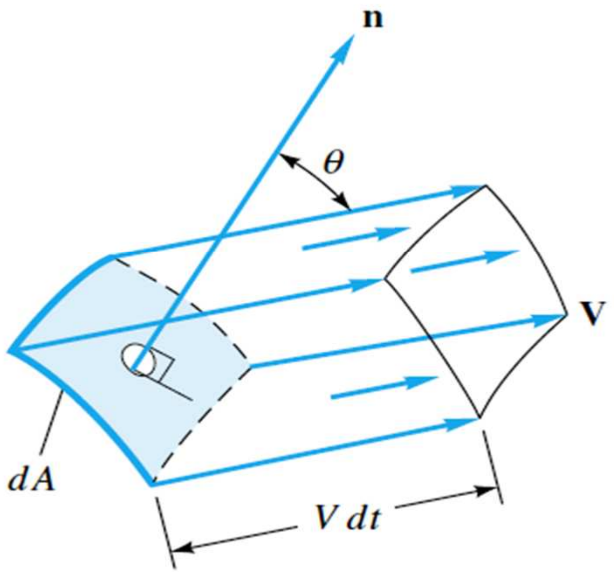




[4]

Volume flow rate

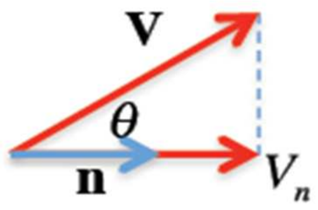




[3]

Inner product:

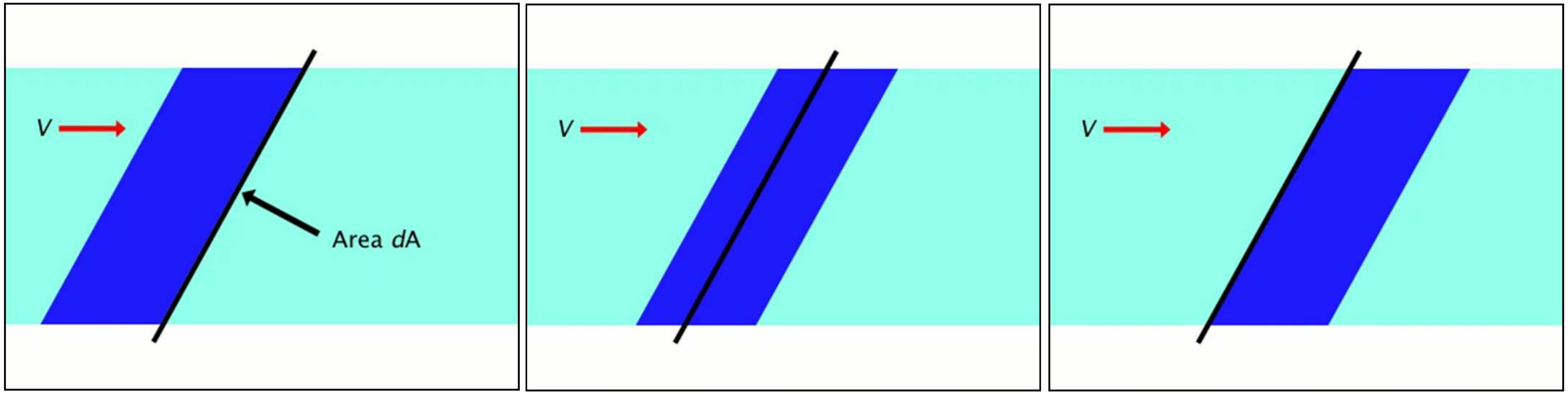
$$\mathbf{V}\cdot\mathbf{n} = V_x n_x + V_y n_y + V_z n_z$$
$$= \|\mathbf{V}\|\cos\theta$$



$$d\mathcal{V} = V\,dt\,dA\cos\theta = (\mathbf{V}\cdot\mathbf{n})\,dA\,dt$$

$$\frac{d\mathcal{V}}{dt} \equiv Q = \int_S (\mathbf{V}\cdot\mathbf{n})\,dA = \int_S V_n\,dA$$

Flux through a surface



[4]

Conservation of mass

$$\int_{CV} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{CS} \rho (\mathbf{V} \cdot \mathbf{n})\, dA = 0$$

Finite number of one-dimensional inlets/outlets:

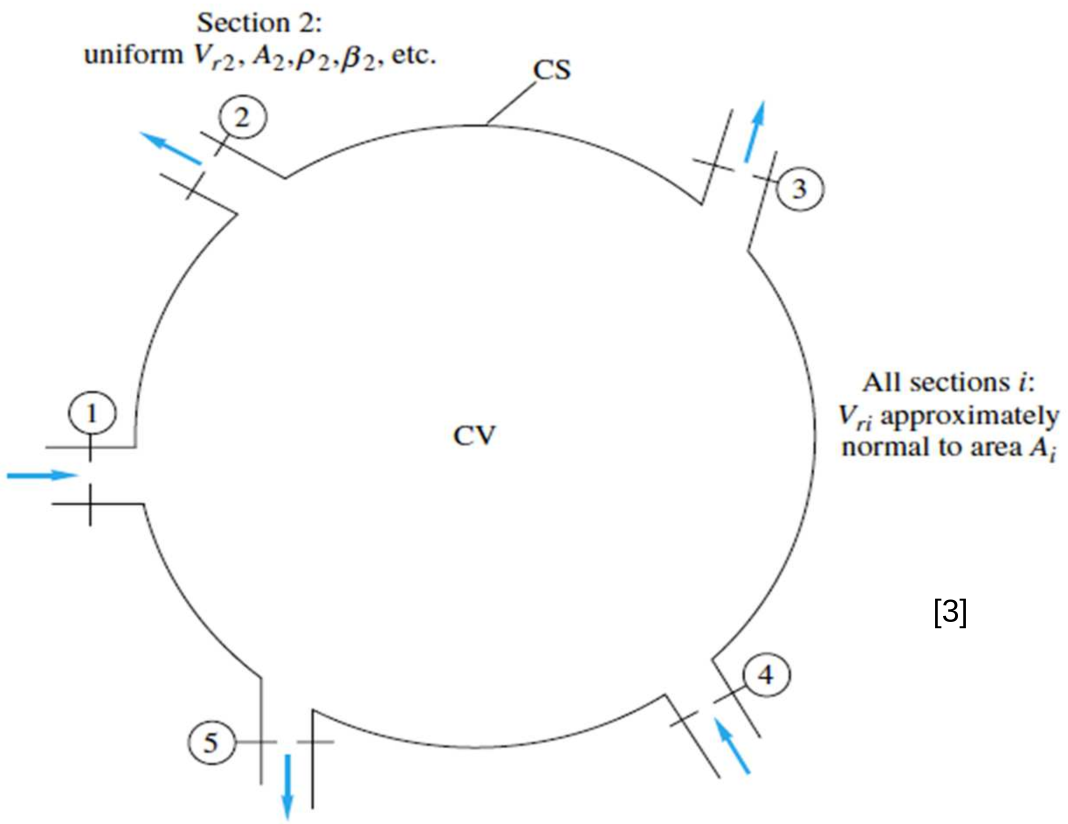
$$\underbrace{\int_{CV} \frac{\partial \rho}{\partial t} d\mathcal{V}}_{\text{change of mass in CV}} + \underbrace{\sum_i (\rho_i A_i V_i)_{\text{out}}}_{\text{mass flux leaving CV}} - \underbrace{\sum_i (\rho_i A_i V_i)_{\text{in}}}_{\text{mass flux entering CV}} = 0$$

ditto, for steady flow $(\partial \rho / \partial t = 0)$:

$$\sum_i (\rho_i A_i V_i)_{\text{out}} = \sum_i (\rho_i A_i V_i)_{\text{in}} \quad \left[\frac{kg}{m^3} \cdot m^2 \cdot \frac{m}{s} = \frac{kg}{s} \right]$$

$$\sum_i \dot{m}_{i,\text{out}} = \sum_i \dot{m}_{i,\text{in}} \quad \Rightarrow \quad \sum_i \dot{m}_i = 0$$

Conservation of mass



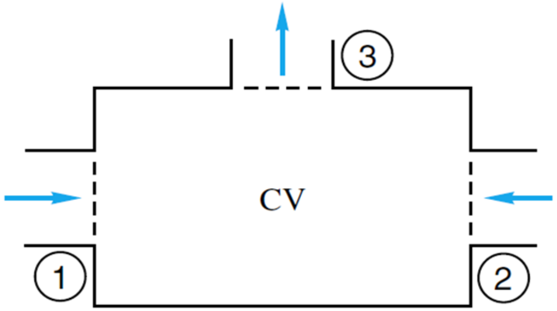
[3]

Example: traffic in Iraq [5]

Several examples

- energy flux and heat loss
  (stationary problem)
- inflating a punch ball (moving control surface)
- jet impinging on an inclined plate
  (conservation of momentum in 2 dimensions)
- hydraulic jump (moving control volume)
- boundary layer (shear stress forces)
- rocket (accelerating control volume)

Example 3.1



A fixed control volume has three one-dimensional boundaries as shown. The flow within the control volume is steady. The flow properties at each section are given in the table.
Find the rate of change of energy of the system which occupies the control volume at this instant.

Section	Type	ρ, kg/m^3	V, m/s	A, m^2	e, J/kg
1	Inlet	800	5.0	2.0	300
2	Inlet	800	8.0	3.0	100
3	Outlet	800	17.0	2.0	150

$B = E$

$\beta = dE/dm = e$

$$\underbrace{\left(\frac{dE}{dt}\right)_{\text{syst}}}_{\substack{\text{rate of change}\\ \text{of energy in system}}} = \underbrace{\int_{CV} \frac{\partial}{\partial t}(\rho e)\, d\mathcal{V}}_{\substack{\text{change of energy}\\ \text{within CV}}} + \underbrace{\int_{CS} \rho e (\mathbf{V}\cdot\mathbf{n})\, dA}_{\substack{\text{flux of energy}\\ \text{through CV boundary}}}$$

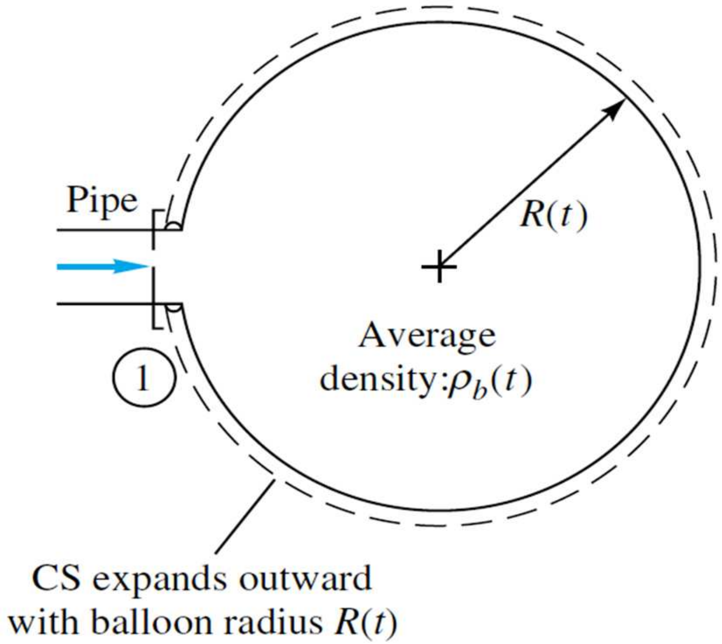
check mass balance!

$$\left(\frac{dE}{dt}\right)_{\text{syst}} = 0 \underbrace{-\rho_1 e_1 A_1 V_1}_{\text{inlet}} \underbrace{-\rho_2 e_2 A_2 V_2}_{\text{inlet}} \underbrace{+\rho_3 e_3 A_3 V_3}_{\text{outlet}} = -0.24 \text{ MW}$$

Example 3.2

The balloon on the right is being filled through (1), where the cross section is A_1, the velocity is V_1, and the fluid density is ρ_1. The average density in the balloon is $\rho_b(t)$.

Find an expression for the rate of change of system mass within the balloon at time t.



$$\underbrace{\left(\frac{dm}{dt}\right)_{syst}}_{\substack{\text{change of}\\ \text{system mass}}} = \underbrace{\frac{d}{dt}\left(\int_{CS} \rho \, d\mathcal{V}\right)}_{\substack{\text{change of}\\ \text{mass in CV}}} + \underbrace{\int_{CS} \rho(\mathbf{V}_r \cdot \mathbf{n})\, dA}_{\substack{\text{flux through}\\ \text{surface of CV}}}$$

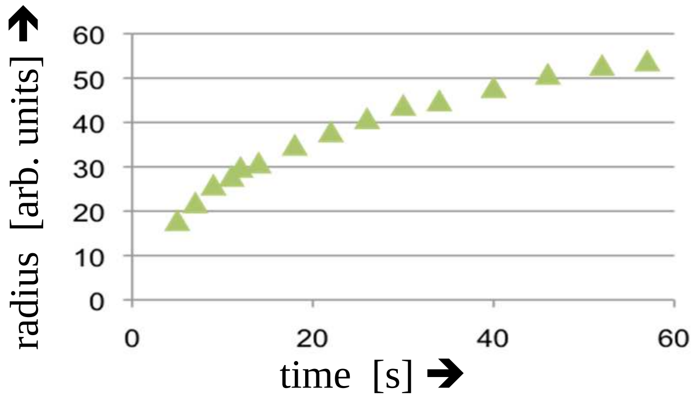
$$0 = \frac{d}{dt}\left(\rho_b \tfrac{4}{3}\pi R^3\right) - \rho_1 A_1 V_1$$

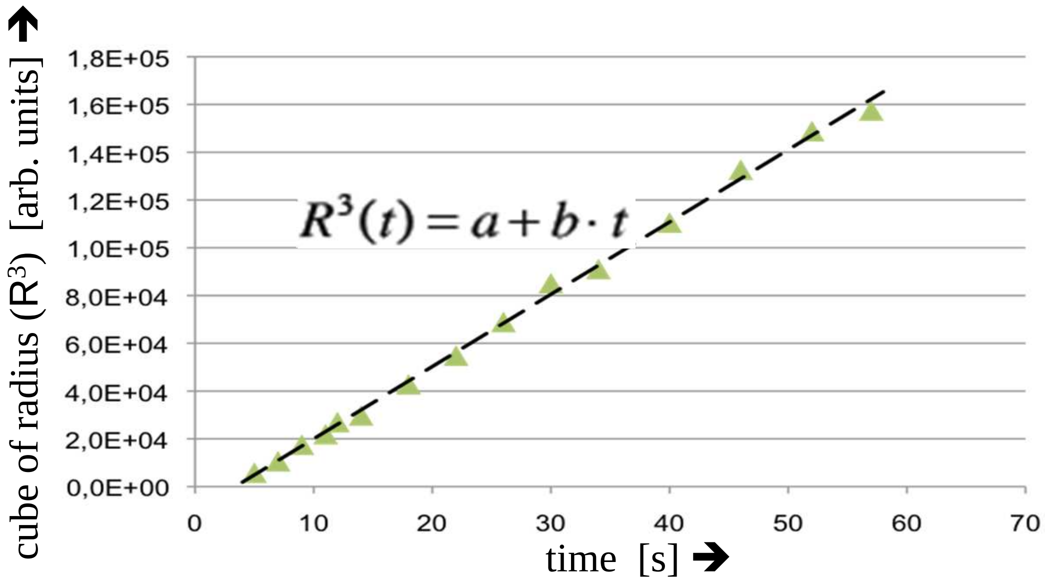
$$\frac{d}{dt}\left(\rho_b R^3\right) = \frac{3}{4\pi}\rho_1 A_1 V_1 \xrightarrow{\rho_b = \rho_1} \frac{d}{dt}R^3 = \frac{3}{4\pi}A_1 V_1 \Rightarrow R(t) = \left(R_0^3 + \frac{3A_1V_1}{4\pi}t\right)^{1/3}$$

Note: ignoring the surface tension of the ballon; this determines the pressure (i.e., air density)

Experiment

[6]

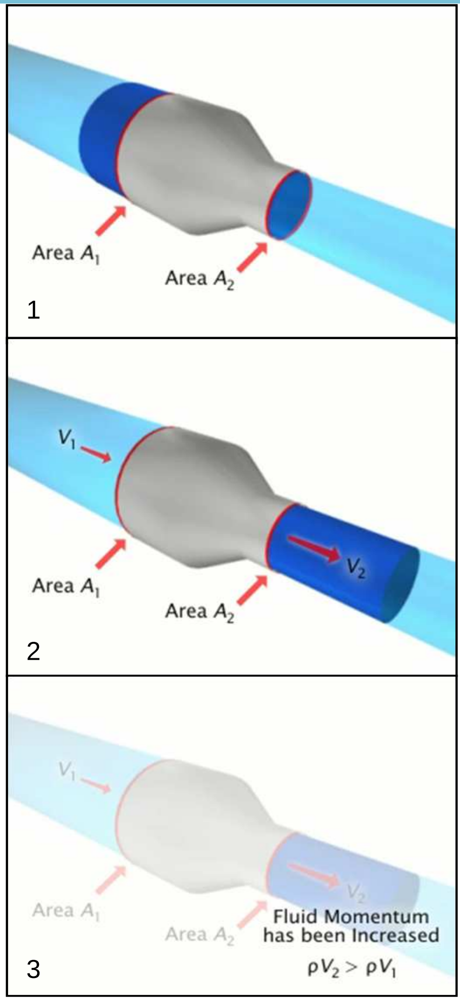




$$R^3(t) = R_0^3 + \frac{3A_1V_1}{4\pi}t$$

$$R^3(t) = \frac{3A_1V_1}{4\pi}(t - t_0)$$

Conservation of momentum



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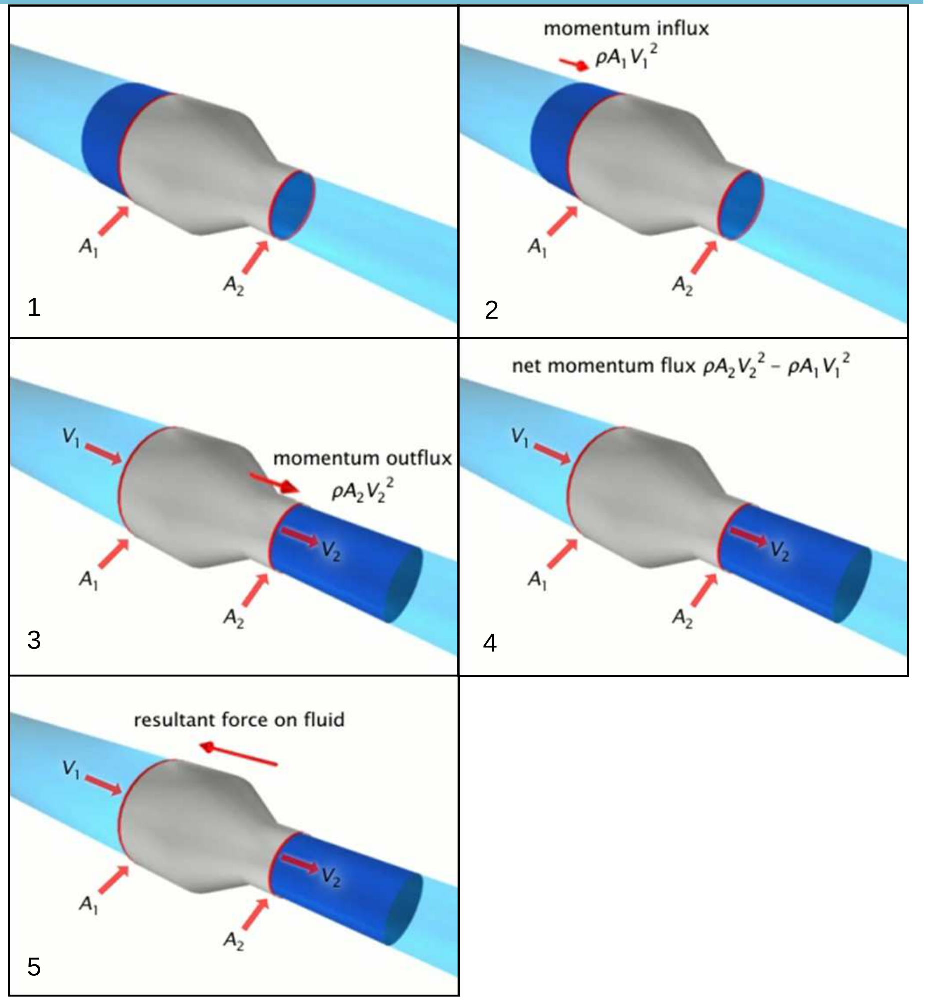

Summary

- Chapter 3: 3.1-3.4
- Examples: 3.1-3.6
- Problems: 3.36, 3.50, 3.51, 3.73, 3.86

Sources

1. Too much pressure!, *http://youtu.be/R8PQTR0vFaY ;* video courtesy of kibba90660
2. Fire hose rodeo – Jackass, *http://youtu.be/jMhD4l_HGcQ*, video courtesy of Jackass
3. Frank M. White, *Fluid Mechanics*, McGraw-Hill Series in Mechanical Engineering
4. Multimedia Fluid Mechanics DVD-ROM, G. M. Homsy, University of California, Santa Barbara
5. Iraqi traffic jam, *http://youtu.be/oRE2eIdYRtY*
6. B2P a blue punchball, *http://youtu.be/I1foEuu1qfE*