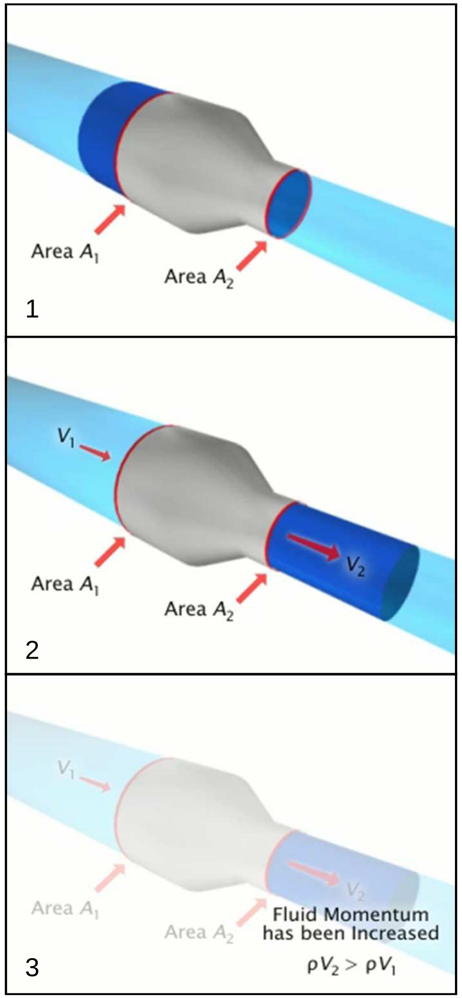
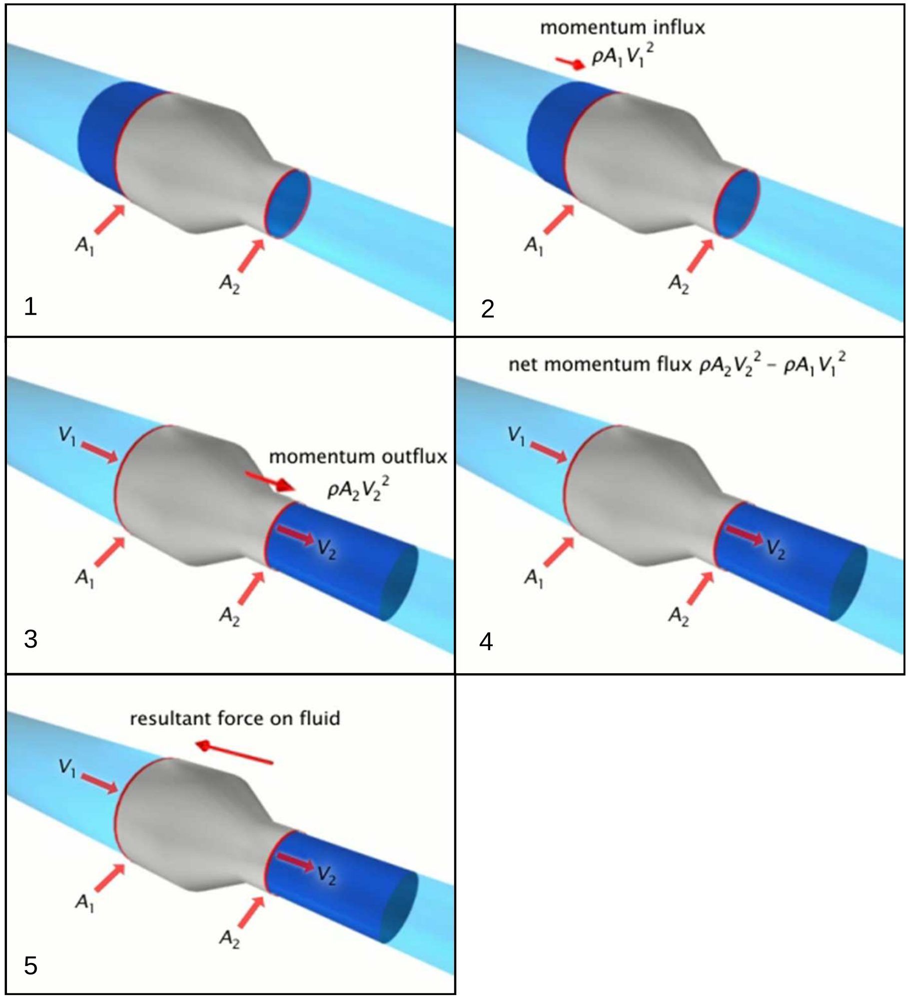
Fluid mechanics (wb1225)

Lecture 4:
control volume analysis
Momentum equation

Conservation of momentum



[1]



Conservation of momentum

$$\mathbf{B} = m\mathbf{V} \quad \text{(momentum)}$$
$$\beta = d\mathbf{B}/dm = \mathbf{V} \quad \text{(velocity)}$$
$$\mathbf{V} = \left(V_x, V_y, V_z\right) \quad \text{(vector!)}$$

$$\frac{d}{dt}(m\mathbf{V})_{\text{syst}} = \underbrace{\sum \mathbf{F}}_{\text{sum of all forces}} = \underbrace{\frac{d}{dt}\left(\int_{CV} \rho \mathbf{V}\, d\mathcal{V}\right)}_{\text{rate of momentum change inside volume}} + \underbrace{\int_{CS} \rho \mathbf{V}\left(\mathbf{V}_r \cdot \mathbf{n}\right) dA}_{\text{flux of momentum through boundary of CV}}$$

momentum flux: $\dot{\mathbf{M}}_{CS} = \int_{CS} \rho \mathbf{V}\left(\mathbf{V} \cdot \mathbf{n}\right) dA \quad \Rightarrow \quad \dot{\mathbf{M}}_{CS} = \sum_i \mathbf{V}_i \underbrace{\left(\rho_i V_{ni} A_i\right)}_{\dot{m}_i} = \sum_i \dot{m}_i \mathbf{V}_i$

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \rho \mathbf{V}\, d\mathcal{V}\right) + \underbrace{\sum_i \left(\dot{m}_i \mathbf{V}_i\right)_{\text{out}}}_{\text{momentum flux out of control volume}} - \underbrace{\sum_i \left(\dot{m}_i \mathbf{V}_i\right)_{\text{in}}}_{\text{momentum flux into control volume}}$$

- pressure forces
- forces due to viscous stresses
- forces by solid bodies that protrude through surface

Example 3.9

$$\mathbf{F}_{\text{vane}} = \dot{m}_2 \mathbf{V}_2 - \dot{m}_1 \mathbf{V}_1$$

given: $\left\|\mathbf{V}_2\right\| = \left\|\mathbf{V}_1\right\| = V$

conservation of mass:

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho A V$$

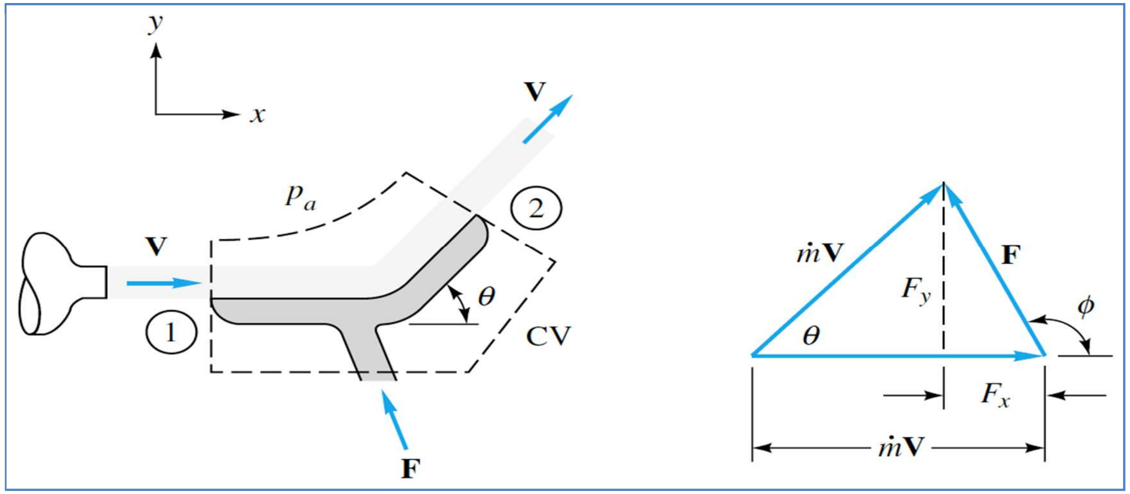
$$F_x = \dot{m} V (\cos\theta - 1) \quad F_y = \dot{m} V \sin\theta$$

with: $\dot{m} V = \rho A V^2$

$$\left\|\mathbf{F}\right\| = \left(F_x^2 + F_y^2\right)^{1/2} = \dot{m} V \left[\sin^2\theta + (\cos\theta - 1)^2\right]^{1/2}$$

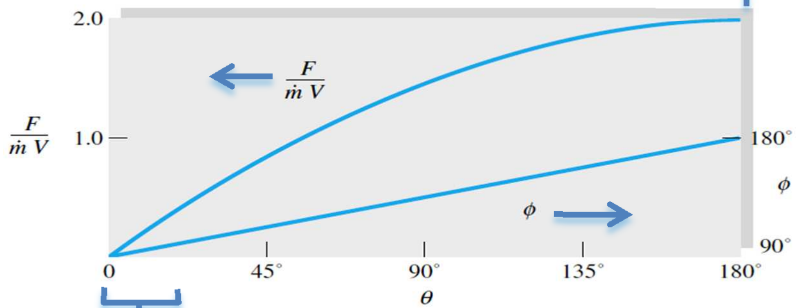
$$= 2\,\dot{m} V \sin\tfrac{1}{2}\theta$$

$$\phi = 180^\circ - \tan^{-1}\frac{F_y}{F_x} = 90^\circ + \frac{\theta}{2}$$



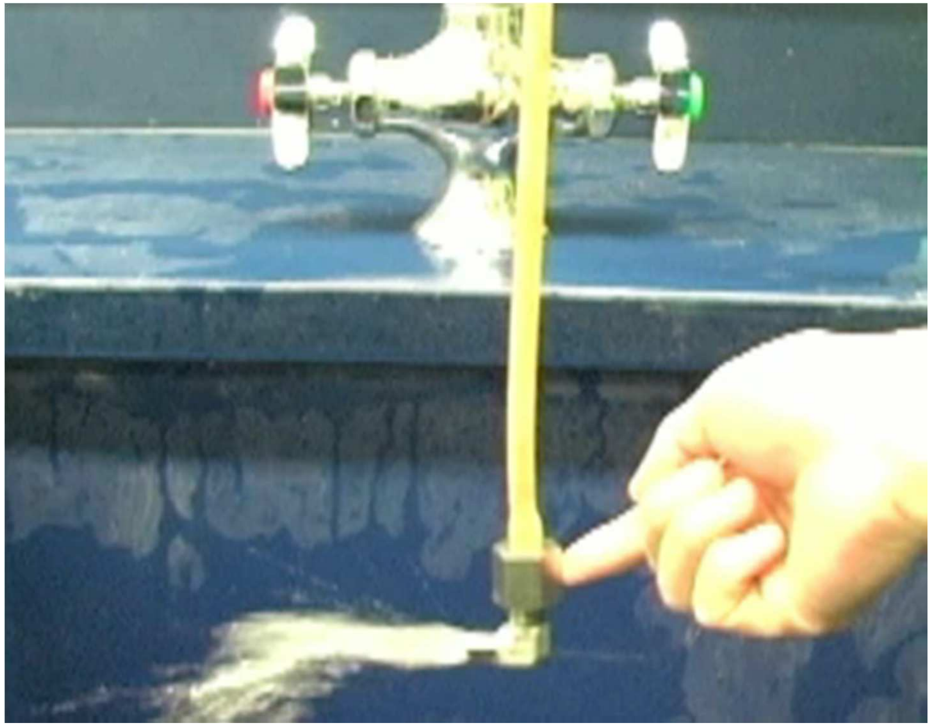
[2]

maximum momentum transfer when jet direction is reversed

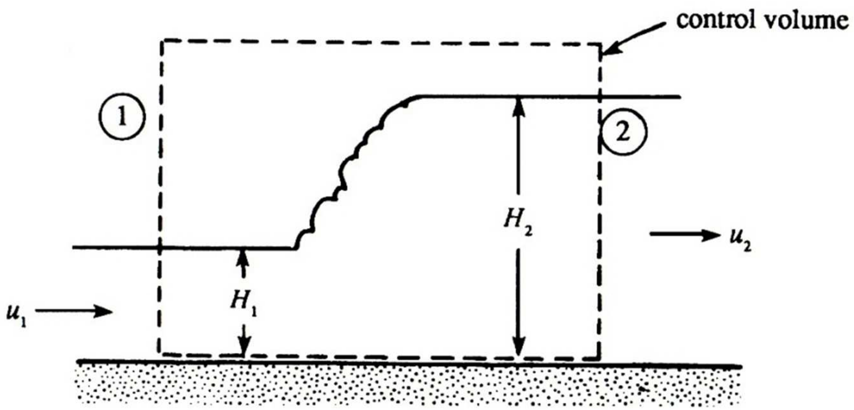


small angle deflection; force is normal to vane, and F ~ θ

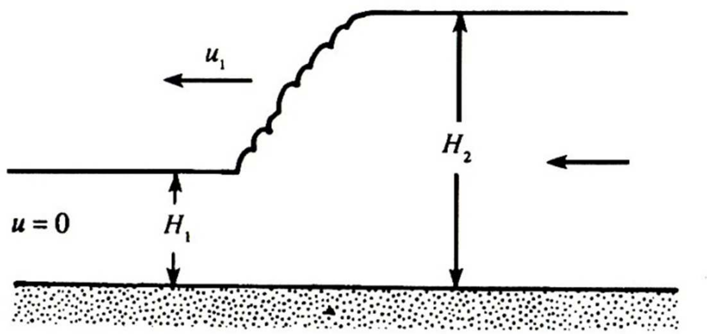
Example


[1]

Hydraulic jump



(b) Stationary



(c) Propagating

continuity:

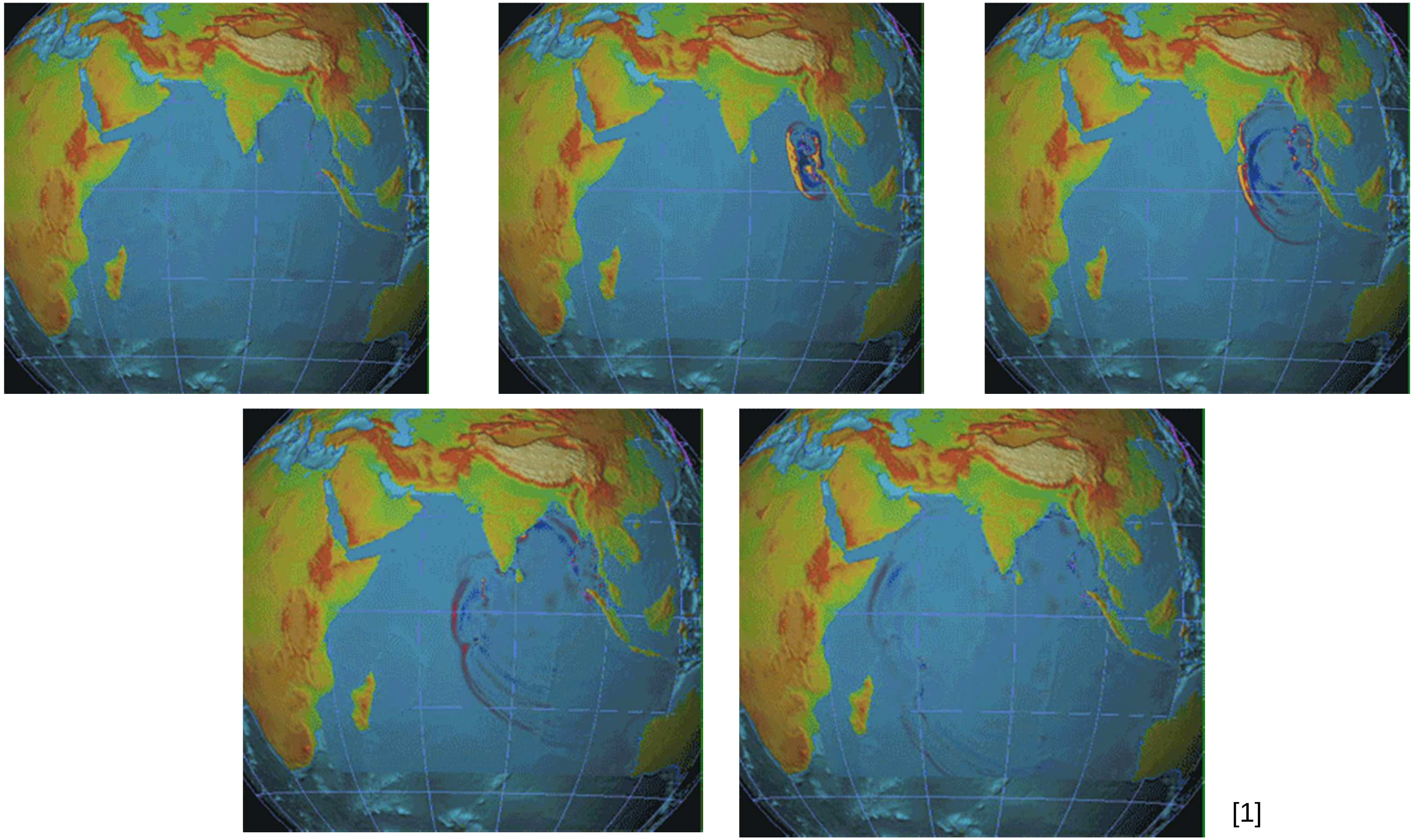
$$Q = u_1 H_1 = u_2 H_2$$

momentum integral:

$$\frac{1}{2}\rho g H_1^2 - \frac{1}{2}\rho g H_2^2 = \rho Q\left(u_2 - u_1\right)$$

$$\frac{\Delta H}{H} \to 0 \quad \Rightarrow \quad u = \sqrt{gH}$$

2004 Tsunami



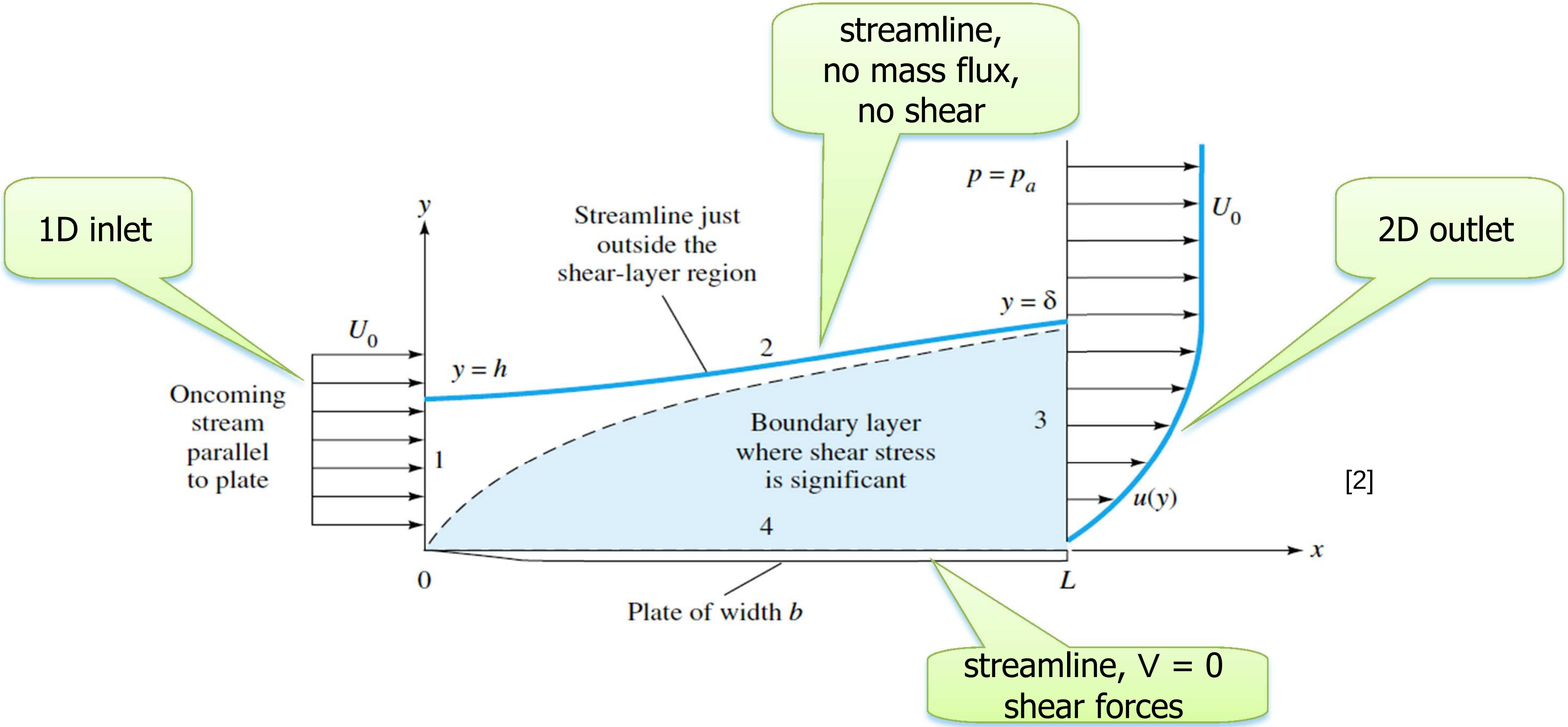
[1]

Hydraulic jump

[1]

Example 3.11

Boundary layer over a flat plate



[2]

Example 3.11

$$\sum F_x = -D = \rho \int_1 u V_n \, dA + \rho \int_3 u V_n \, dA$$

$$= \rho \int_0^h U_0 (-U_0) b \, dy + \rho \int_0^\delta u(y)(+u(y)) b \, dy$$

$$\Rightarrow \quad D = \rho U_0^2 bh - \rho b \int_0^\delta u^2(y) dy$$

$h \rightarrow \delta$? $\Rightarrow$ conservation of mass $\quad U_0 h = \int_0^\delta u(y) dy$

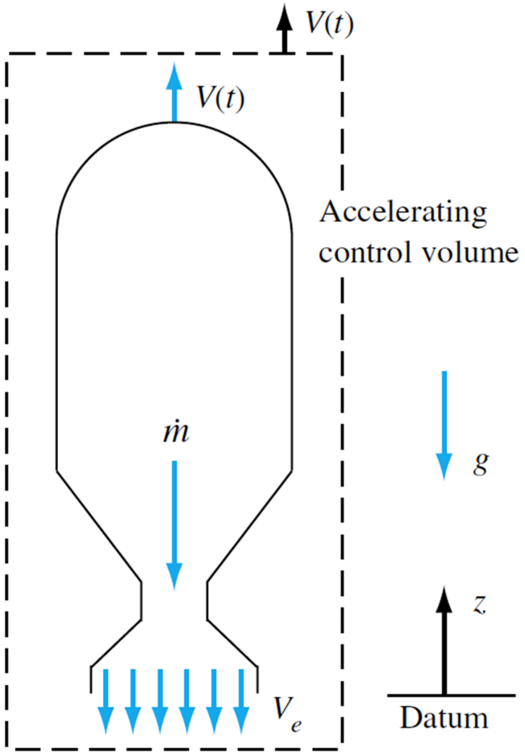
$$\boxed{D = \rho b \int_0^\delta \underbrace{u(U_0 - u)}_{\text{momentum defect}} dy}$$

Theodore von Kármán
(1921)

momentum defect

$$u(y) \cong U_0 \left(2\frac{y}{\delta} - \frac{y^2}{\delta^2} \right), \quad 0 \leq y \leq \delta \quad \Rightarrow \quad D = \tfrac{2}{15} \rho U_0^2 b \delta$$

Example 3.12



[2]

$$\sum F_z - \int a_{\text{rel}}\, dm = \frac{d}{dt}\left(\int_{CV} w\, d\dot{m}\right) + (\dot{m}w)_e$$

$$-mg - \quad m\frac{dV}{dt} \quad = \quad 0 \quad + \ \dot{m}V_e$$

$$\text{with} \quad m = m(t) = M_0 - \dot{m}\,t$$

$$\int_0^V dV = \dot{m}V_e \int_0^t \frac{dt}{M_0 - \dot{m}\,t} - g\int_0^t dt, \quad V(t=0) = 0$$

$$V(t) = -V_e \ln\left(1 - \frac{\dot{m}\,t}{M_0}\right) - g\,t$$

Example: Pelton wheel

[3]

Summary

- Chapter 3: 3.1-3.4
- Examples: 3.1-3.6
- Problems: 3.36, 3.50, 3.51, 3.73, 3.86

Source

1. Multimedia Fluid Mechanics DVD-ROM, G. M. Homsy, University of California, Santa Barbara
2. Frank M. White, *Fluid Mechanics*, McGraw-Hill Series in Mechanical Engineering
3. Pelton Turbine, http://www.eumarine.com