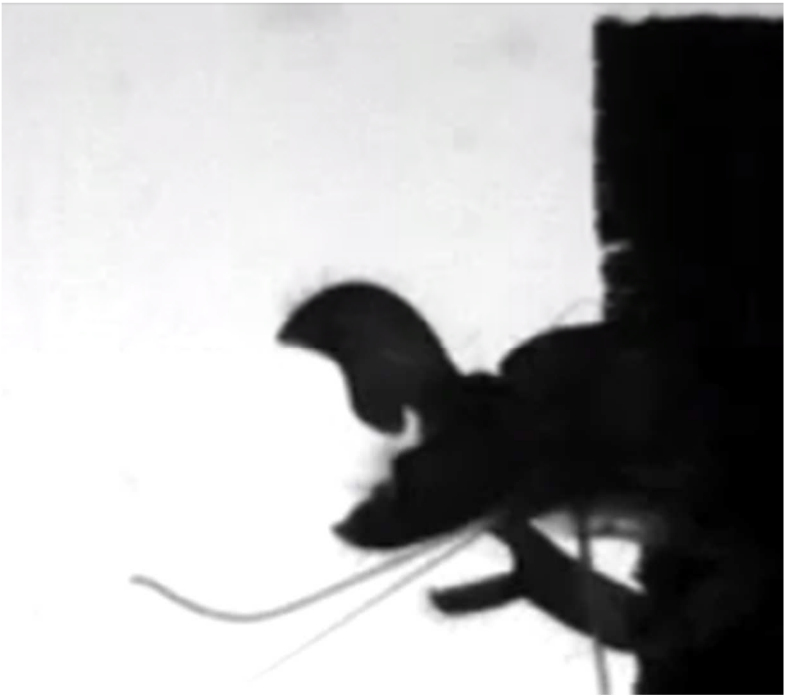
Fluid mechanics (wb1225)

Lecture 5: energy equation

Snapping shrimp



[1]

The energy equation

$$B = E \qquad \frac{dE}{dt} = \dot{Q} - \dot{W}$$

$$\beta = \frac{dE}{dm} = e \qquad e = e_{\text{internal}} + e_{\text{kinetic}} + e_{\text{potential}} + e_{\text{other}}$$

$$\dot{Q} - \dot{W} = \frac{dE}{dt} = \frac{d}{dt}\left(\int_{CV} \rho\, e\, d\mathcal{V}\right) + \int_{CS} \rho\, e \underbrace{(\mathbf{V} \cdot \mathbf{n})}_{V_n} dA$$

$$e = \hat{u} + \tfrac{1}{2}V^2 + gz$$

$$\dot{W} = \dot{W}_{\text{shaft}} + \dot{W}_{\text{press}} + \dot{W}_{\text{visc}} + \dot{W}_{\text{other}}$$

$\dot{W}_{\text{shaft}} =$ work by machine part

$\dot{W}_{\text{press}} = \int_{CS} p(\mathbf{V} \cdot \mathbf{n})\, dA$

$\dot{W}_{\text{visc}} = -\int \tau \cdot \mathbf{V}\, dA$

$\dot{W}_{\text{other}} =$ e.g., electromagnetic work, etc.

solid surface : $\mathbf{V} = \mathbf{0}$ (no-slip condition)
inlet/outlet : normal stresses only (small)
streamline : only important viscous stresses

The energy equation (cont'd)

$$\dot{W} = \dot{W}_{\text{shaft}} + \int_{CS} p(\mathbf{V}\cdot\mathbf{n})\,dA - \int_{CS} (\tau\cdot\mathbf{V})_{SS}\,dA$$

$$\dot{Q} - \dot{W}_{\text{shaft}} - (\dot{W}_{\text{visc}})_{SS} = \frac{\partial}{\partial t}\left(\int_{CV} \rho e \, d\mathcal{V}\right) + \int_{CS} \rho\left(e + \frac{p}{\rho}\right)(\mathbf{V}\cdot\mathbf{n})\,dA$$

enthalpy:

$$\hat{h} = \hat{u} + \frac{p}{\rho}$$

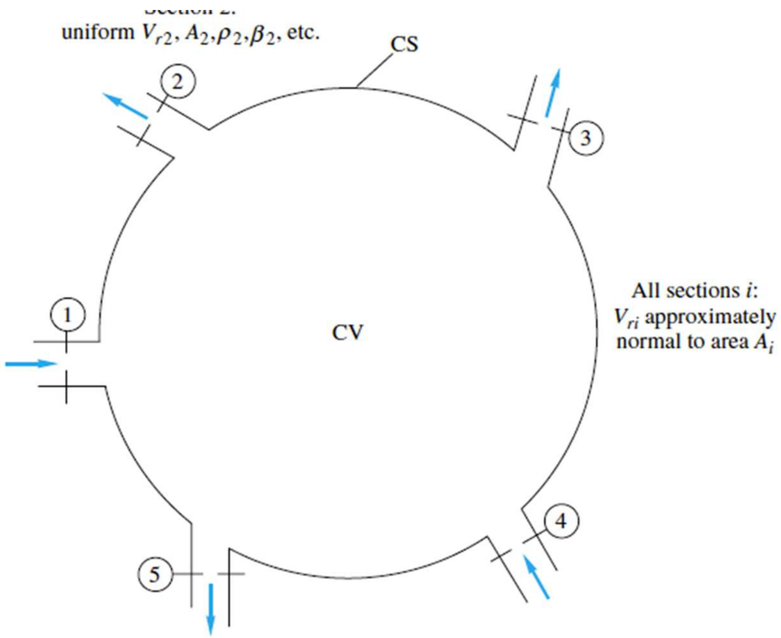
$$\dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{visc}} = \frac{\partial}{\partial t}\left[\int_{CV} \rho\left(\hat{u} + \tfrac{1}{2}V^2 + gz\right) d\mathcal{V}\right] + \int_{CS} \rho\left(\hat{h} + \tfrac{1}{2}V^2 + gz\right)(\mathbf{V}\cdot\mathbf{n})\,dA$$

small

One-dimensional stationary energy flux equation

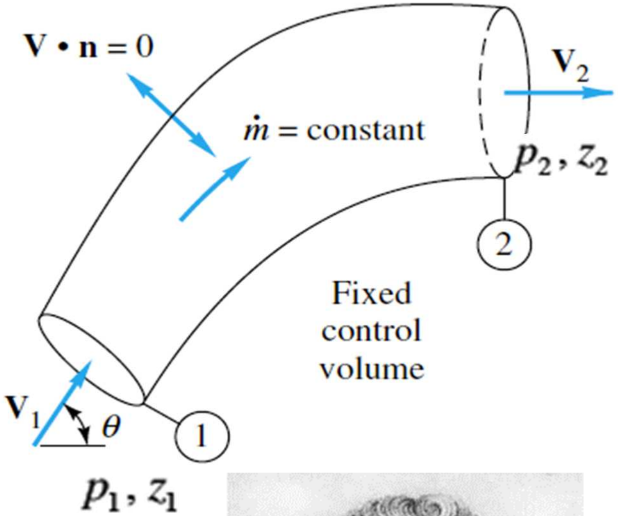
$$\int_{CS}\left(\hat{h}+\tfrac{1}{2}V^2+gz\right)\rho\underbrace{(\mathbf{V}\cdot\mathbf{n})}_{V_n}dA=$$

$$\sum\left(\hat{h}+\tfrac{1}{2}V^2+gz\right)_{\text{out}}\dot{m}_{\text{out}}-\sum\left(\hat{h}+\tfrac{1}{2}V^2+gz\right)_{\text{in}}\dot{m}_{\text{in}}$$



enthalpy: $\hat{h}=\hat{u}+\dfrac{p}{\rho}$

Bernoulli's equation



Daniel Bernoulli
(1700-1782)

conservation of energy for stationary flow in a stream tube:

- stationary flow
- no mechanical work
- no viscous friction

$$\left\{\begin{array}{l} \partial/\partial t = 0 \\ \dot{W}_{\text{shaft}} = 0 \\ \dot{W}_{\text{visc}} = 0 \end{array}\right.$$

and:

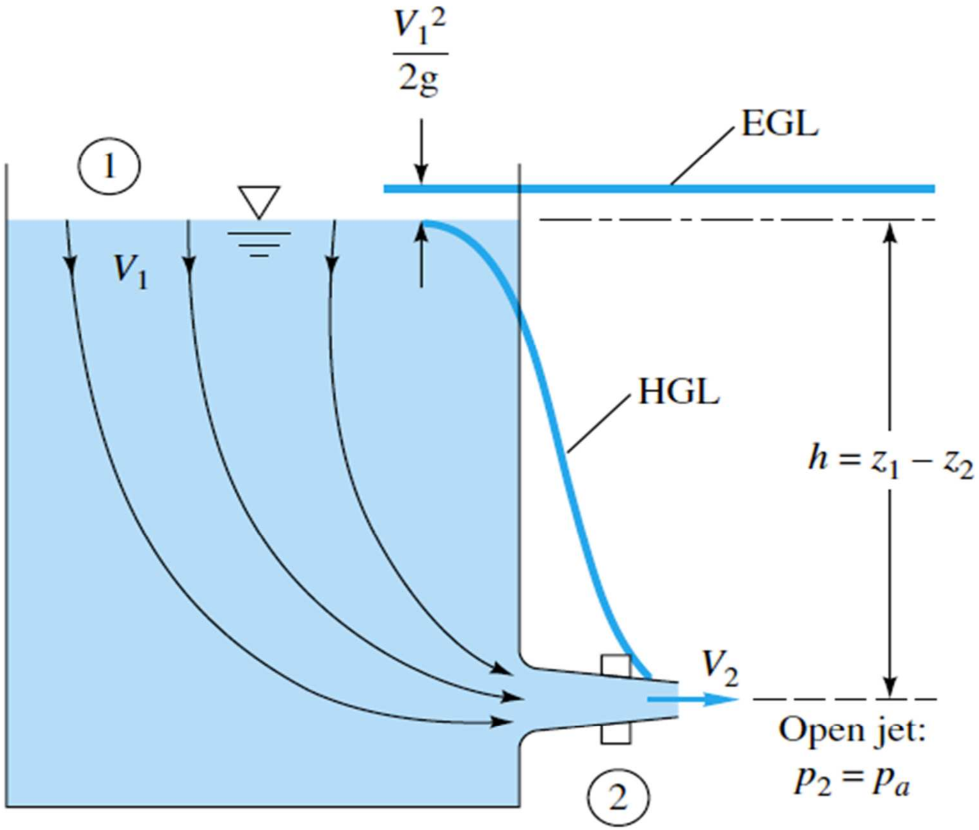
- incompressible fluid
- motion along streamline
- no heat transfer

$$\left\{\begin{array}{c} \rho = \text{constant} \\ \hat{u}_1 = \hat{u}_2 \\ \dot{q} = 0 \end{array}\right.$$

$$\underbrace{\frac{p_1}{\rho} + \tfrac{1}{2}V_1^2 + gz_1}_{\text{total pressure}} = \frac{p_2}{\rho} + \tfrac{1}{2}V_2^2 + gz_2 = \text{constant}$$

$$p_t = p + \tfrac{1}{2}\rho V^2 + \rho g z$$

Example 3.21



[2]

conservation of mass: $A_1 V_1 = A_2 V_2$

Bernoulli's equation:

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + g z_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + g z_2$$

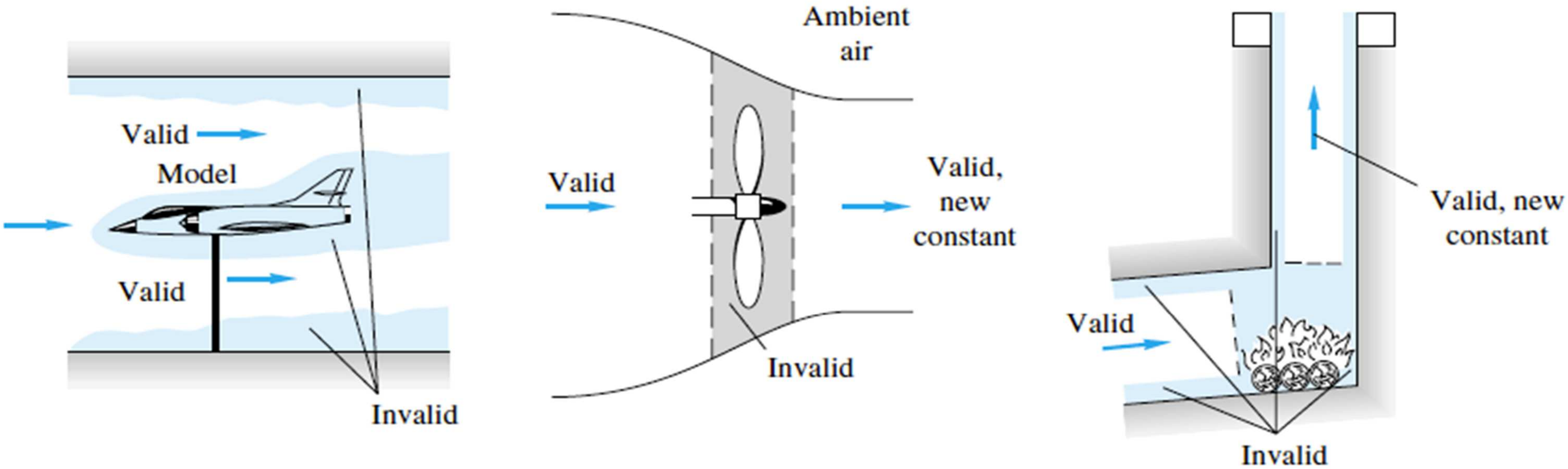
$$p_1 = p_2 = p_{atm}$$

$$V_2^2 - V_1^2 = 2g(z_1 - z_2) = 2gh$$

$$V_2^2 = \frac{2gh}{1 - A_2^2 / A_1^2}$$

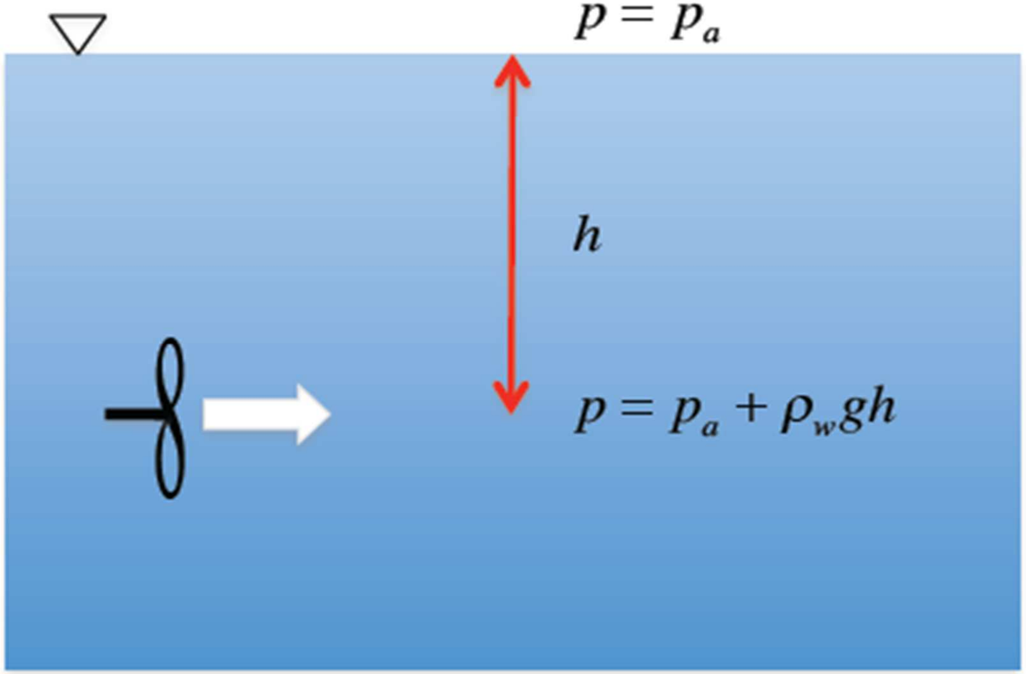
$$V_2 \cong \sqrt{2gh}$$

Validity of Bernoulli equation



[2]

Cavitation



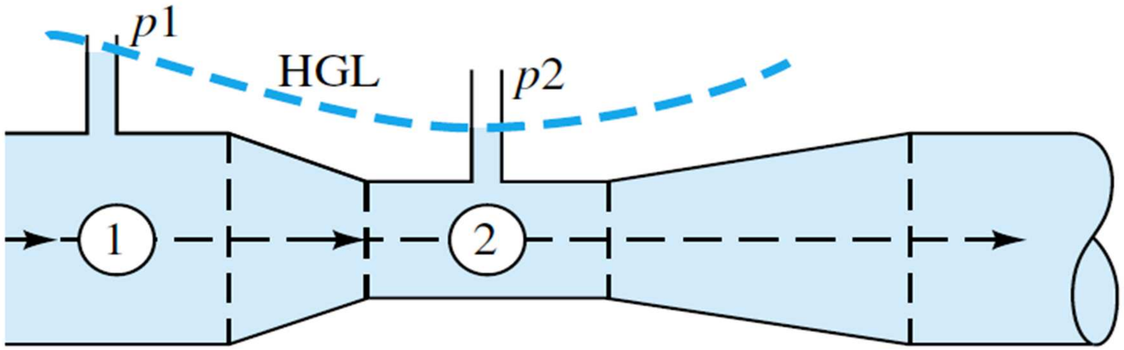
[2]

$$p_a = p - \rho_w g h + \tfrac{1}{2}\rho V^2$$

$$V = \sqrt{2\frac{p_a}{\rho_w} + 2gh}$$

$$\left.\begin{aligned} p_a &= 10^5 \text{ Pa} \\ \rho_w &= 10^3 \text{ kg/m}^3 \\ g &= 10 \text{ m/s}^2 \\ h &= 5 \text{ m} \end{aligned}\right\} \quad V = 17 \text{ m/s} \quad (62 \text{ km/h})$$

Example 3.23

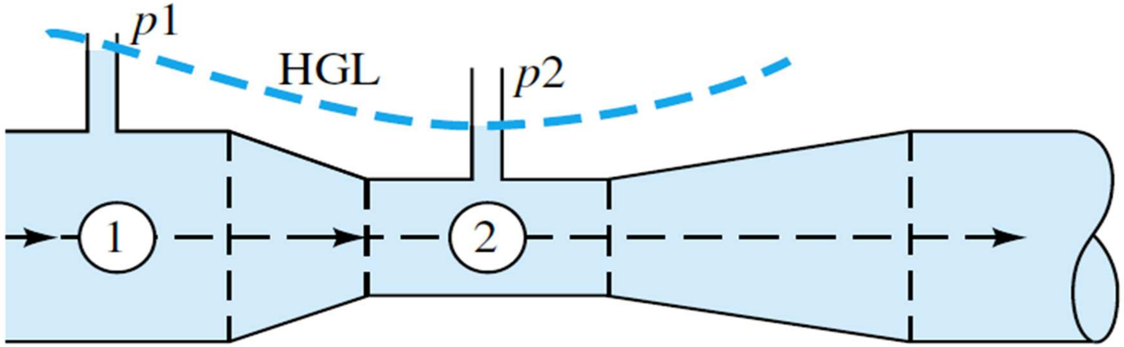


[2]

A constriction in a pipe will cause the velocity to rise and the pressure to fall at section 2 in the throat. The pressure difference is a measure of the flow rate through the pipe.

The smoothly necked-down system shown is called a venturi tube.

Find an expression for the mass flux in the tube as a function of the pressure change



along the centerline:

$$\frac{p_1}{\rho} + \tfrac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \tfrac{1}{2}V_2^2 + gz_2$$

$z_1 = z_2$ (horizontal tube)

Solve for V_2 :

$$V_2^2 - V_1^2 = \frac{2\Delta p}{\rho} \quad \text{with } \Delta p = p_1 - p_2$$

continuity equation:

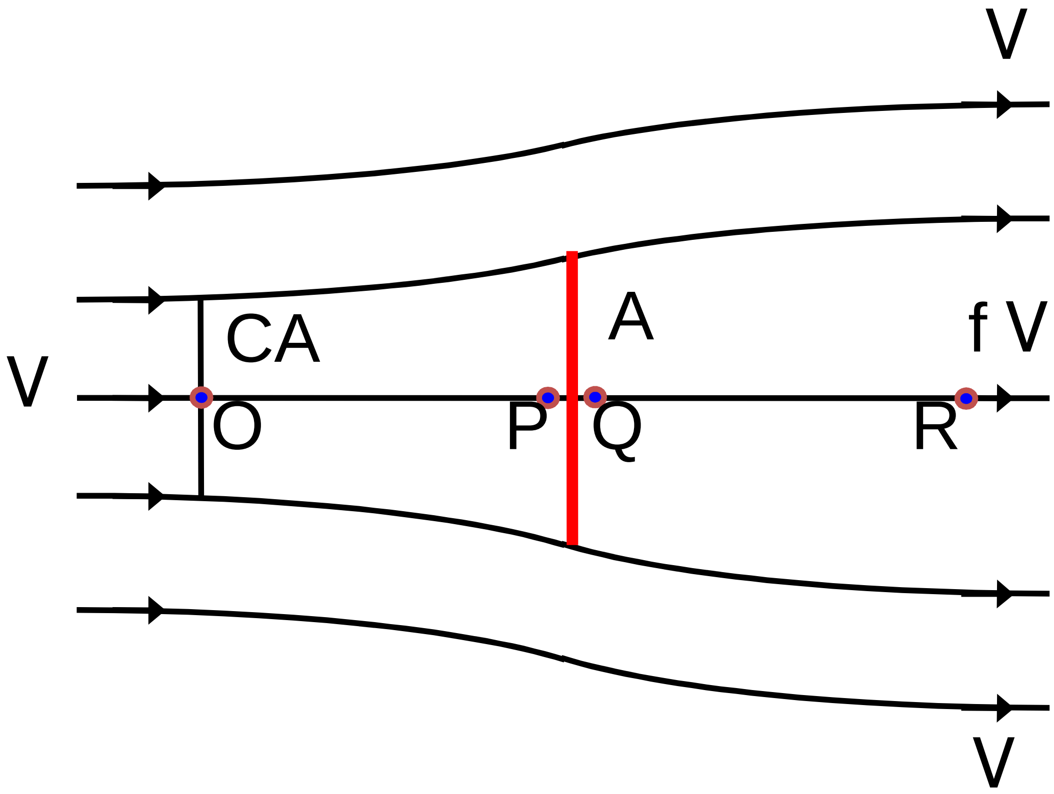
$$A_1V_1 = A_2V_2 \;\Rightarrow\; V_1 = \beta^2 V_2 \qquad \beta = \frac{D_2}{D_1}$$

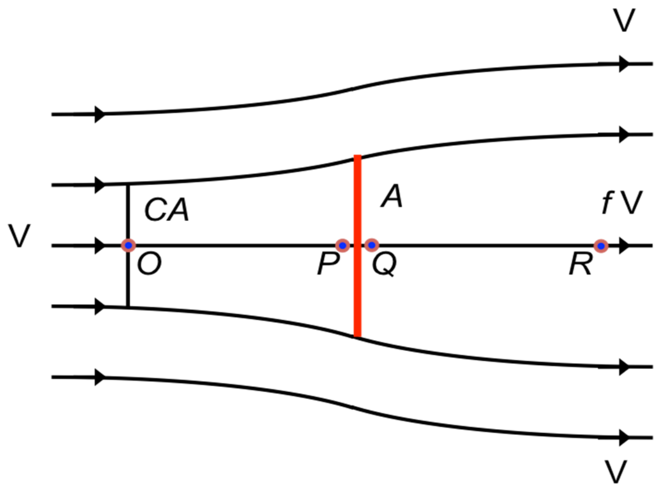
$$V_2 = \left[\frac{2\Delta p}{\rho\left(1-\beta^4\right)}\right]^{1/2} \;\Rightarrow\; \dot{m} = \rho A_2 V_2 = A_2\left(\frac{2\rho\,\Delta p}{1-\beta^4}\right)^{1/2}$$

Wind turbine

[3]

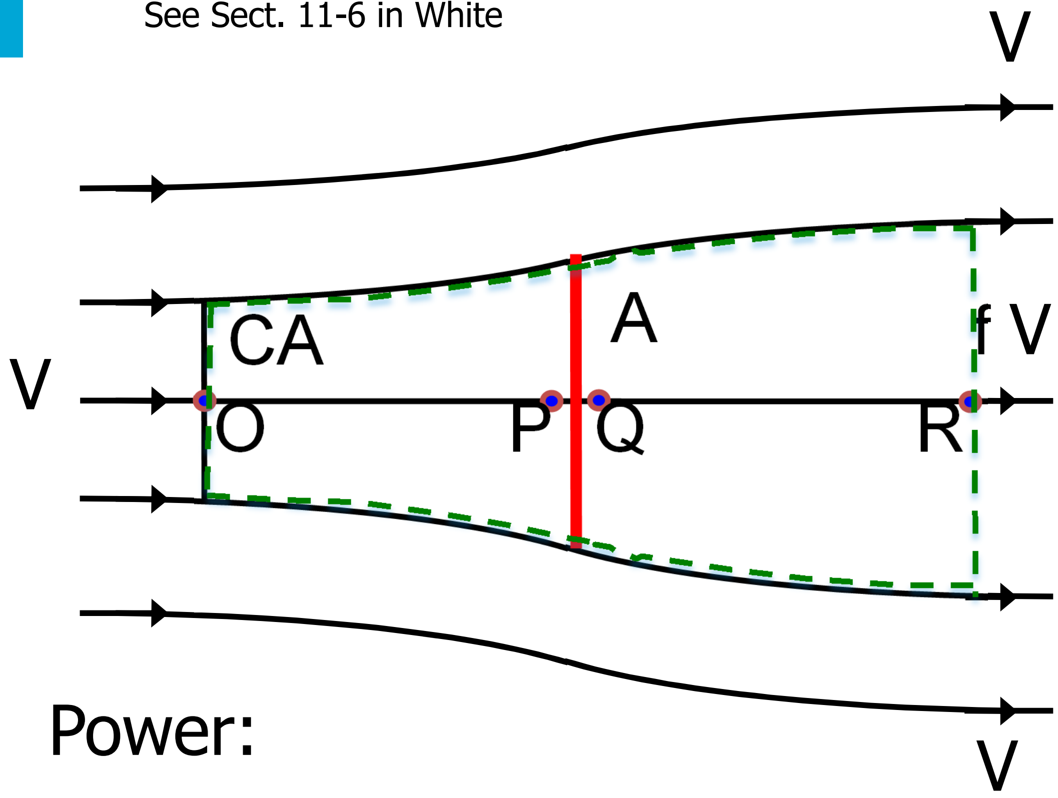
Wind turbine




V
CA
A
f V
V
O
P
Q
R
V

Wind turbine

See Sect. 11-6 in White



$f < 1 \quad V_R = fV$

area at R: $\quad CA/f$

thrust $(F = \Delta p\, A)$:

$(p_P - p_Q)A = \frac{1}{2}\rho V^2 A(1 - f^2)$

momentum change $(F = \dot{m}V)$:

$\rho V^2 CA(1 - f)$

balance of forces:

$C = \frac{1}{2}(1 + f)$

Power:

$$\dot{W} = FV_P = \frac{1}{4}\rho V^3 A(1 + f)(1 - f^2) \Rightarrow \boxed{\dot{W}_{\max} = \frac{8\rho A V^3}{27}} \quad f = 1/3$$

Summary

- Chapter 3: 3.6, 3.7
- Examples: 3.20, 3.21, 3.23, 3.24
- Problems: (see BlackBoard)

- Study guideline:
  - practice …
  - practice …
  - practice …

Source

1. Snapping shrimp, *http://youtu.be/ONQlTMUYCW4*
2. Frank M. White, *Fluid Mechanics*, McGraw-Hill Series in Mechanical Engineering
3. Wind turbines of Horns Rev wind farm, Denmark, photo courtesy of Vattenfall