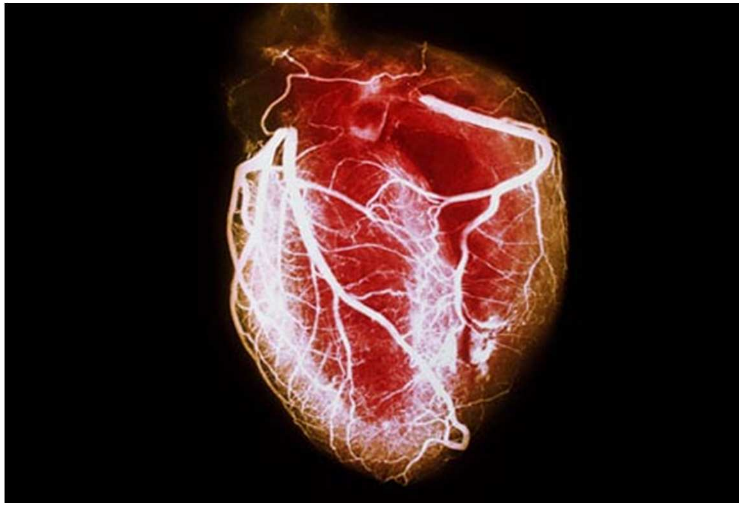
Fluid mechanics (wb1225)

Lecture 8: flows through pipes and ducts

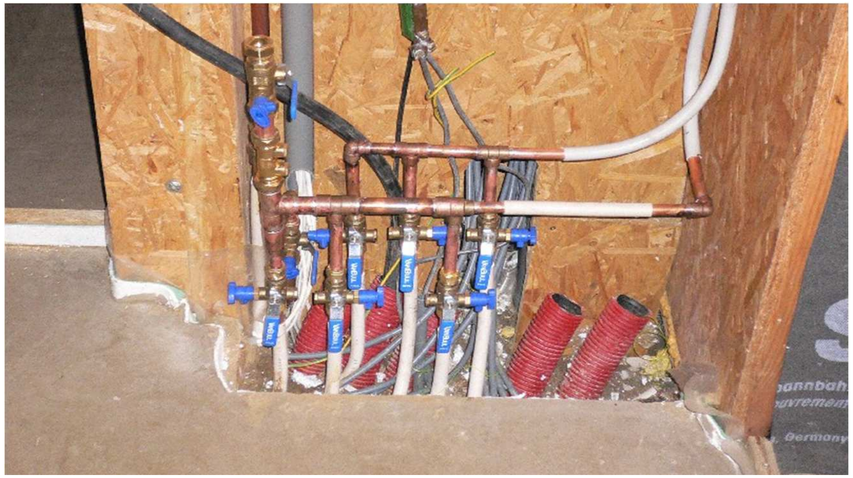
Examples of pipe flows

[1]

[2]

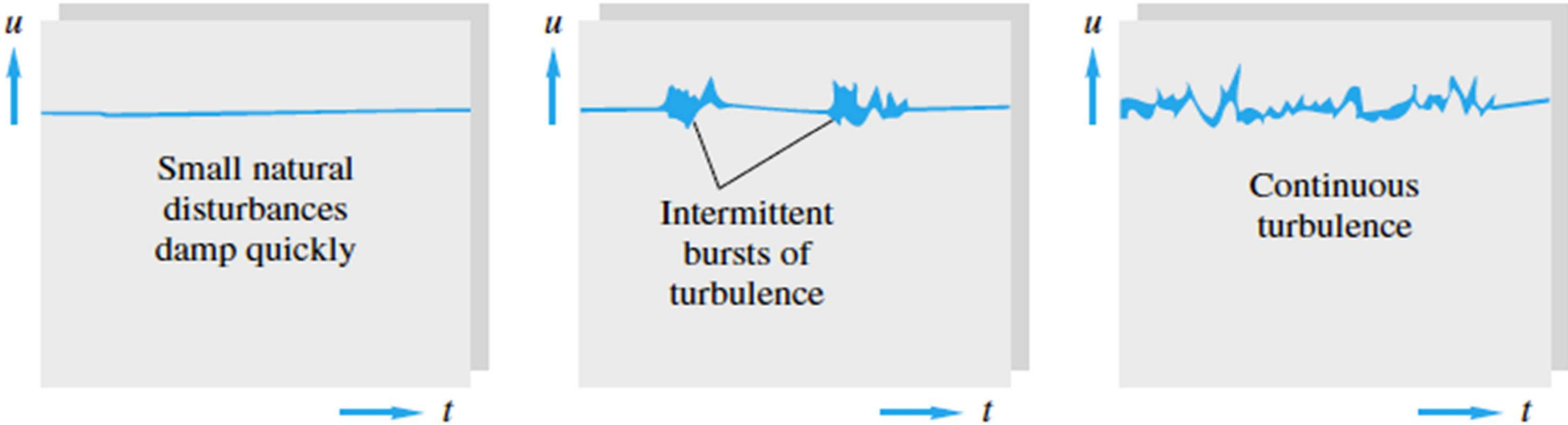


[3]



[4]

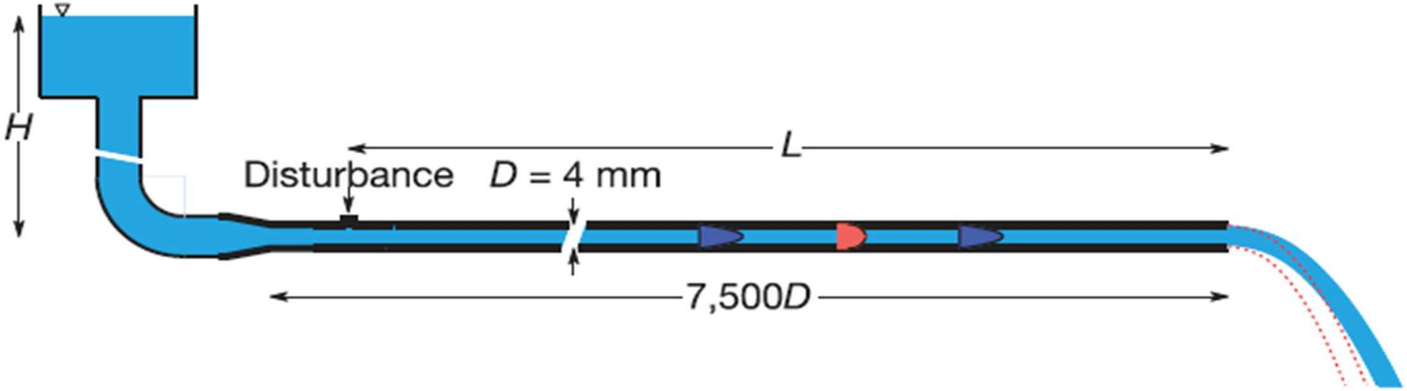
Flow states in pipe flow



[5]

$$\mathrm{Re}_{\mathrm{crit}} \approx 2300$$

Laminar vs turbulent



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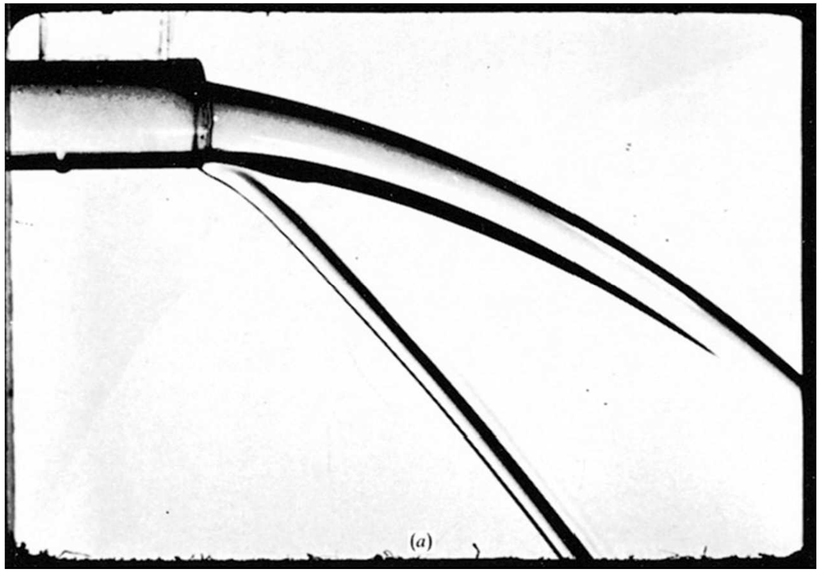

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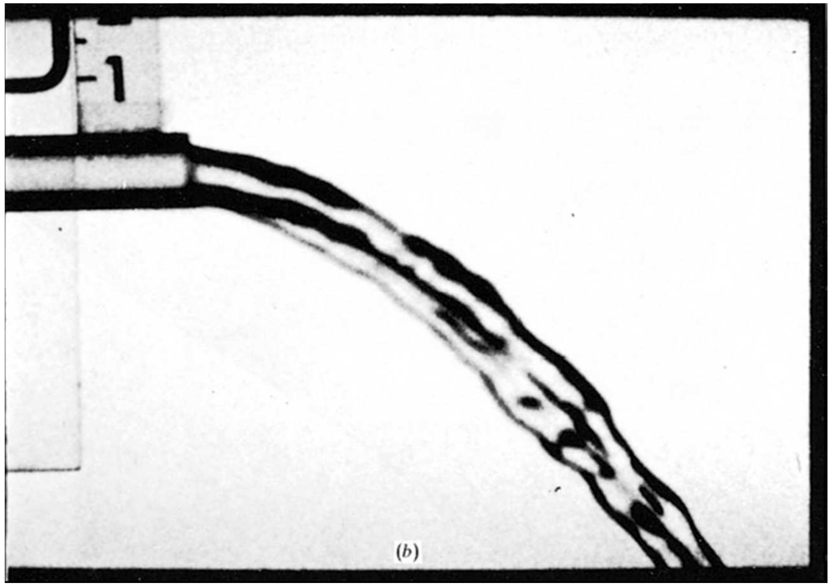
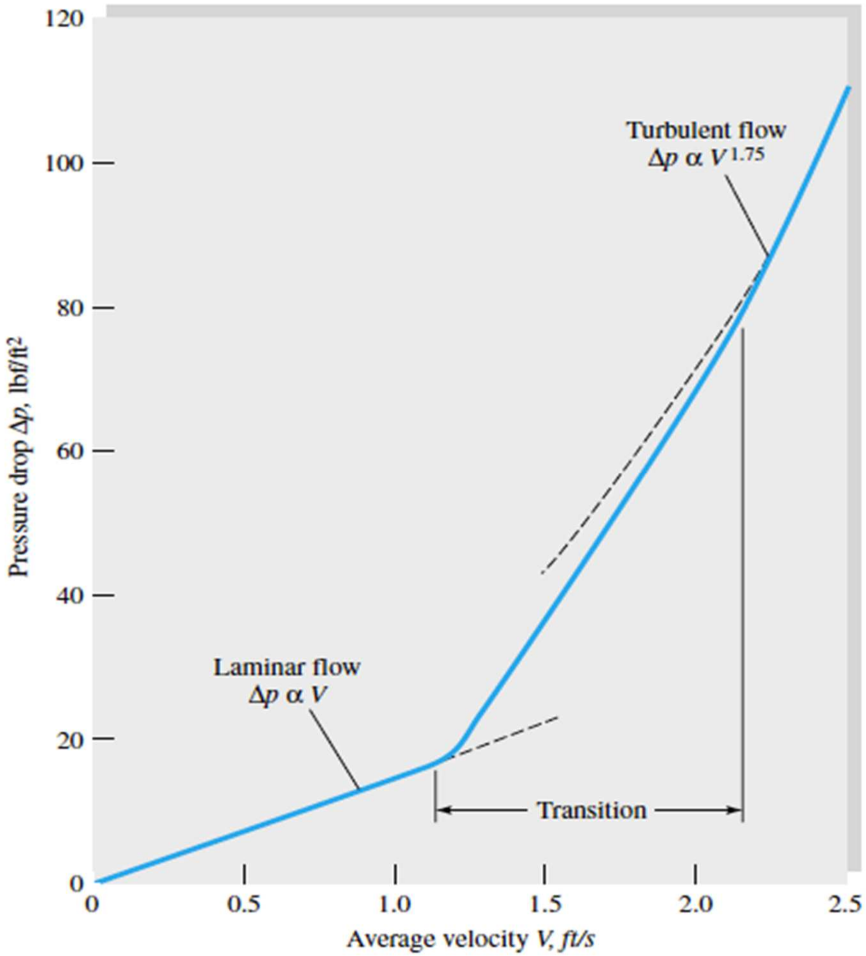
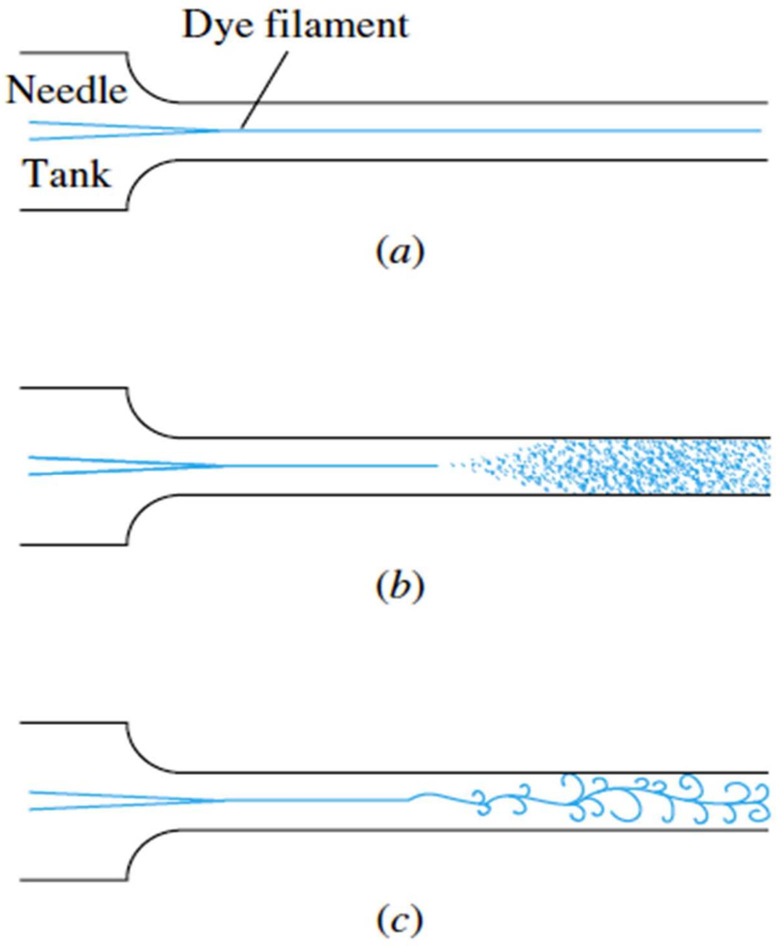


Pipe flow transition





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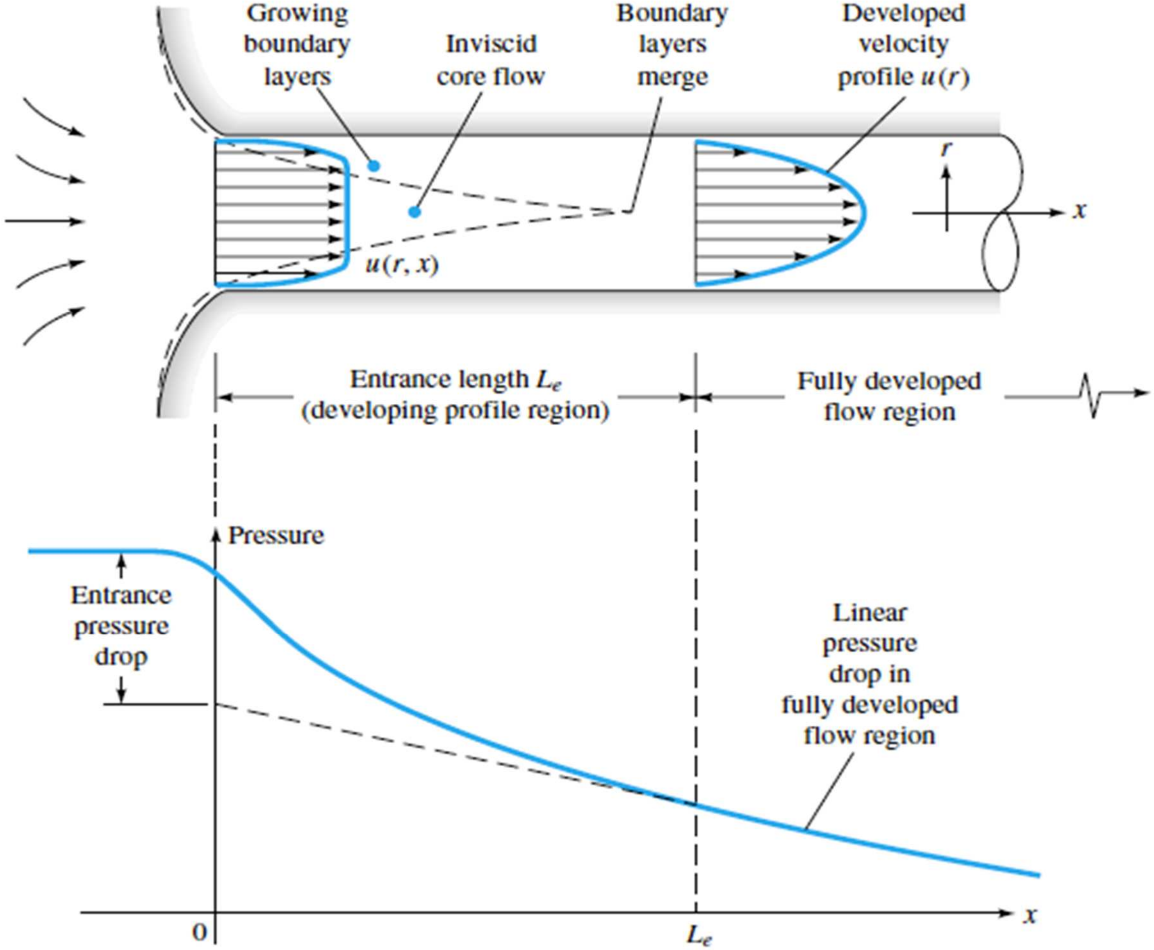
Pipe entrance length



[5]

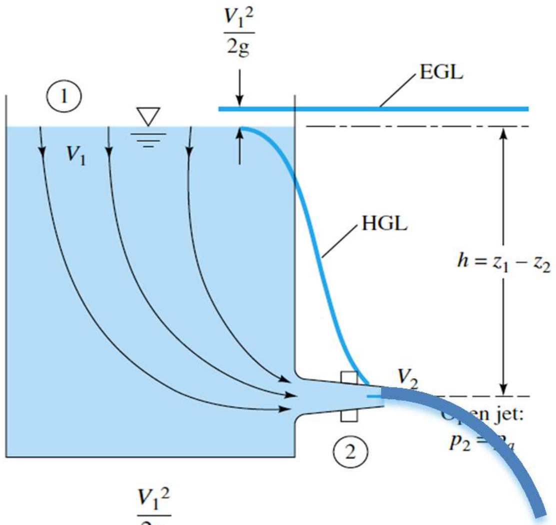
laminar flow:

$$\frac{L_e}{d} \approx 0.06\,\mathrm{Re}$$

turbulent flow:

$$\frac{L_e}{d} \approx 4.4\,\mathrm{Re}^{1/6}$$

Flow with energy losses



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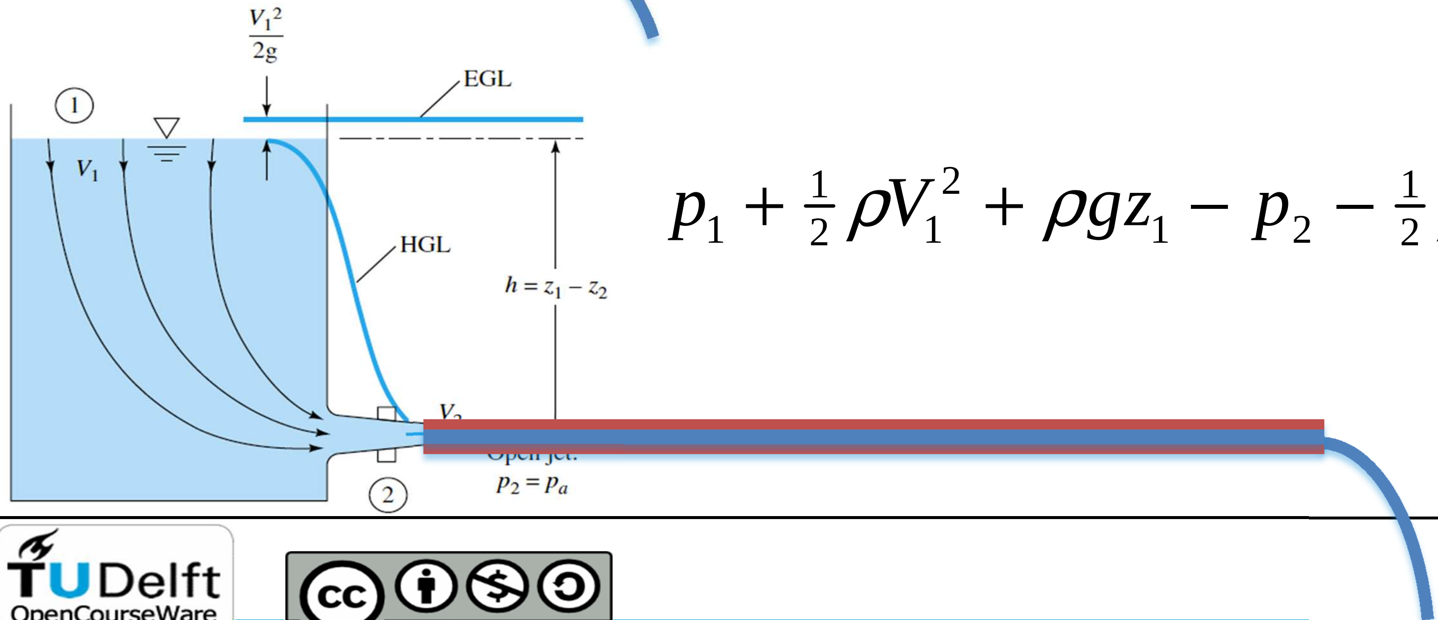
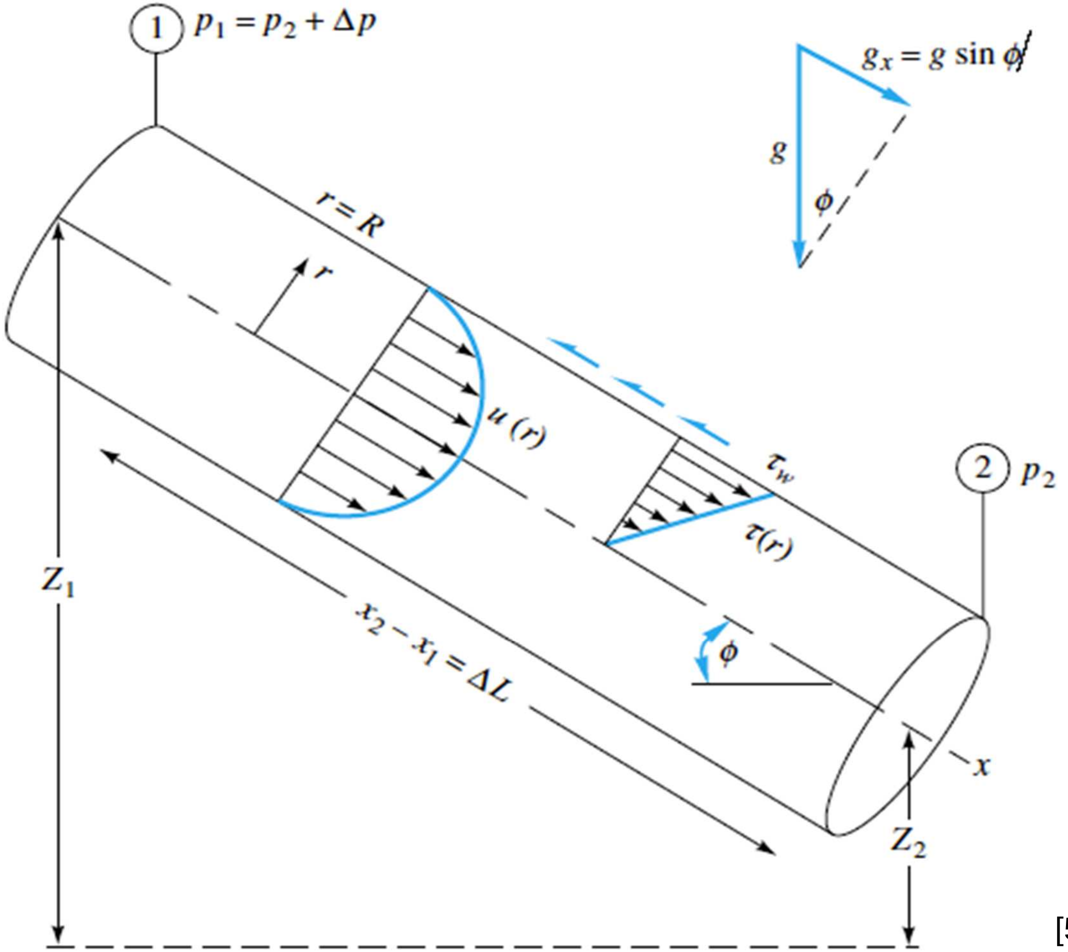
$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g z_1 - p_2 - \tfrac{1}{2}\rho V_2^2 - \rho g z_2 = 0$$



$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g z_1 - p_2 - \tfrac{1}{2}\rho V_2^2 - \rho g z_2 = \Delta p_{\mathrm{loss}}$$

Flow in a circular pipe



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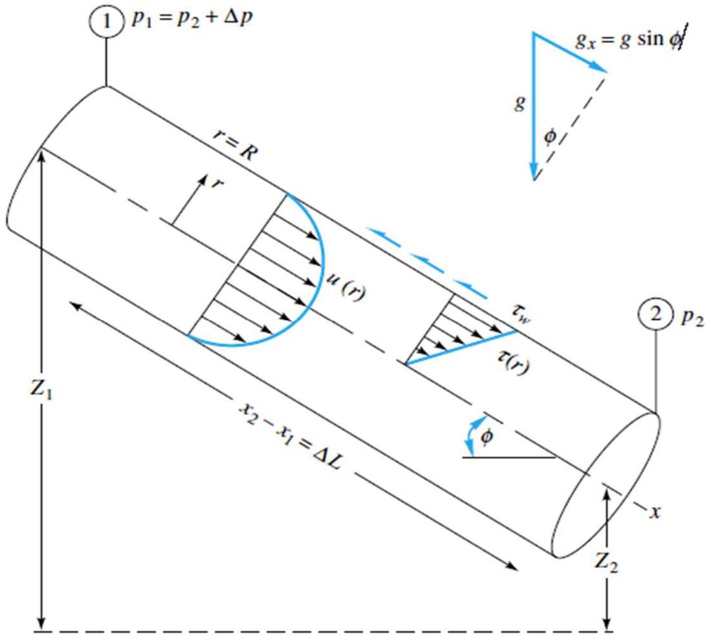
Control volume analysis



[5]

continuity: $Q_1 = Q_2 \quad \Rightarrow \quad V_1 = V_2$

steady-flow energy equation:

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \rho g z_2 + \rho g h_f$$

$$h_f = \left(z_1 + \frac{p_1}{\rho g}\right) - \left(z_2 + \frac{p_2}{\rho g}\right) = \Delta\left(z + \frac{p}{\rho g}\right) \quad \Rightarrow$$

$$h_f = \Delta z + \frac{\Delta p}{\rho g}$$

momentum equation:

$$\Delta p\, \pi R^2 + \rho g \left(\pi R^2\right)\Delta L \sin\phi - \tau_w (2\pi R)\Delta L = \dot{m}\left(V_2 - V_1\right) = 0$$

$$\Delta z + \frac{\Delta p}{\rho g} = h_f = \frac{2\tau_w}{\rho g}\frac{\Delta L}{R} \qquad \text{with: } \Delta z = \Delta L \sin\phi$$

Dimensional analysis

$$h_f = \frac{2\tau_w}{\rho g}\frac{\Delta L}{R}, \quad \tau_w = F(\rho, V, \mu, d, \varepsilon), \quad \varepsilon = \text{wall roughness}$$

$$\frac{8\tau_w}{\rho V^2} = f = F\left(\mathrm{Re}, \frac{\varepsilon}{d}\right)$$

$$\rho g h_f = f\frac{L}{d}\tfrac{1}{2}\rho V^2,$$

f = Darcy friction factor

Equations of motion

continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(ru_r\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(u_\theta\right)+\frac{\partial u_x}{\partial x}=0 \;\Rightarrow\; \frac{1}{r}\frac{\partial}{\partial r}\left(ru_r\right)=0 \;\Rightarrow\; ru_r=\text{const}$$

$$\Rightarrow\; \text{BC: } u_r=0,\ r=R \;\Rightarrow\; u_r=0$$

momentum equation: $u=u_x(r)$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}=-\frac{dp}{dx}+\rho g_x+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau\right)$$

$$\Rightarrow\quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\tau\right)=\frac{d}{dx}\left(p-\rho g x\sin\phi\right)=\frac{d}{dx}\big(\underbrace{p-\rho g z}_{p'}\big)$$

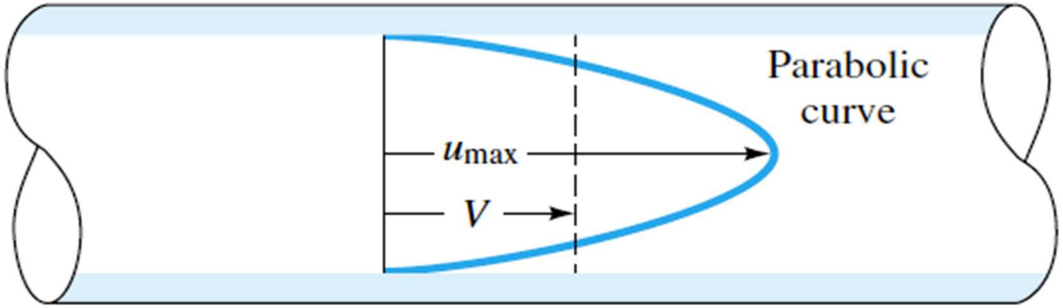
$$\Rightarrow\quad \tau(r)=\tfrac{1}{2}r\frac{dp'}{dx}$$

Laminar flow solution

$$\tau = \mu \frac{du}{dr} = \tfrac{1}{2} rK \qquad K = \frac{dp'}{dx}$$

$$\text{integrate: } u(r) = \tfrac{1}{4} r^2 \frac{K}{\mu} + C_1 \qquad \text{BC:}\, u = 0,\, r = R \quad \Rightarrow C_1 = -\frac{R^2 K}{4\mu}$$

$$u(r) = \frac{-K}{4\mu}\left(R^2 - r^2\right)$$



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$$u_{max} = \frac{-KR^2}{4\mu}$$

$$Q = \int_0^R u(r)\, 2\pi r\, dr \quad \Rightarrow \quad Q = \frac{-K\pi R^4}{8\mu} \qquad V = \frac{Q}{A} = \frac{Q}{\pi R^2} = \tfrac{1}{2} u_{max}$$

Laminar flow solution (cont'd)

$$Q = \frac{-K\pi R^4}{8\mu} \quad \Rightarrow \quad Q = \frac{\pi R^4}{8\mu}\frac{\Delta p}{L} \quad \Rightarrow \quad \Delta p = \frac{8\mu L Q}{\pi R^4}$$

$$\tau_w = \left|\mu \frac{du}{dr}\right|_{r=R} = \frac{2\mu u_{\max}}{R} = \tfrac{1}{2} R \frac{\Delta p}{L}$$

$$u(r) = u_{\max}\left(1 - r^2/R^2\right)$$

$$u_{\max} = -KR^2/4\mu = \Delta p R^2/4\mu L$$

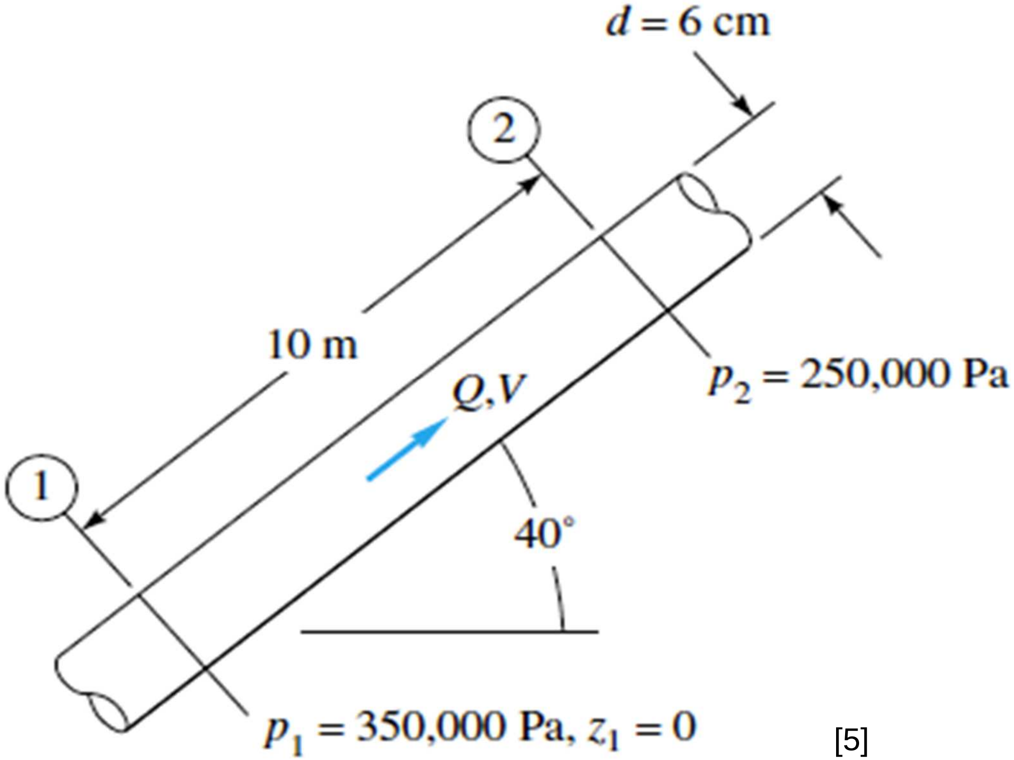
$$\Delta p = f \frac{L}{d} \tfrac{1}{2}\rho V^2$$

$$\Delta p = \frac{8\mu L Q}{\pi R^4} = \frac{8\mu L}{\pi R^4}\pi R^2 V = \frac{8\cdot 2\cdot 2\mu}{2R}\frac{L}{2R}\frac{\pi R^2}{\pi R^2}\frac{\tfrac{1}{2}\rho V^2}{\tfrac{1}{2}\rho V}$$

$$= \frac{64}{\rho V d/\mu}\frac{L}{d}\tfrac{1}{2}\rho V^2 \quad \Rightarrow \quad f = \frac{64}{\mathrm{Re}}$$

Example 6.4

An oil with ρ = 900 kg/m^3 and ν = 2x10^{-4} m^2/s flows upward through an inclined pipe as shown in the diagram. The pressure and elevation ar given at sections 1 and 2, 10 m apart.



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Assuming steady laminar flow:

a) verify that the flow is up;

b) compute h_f between 1 and 2;

c) compute Q, V, and Re_d.

d) Is the flow laminar?

Bore diameter

$$u = -\frac{K}{4\mu}\left(R^2 - r^2\right) \qquad Q = \int_0^a u(r)\,2\pi r\,dr = -\frac{K\pi}{8\mu}R^4 = \frac{\Delta P / L}{128\mu/\pi}D^4$$

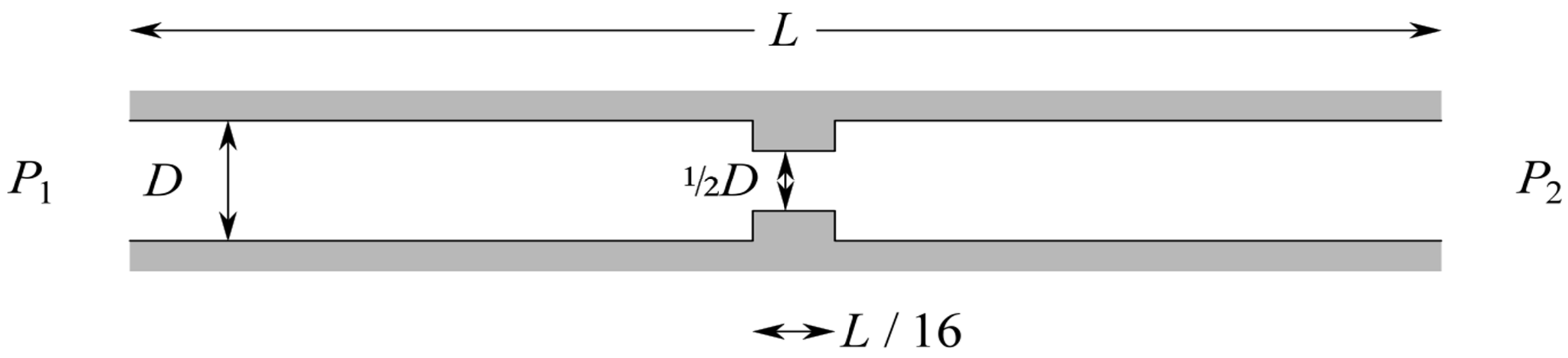
$$\Delta P = \frac{128}{\pi}\mu\frac{L}{D}\frac{Q}{D^3} = \frac{128}{\cancel{\pi}}\mu\frac{L}{D}\frac{\cancel{\pi}/4}{D^{\cancel{3}}}\cancel{D^2}U = \frac{128\times 2}{4}\frac{\nu}{UD}\frac{L}{D}\tfrac{1}{2}\rho U^2$$

$$= \frac{64}{\mathrm{Re}}\frac{L}{D}\tfrac{1}{2}\rho U^2$$

$$Q = \frac{\Delta P / L}{128\mu/\pi}D^4 \quad \Rightarrow \begin{cases} \delta Q \sim 0.1\% \\ \delta P \sim 0.1\% \end{cases} \quad \Rightarrow \frac{\delta D}{D} \cong \frac{1}{4}\left(\frac{\delta Q}{Q} + \frac{\delta P}{P}\right) \cong 0.05\%$$

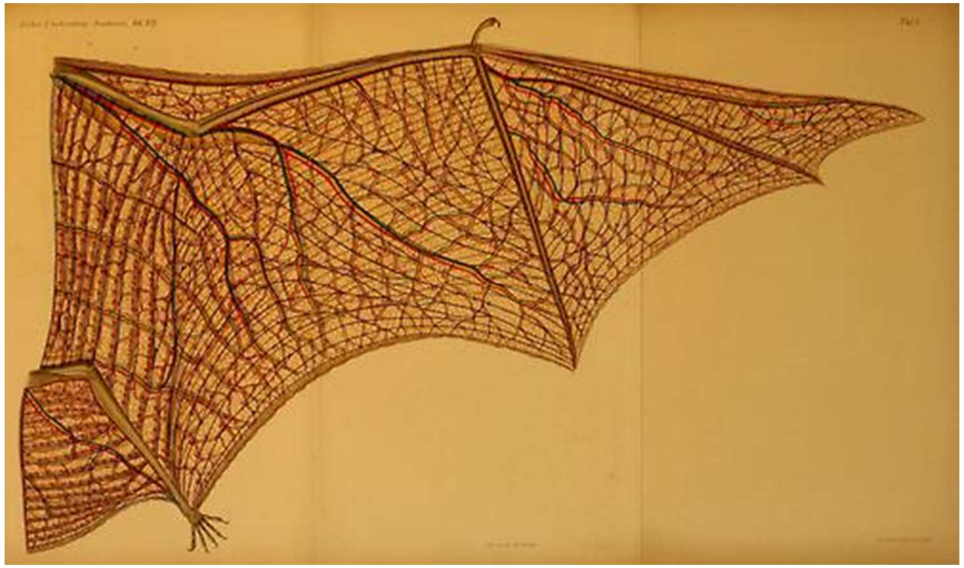
$$D = 150\ \mu\mathrm{m} \quad \Rightarrow \quad \delta D \cong 75\ \mathrm{nm}$$

Stenosis



[5]

Vascular networks



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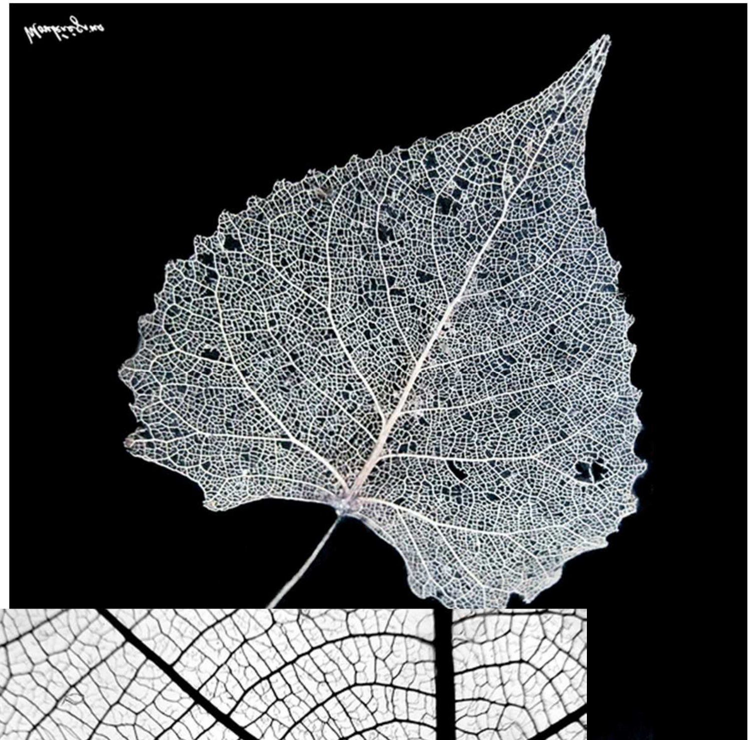
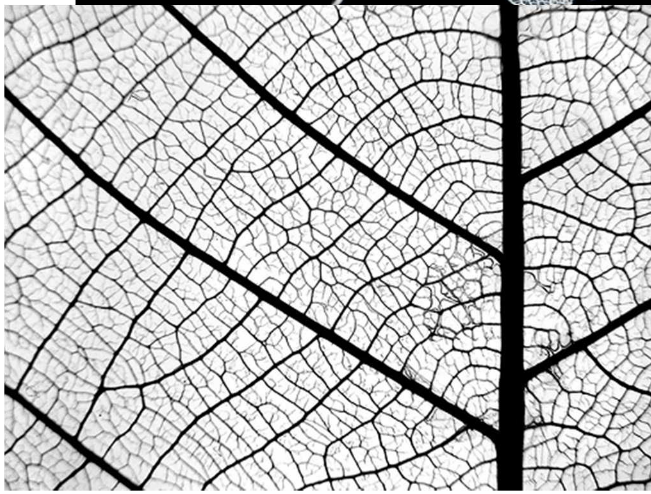
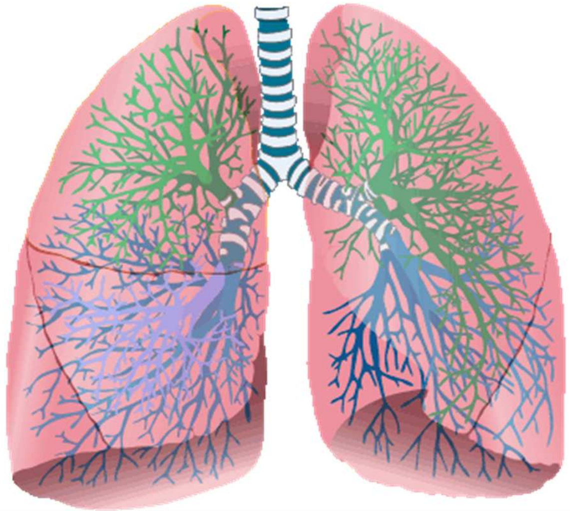


[9]

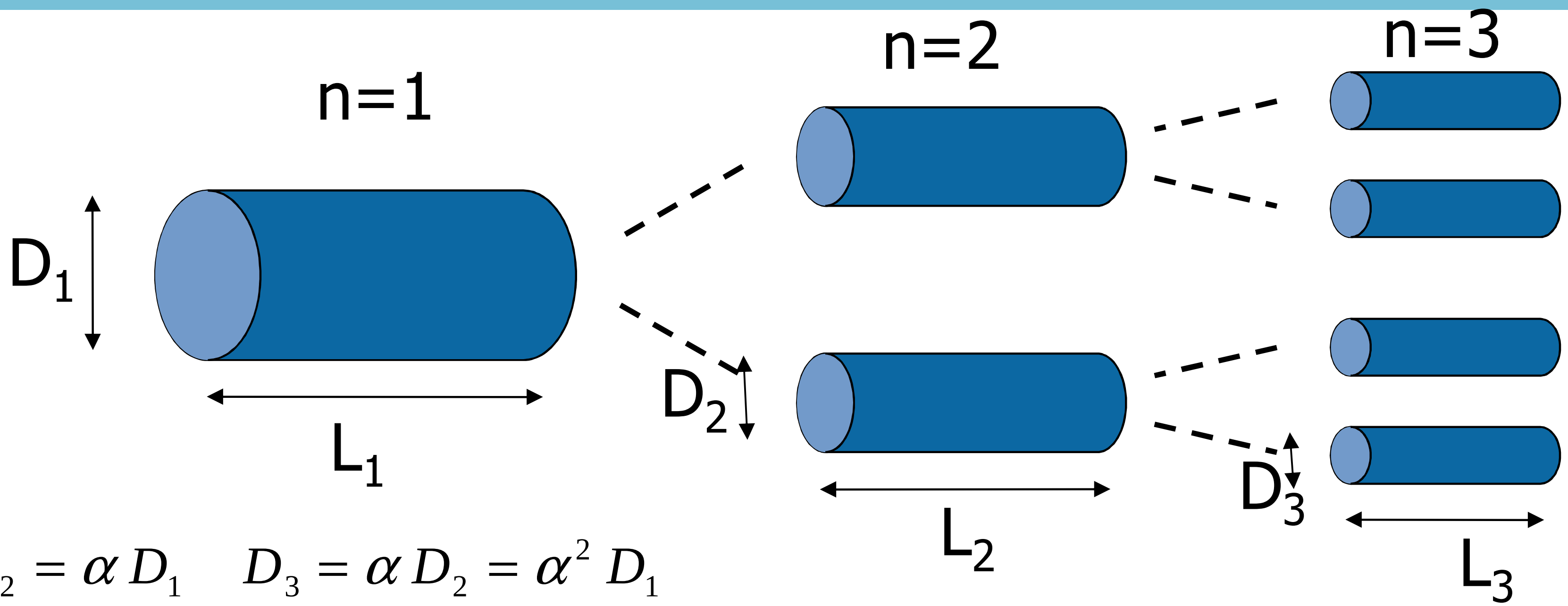


[10]



[8]

Tube networks



$$D_2 = \alpha D_1 \qquad D_3 = \alpha D_2 = \alpha^2 D_1$$

$$L_2 = \beta L_1 \qquad L_3 = \beta L_2 = \beta^2 L_1$$

$$Q = Q_1 = 2\,Q_2 = 4\,Q_3 = \cdots$$

$$\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3 + \cdots$$

$$\Delta P = \frac{128\eta Q_1 L_1}{\pi D_1^4} + \frac{128\eta Q_2 L_2}{\pi D_2^4} + \frac{128\eta Q_3 L_3}{\pi D_3^4} + \ldots$$

$$= \frac{128\eta}{\pi}\left(\frac{Q_1 L_1}{D_1^4} + \frac{Q_2 L_2}{D_2^4} + \frac{Q_3 L_3}{D_3^4} + \ldots \right)$$

$$= \frac{128\eta Q_1}{\pi}\left(\frac{L_1}{D_1^4} + \tfrac{1}{2}\frac{L_2}{D_2^4} + \tfrac{1}{4}\frac{L_3}{D_3^4} + \ldots \right)$$

$$= \frac{128\eta Q_1 L_1}{\pi D_1^4}\left(1 + \tfrac{1}{2}\frac{L_2/L_1}{D_2^4/D_1^4} + \tfrac{1}{4}\frac{L_3/L_1}{D_3^4/D_1^4} + \ldots \right)$$

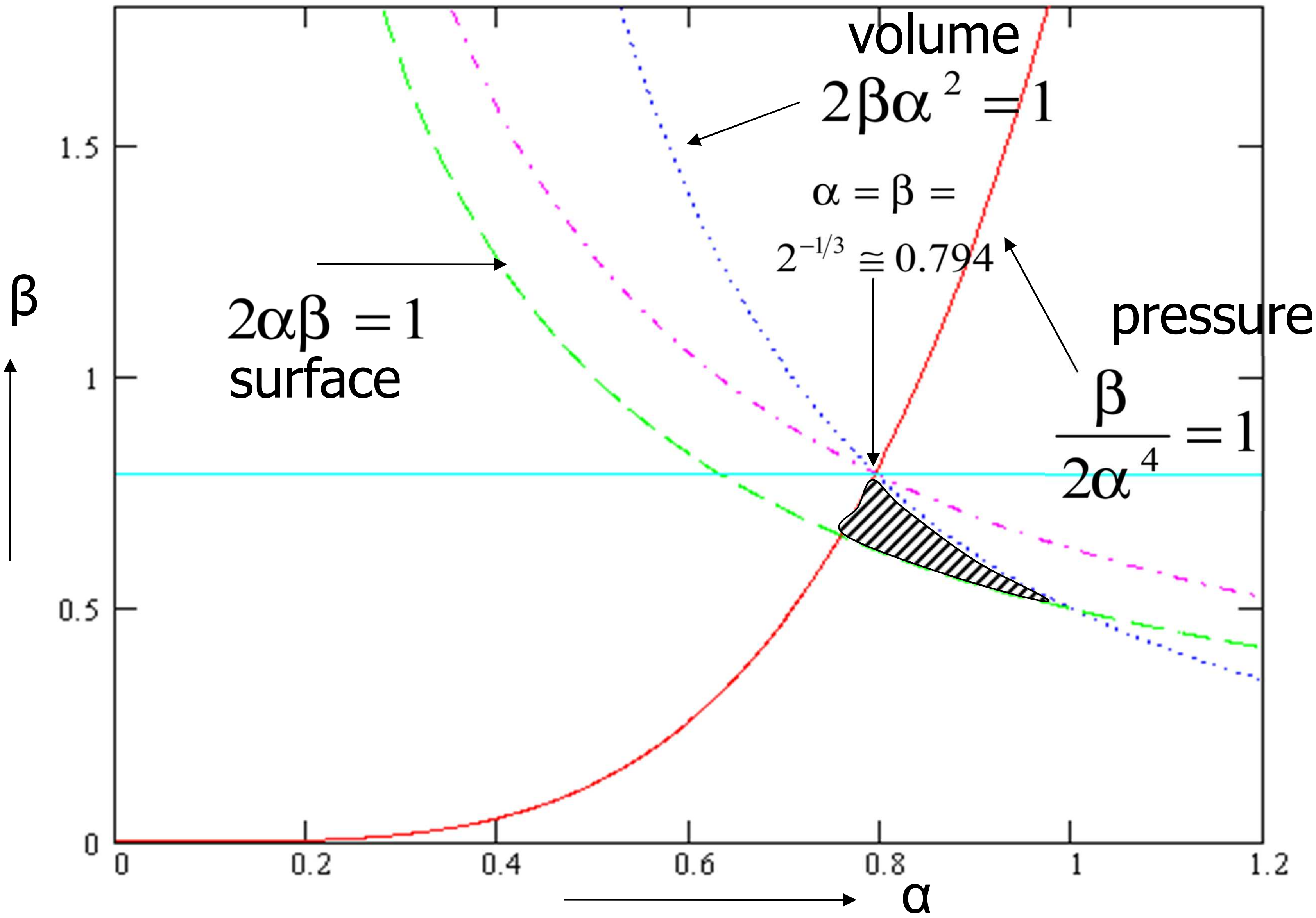
$$= \frac{128\eta Q_1 L_1}{\pi D_1^4}\left(1 + \tfrac{1}{2}\frac{\beta}{\alpha^4} + \tfrac{1}{4}\frac{\beta^2}{\alpha^8} + \ldots \right)$$

$$= \Delta P_1 \cdot \left[1 + \tfrac{1}{2}\frac{\beta}{\alpha^4} + \left(\tfrac{1}{2}\frac{\beta}{\alpha^4}\right)^2 + \ldots \right] = \Delta P_1 \cdot \frac{1}{1 - \tfrac{1}{2}\frac{\beta}{\alpha^4}} \quad \Rightarrow \quad \tfrac{1}{2}\frac{\beta}{\alpha^4} < 1$$

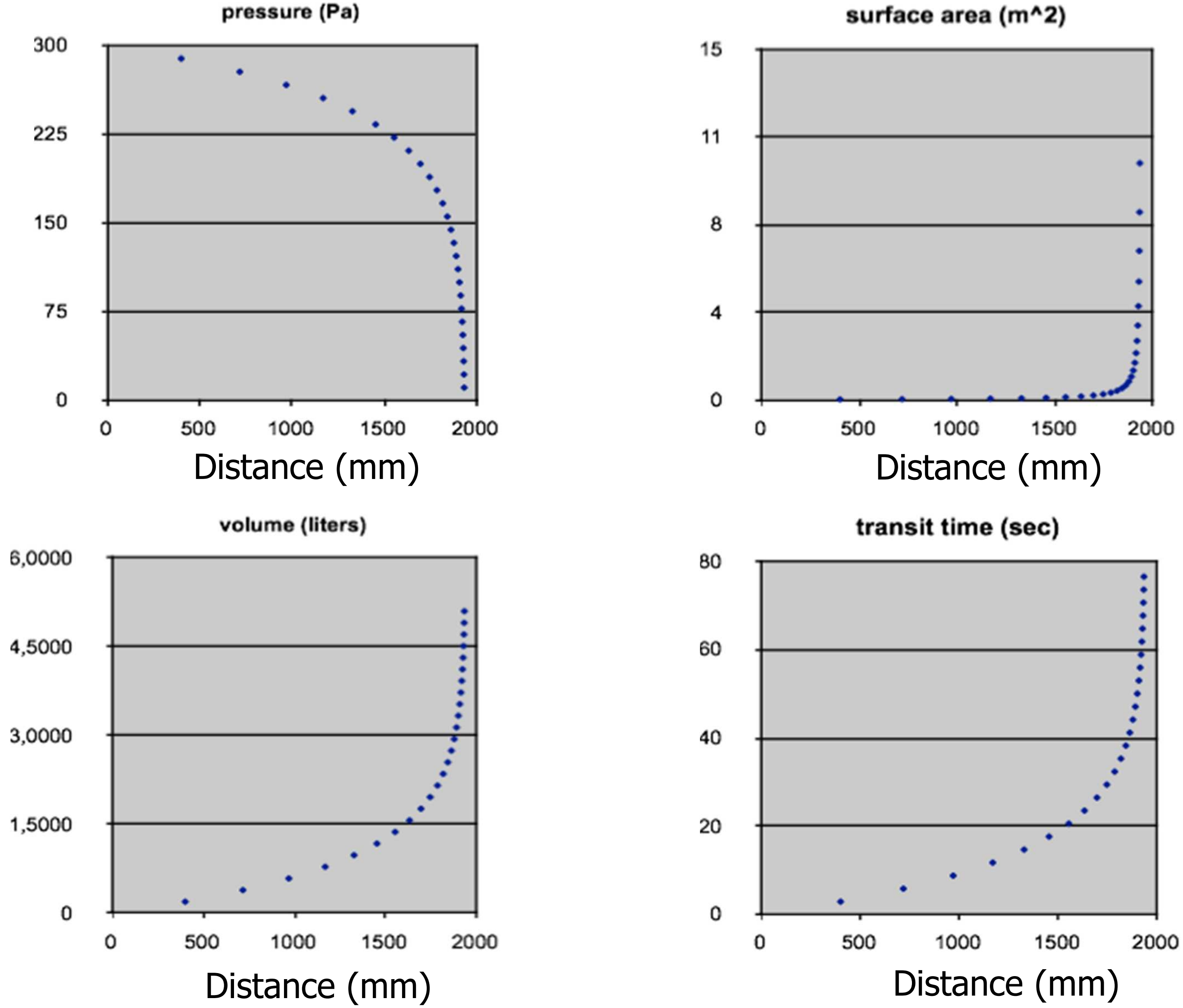
$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = ?$$

$$\sum_{n=0}^{N} a^n = \frac{1 - a^{N+1}}{1 - a}$$

Optimal Value



Vascular network



[5]

Summary

- Chapter 6: 6.1-6.2, 6.4
- Examples: 6.4-6.5
- Problems: see BlackBoard

Source

1. Trans Alaska Oil Pipeline, *http://www.bloomberg.com,* photo courtesy of Daniel Acker/Bloomberg
2. Suncor refinery, Montreal, *http://www.flickr.com/photos/manualcrank/4467585510/*, photo courtesy of manuel crank
3. Heart Disease, *http://bruisedfruits.net/3178/heart-disease.html*
4. Waterleiding verdeler (waterworks distributor), *http://www.hetkapottehuis.nl/album2/2005/2005-10-05/slides/Waterleiding%20verdeler.html*, photo courtesy of Het Kapotte Huis
5. Frank M. White, *Fluid Mechanics*, McGraw-Hill Series in Mechanical Engineering
6. Multimedia Fluid Mechanics DVD-ROM, G. M. Homsy, University of California, Santa Barbara
7. The vascular system of a bat wing, *http://archive.org/stream/archivfrmikros07berl#page/n400/mode/1up,* photo courtesy of Internet Archive
8. Lungs, *http://commons.wikimedia.org/wiki/File:Lungs_(animated).gif*, photo courtesy of Mikael Häggström
9. Poplar leaf - Feuille de tremble, *http://www.flickr.com/photos/monteregina/4998677623/,* photo courtesy of monteregina
10. Structure in leaf, *http://dsdn104robchesney.blogspot.nl/2011/05/precedent-images.html,* photo courtesy of Rob Chesney