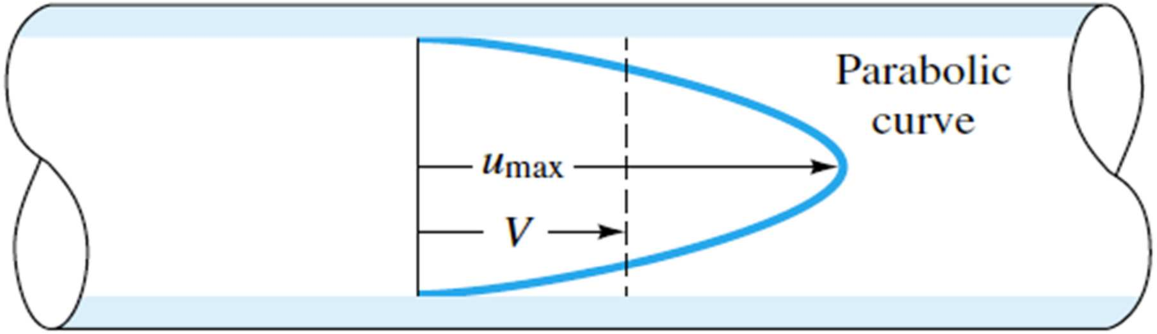
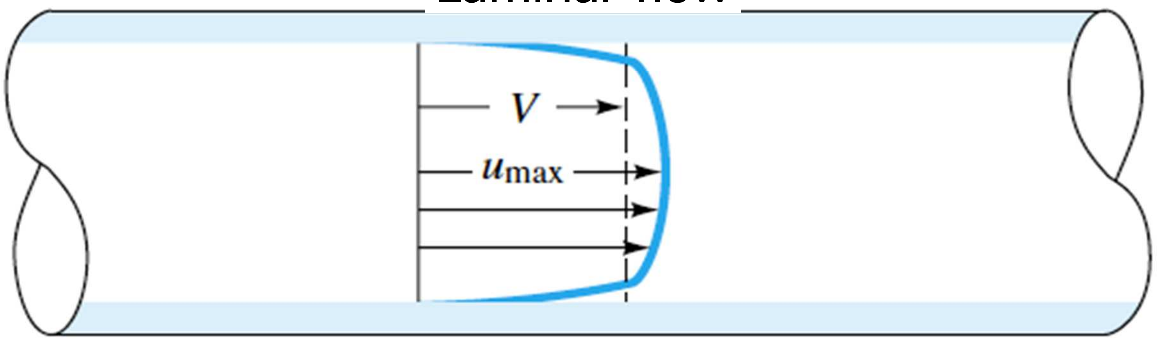
Fluid mechanics
(wb1225)

Lecture 9:
flow losses

Laminar vs turbulent pipe flow



(a)
Laminar flow



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Turbulent flow

Friction factor for turbulent flow

$$u(r) = u_0 \left(1 - \frac{r}{R}\right)^{1/n}$$

Re =	4x10³	2x10⁴	1x10⁵	1x10⁶	2x10⁶	3x10⁶
n =	6.0	6.6	7.0	8.8	10	10

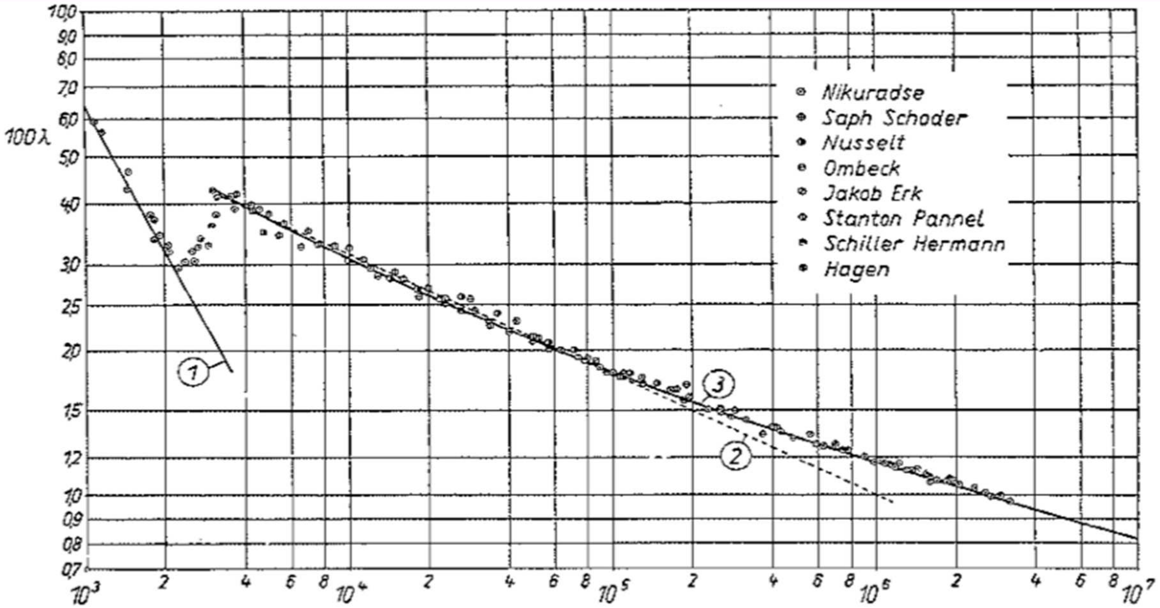
$$\frac{V}{u_0} = \frac{2n^2}{(2n+1)(n+1)} = \frac{49}{60} = 0.817 \quad \text{for } n = 7$$

$$f = \frac{8\tau_w}{\rho V^2} \Rightarrow \frac{f}{4} = \frac{\rho u_*^2}{\frac{1}{2}\rho V^2} = 2\left(\frac{u_*}{V}\right)^2 \quad \text{Fanning friction factor}$$

$$\frac{u(y)}{u_*} = C\left(\frac{y u_*}{\nu}\right)^{1/7} \Rightarrow \frac{u_0}{u_*} = C\left(\frac{R u_*}{\nu}\right)^{1/7} \Rightarrow \frac{V}{u_*} = C'\left(\frac{R u_*}{\nu}\right)^{1/7}$$

$$\left(\frac{V}{u_*}\right)^7 = C''\frac{R u_*}{\nu} \Rightarrow \frac{V^8}{u_*^8} = C''\frac{RV}{\nu} \Rightarrow 4f = 2\left(\frac{u_*}{V}\right)^2 = C \cdot \mathrm{Re}^{-1/4}$$

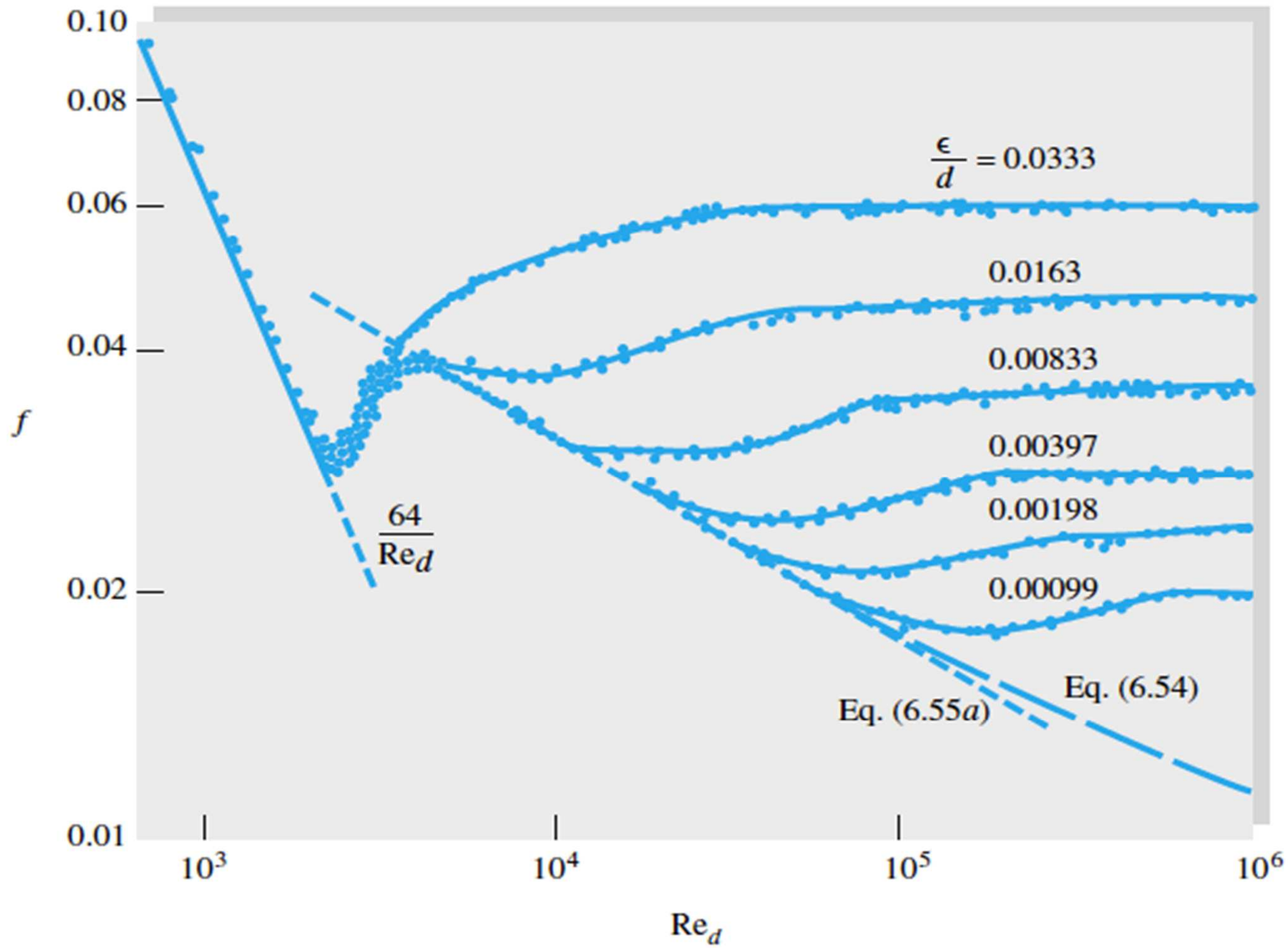
Moody diagram



$$f = \begin{cases} 64/\mathrm{Re} & \mathrm{Re} < 2300 & \text{Hagen-Poiseuille} \\ 0.316\,\mathrm{Re}^{-1/4} & 4000 < \mathrm{Re} < 10^5 & \text{Blasius} \\ \left[1.8\log\left(\mathrm{Re}/6.9\right)\right]^{-2} & \mathrm{Re} > 10^5 & \text{Colebrook} \end{cases}$$

$$\text{implicit:} \quad \frac{1}{\sqrt{f}} = 2.0\log\left(\mathrm{Re}\sqrt{f}\right) - 0.8 \quad \text{Prandtl}$$

Effect of roughness



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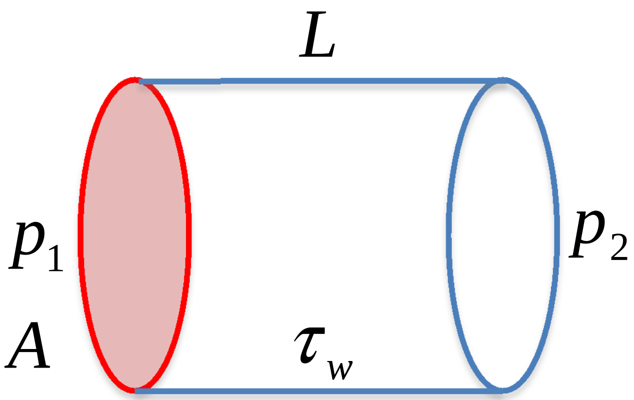
Hydraulic diameter



momentum balance:

$$\underbrace{\tfrac{1}{4}\pi D^2}_{\text{area}} \cdot \Delta p = \tau_w \cdot \underbrace{\pi D}_{\text{perim.}} \cdot L \quad \Rightarrow \quad \frac{\Delta p}{L} = \frac{4\tau_w}{D_h}$$

hydraulic diameter: $D_h = 4\,\dfrac{\text{surface area}}{\text{wetted perimeter}}$

flow between 2 parallel plates at distance $2h$:

$$f = \frac{24}{\mathrm{Re}_h} \quad \Rightarrow \quad f = \frac{96}{\mathrm{Re}_{D_h}} \qquad D_h = 4\,\frac{2hw}{4h + 2w} = 4h \quad \text{for } w \to \infty$$

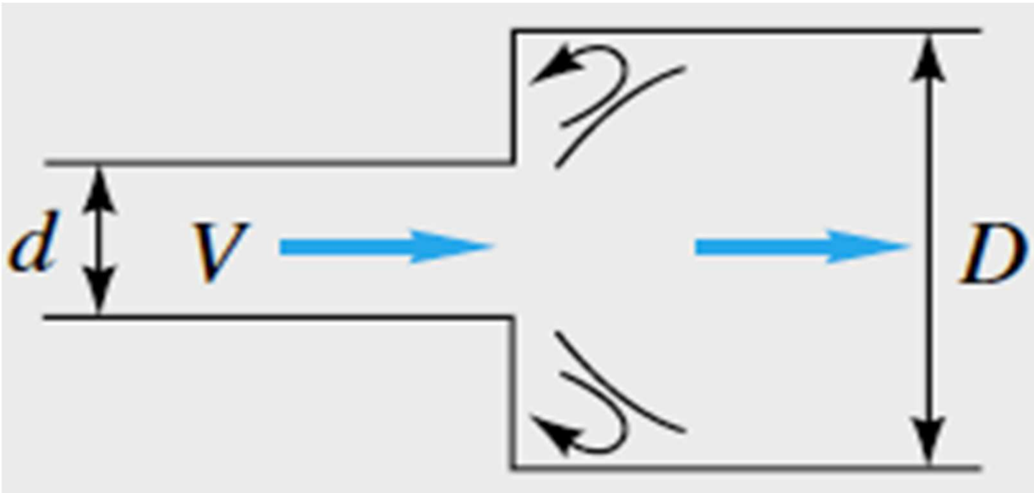
cf. pipe flow: $f = \dfrac{64}{\mathrm{Re}}$

Other losses in pipe systems

- entrance / exit
- sudden expansion / contraction
- bends / elbows / tees
- valves
- gradual expansion / contraction

$$K = \frac{\Delta P}{\frac{1}{2}\rho V^2} \quad \Rightarrow \quad \Delta P_{\text{tot}} = \tfrac{1}{2}\rho V^2 \left(f\frac{L}{D} + \sum_i K_i \right)$$

Sudden expansion



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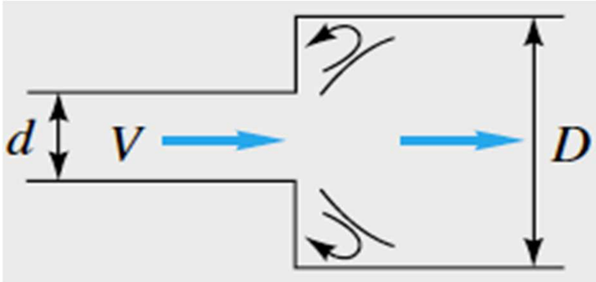
$$\Delta p = K \cdot \tfrac{1}{2} \rho V^2 \cdot S$$

Sudden expansion

conservation of mass: $S_1 V_1 = S_2 V_2 \quad \Rightarrow V_2 = \dfrac{S_1}{S_2} V_1$



momentum balance:

$$p_1 S_1 + p_1 (S_2 - S_1) + \dot{m} V_1 = p_2 S_2 + \dot{m} V_2 \quad \text{with } \dot{m} = \rho S_1 V_1 = \rho S_2 V_2$$

$$p_1 S_2 = p_2 S_2 + \rho S_2 V_2 (V_2 - V_1)$$

$$p_1 = p_2 + \rho (V_2^2 - V_2 V_1)$$

Energy equation:

$$p_1 = p_2 + \tfrac{1}{2}\rho (V_2^2 - V_1^2) + K \tfrac{1}{2}\rho V_1^2$$

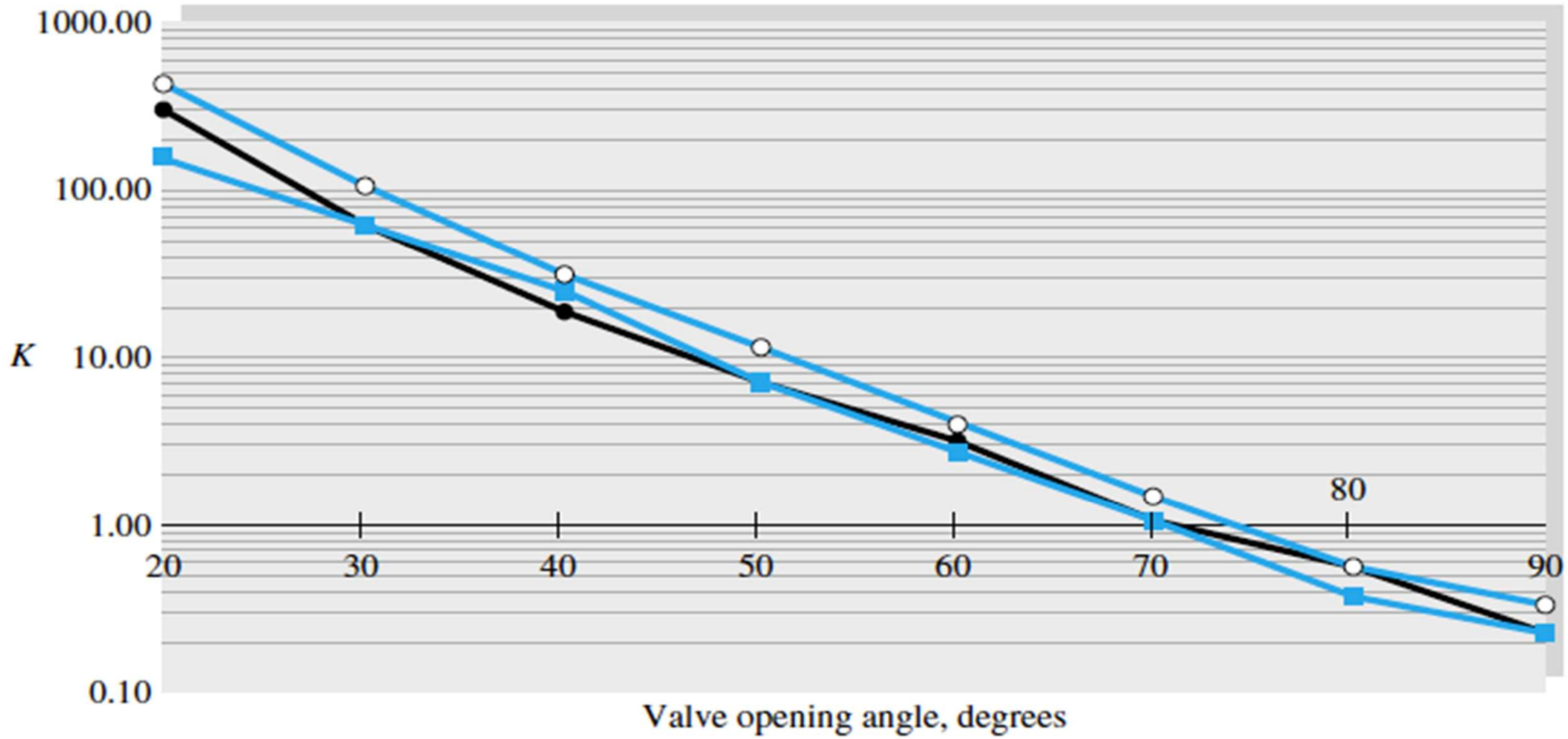
$$0 = \ 0 \ + \tfrac{1}{2}\rho (2V_2^2 - 2V_2 V_1) - \tfrac{1}{2}\rho (V_2^2 - V_1^2) - K \tfrac{1}{2}\rho V_1^2$$

$$K \tfrac{1}{2}\rho V_1^2 = \tfrac{1}{2}\rho (2V_2^2 - 2V_2 V_1 - V_2^2 + V_1^2)$$

$$K \tfrac{1}{2}\rho V_1^2 = \tfrac{1}{2}\rho (V_2^2 - 2V_2 V_1 + V_1^2)$$

$$K \tfrac{1}{2}\rho V_1^2 = \tfrac{1}{2}\rho V_1^2 \left(\frac{V_2^2}{V_1^2} - 2\frac{V_2}{V_1} + 1 \right) \quad \Rightarrow K = \left(\frac{V_2}{V_1} - 1 \right)^2 = \left(\frac{S_1}{S_2} - 1 \right)^2 = \left(\frac{d^2}{D^2} - 1 \right)^2$$

Commercial valve



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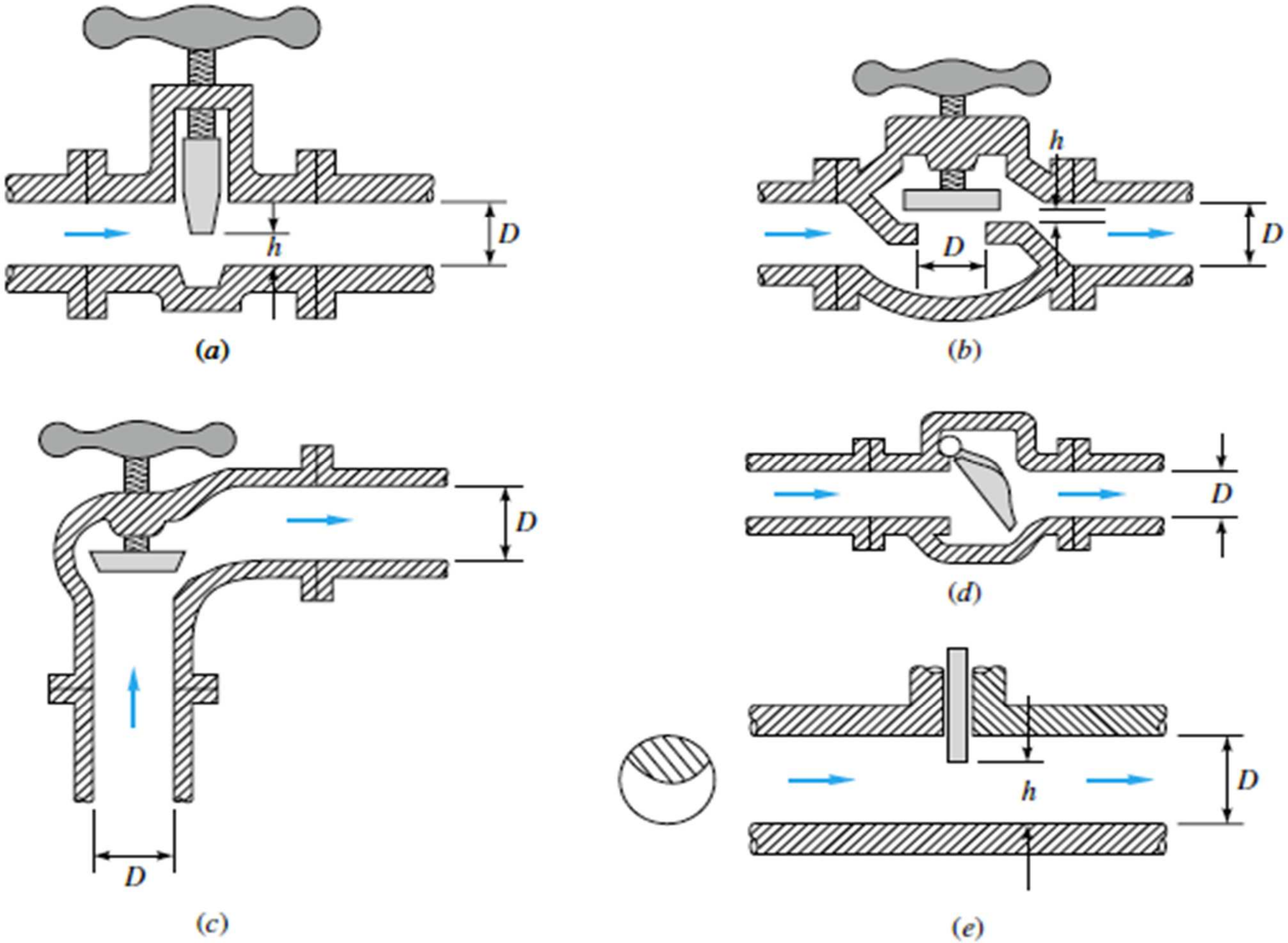
Other valves



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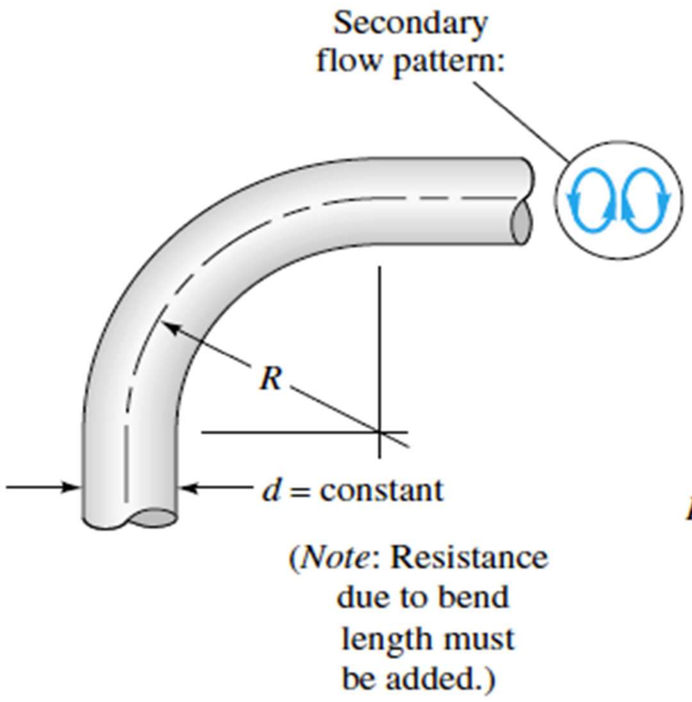
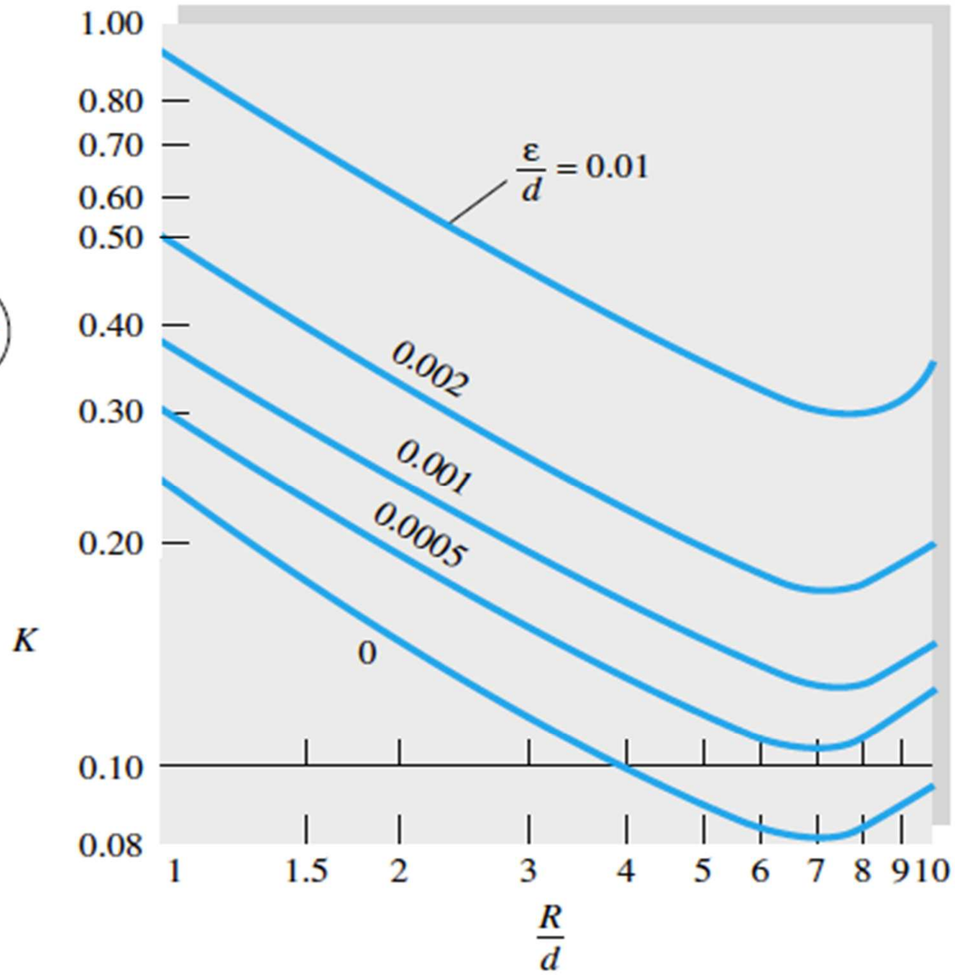
Bends


Secondary
flow pattern:
R
d = constant
(Note: Resistance
due to bend
length must
be added.)


1.00
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.08
K
ε/d = 0.01
0.002
0.001
0.0005
0
1
1.5
2
3
4
5
6
7
8
9
10
R/d

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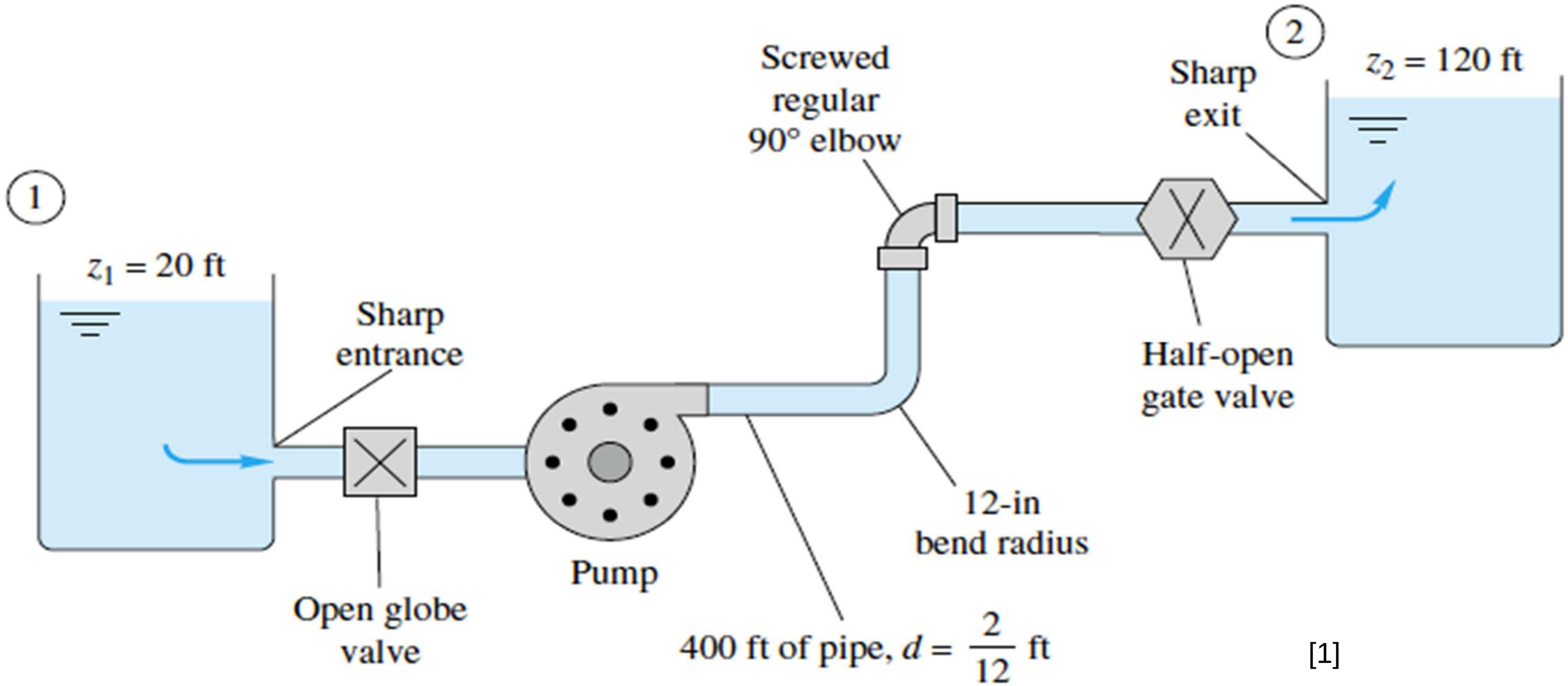
Loss coefficient table

	Nominal diameter, in								
	Screwed				Flanged				
	$\frac{1}{2}$	1	2	4	1	2	4	8	20
Valves (fully open):									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows:									
45° regular	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° regular	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees:									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

Example 6-16

Water is pumped between two reservoirs at 5.4 liter/sec through 120 m of a 5-cm diameter pipe with several minor losses. The roughness ratio is 0.001. Compute the required power for the pump.



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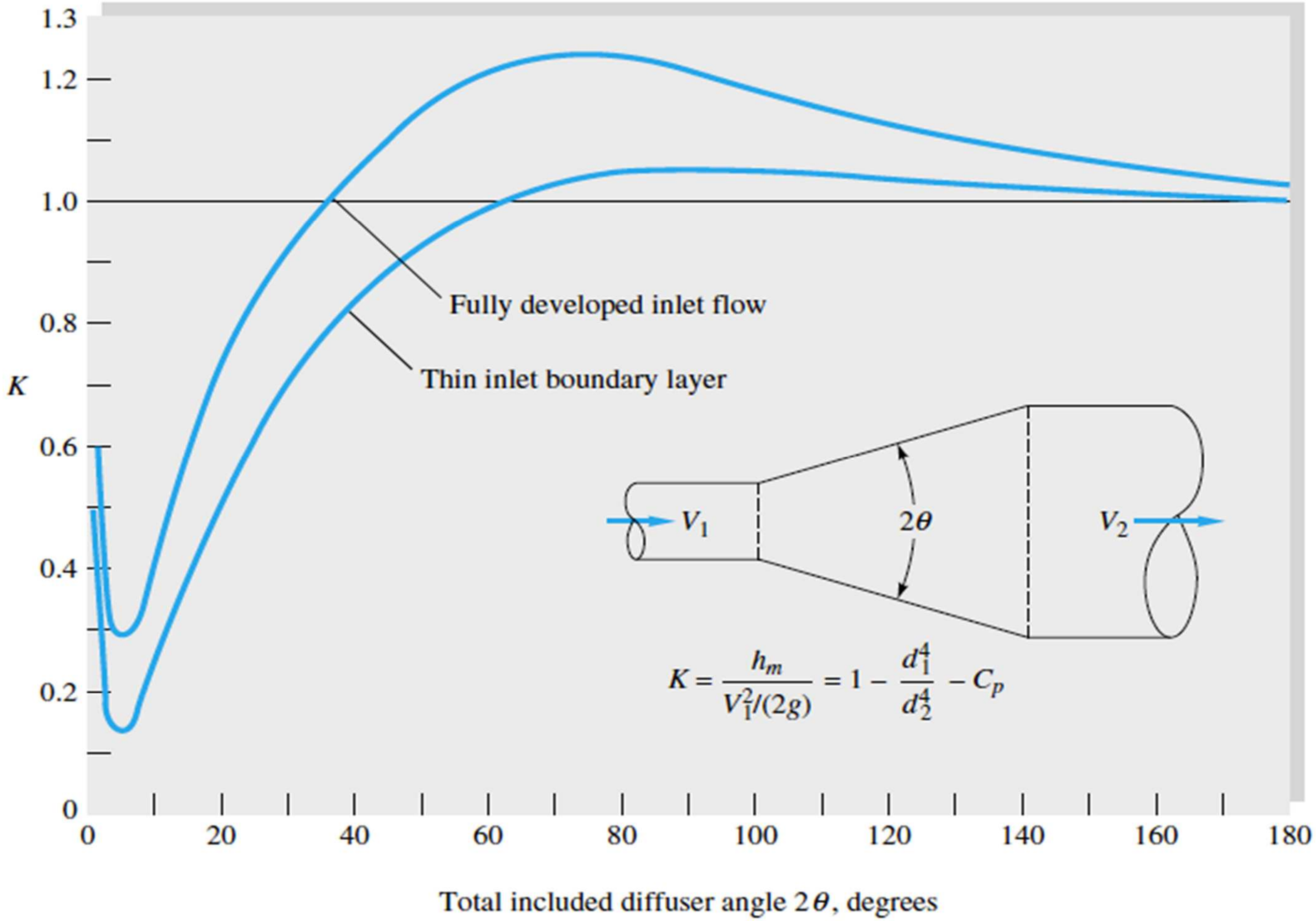
Diffuser



1.3
1.2
1.0
0.8
K
0.6
0.4
0.2
0
0
20
40
60
80
100
120
140
160
180
Fully developed inlet flow
Thin inlet boundary layer
V_1
2θ
V_2
$K = \frac{h_m}{V_1^2/(2g)} = 1 - \frac{d_1^4}{d_2^4} - C_p$
Total included diffuser angle 2θ, degrees

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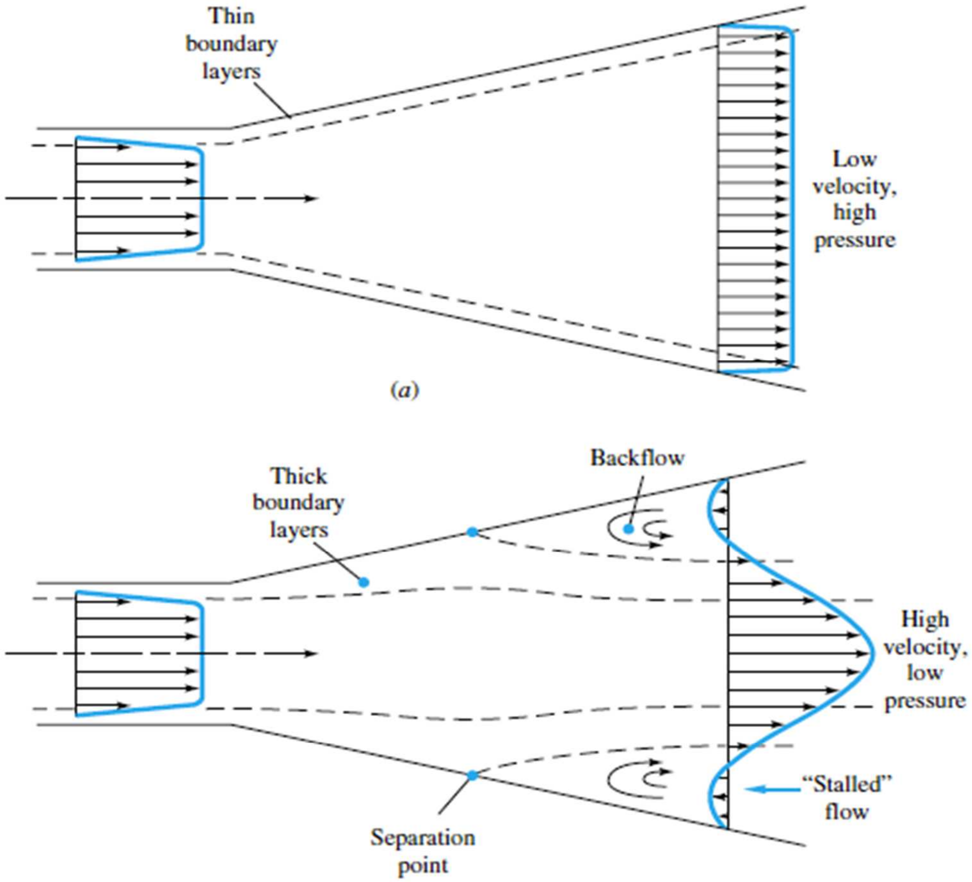
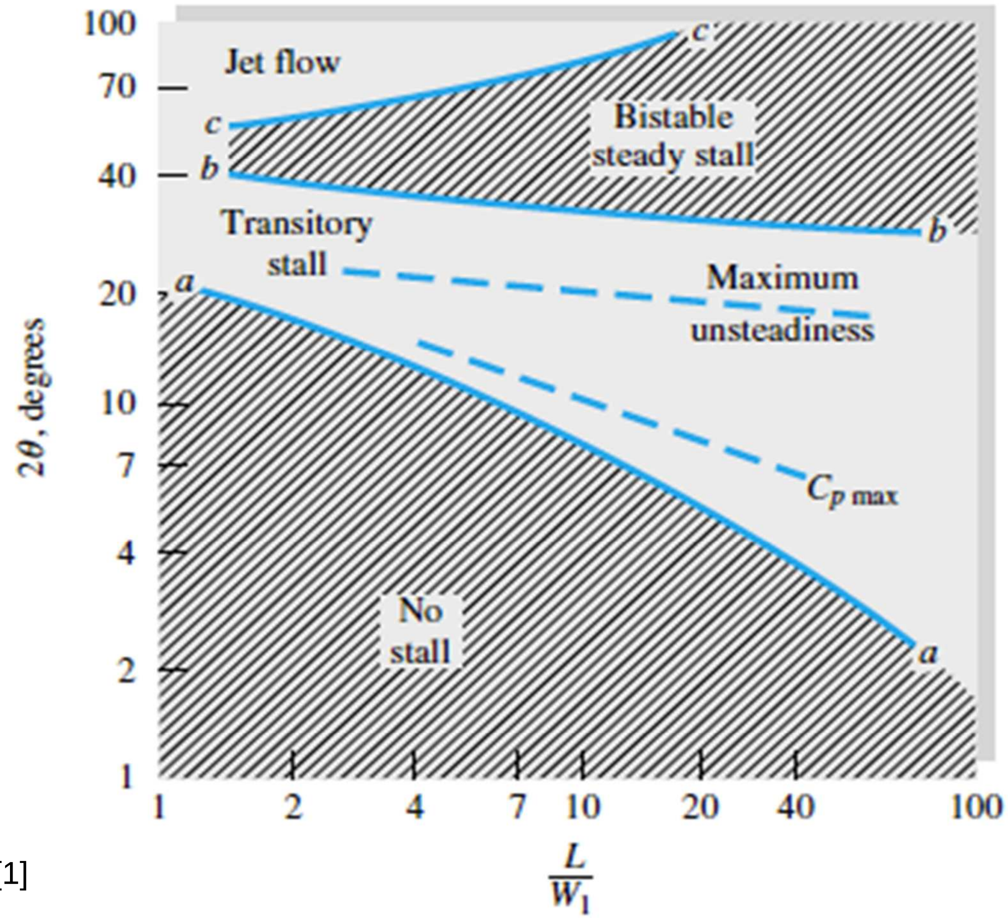
Diffuser performance





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Summary

- Chapter :
- Examples:
- Problems:

Source

1. Frank M. White, *Fluid Mechanics*, McGraw-Hill Series in Mechanical Engineering