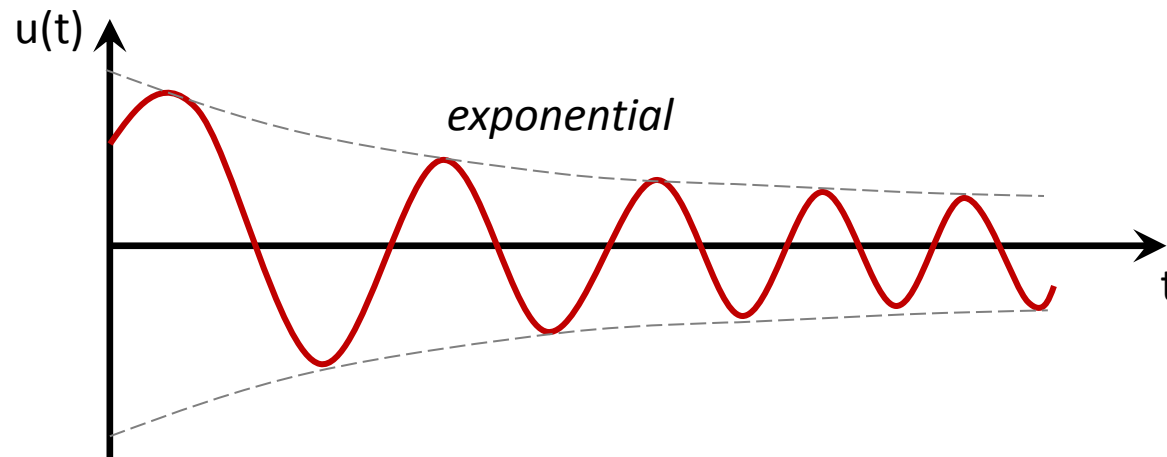


# Damping models

# types of damping

## ***viscous*** damping

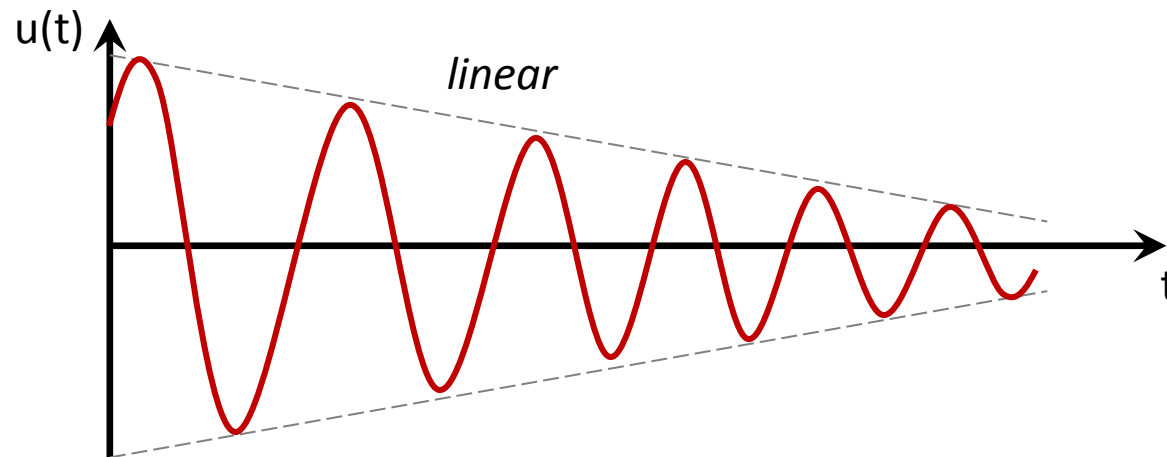
- results from slow moving structures in fluids and gases, e.g.
  - friction bearings
  - hydraulic dampers
  - shock absorbers, ...
- damping force is *proportional* to velocity



# types of damping

## ***Coulomb*** damping

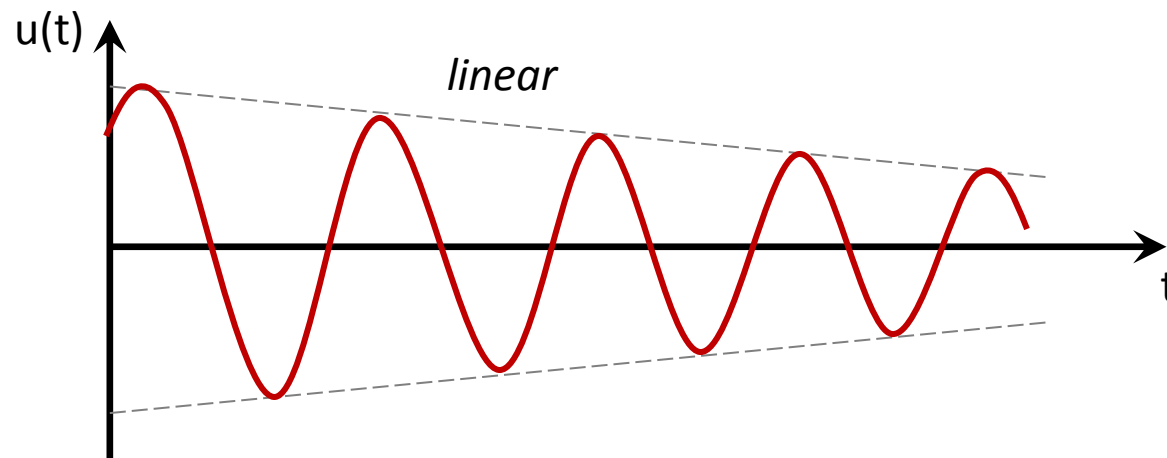
- results from friction on a dry surface, depending on
  - contact pressure
  - friction coefficient
- damping behavior is *constant*
- typical examples are bolt & rivet connections



# types of damping

## ***hysteresis*** damping

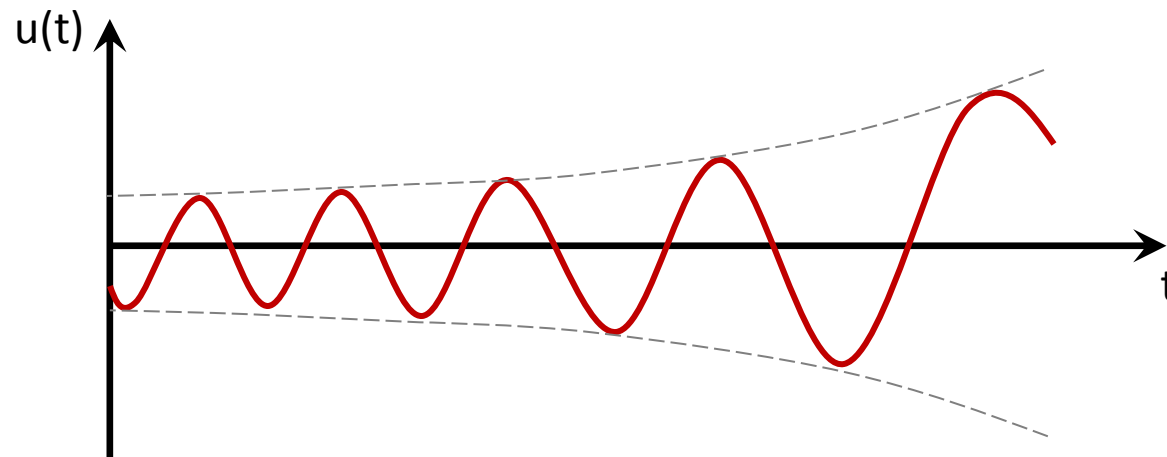
- results from internal friction of material (*structure damping*)
- damping forces are functions of displacements  $\mathbf{u}$  and strains  $\boldsymbol{\varepsilon}$
- often measured experimentally from the hysteresis of a load-displacement diagram using harmonic excitation
- damping force (absolute value of) is proportional to elastic forces



# types of damping

## *negative* damping

- appears at e.g. flutter of aircrafts
- energy input instead of energy dissipation
- energy input from streaming fluids
- results in excitation of the structure



## properties

- in general several damping effects are present in *real structures*
- damping models are often highly complex
- simplification is necessary with regard to the solution process of the governing damped equations of motion
- equivalent viscous damping models often replace complex damping models (equivalent = same energy dissipation)
- damping matrix can be derived analogue to consistent mass matrix

$$\mathbf{C} = \int_{\Omega} \mu \mathbf{N} \mathbf{N}^T d\Omega$$

## properties

- damping matrix  $\mathbf{C} = \int_{\Omega} \mu \mathbf{N} \mathbf{N}^T d\Omega$
- symmetric, often sparse
- without relation to  $\mathbf{K}$  and  $\mathbf{M} \rightarrow$  **non-proportional damping**
  - decoupling of equations of motion difficult/not possible

- alternative approach  $\rightarrow$  **proportional damping**
  - e.g. Rayleigh damping

$$\mathbf{C} = \alpha \mathbf{K} + \beta \mathbf{M}$$

- pro: eigenvectors of undamped problem diagonalize  $\mathbf{C}$
- con: use of only two free parameters  $\alpha, \beta$

## properties

- alternative approach → **proportional damping**
  - e.g. Rayleigh damping

$$\mathbf{C} = \alpha \mathbf{K} + \beta \mathbf{M}$$

- often choice of  $\alpha, \beta$  depending on the *damping ratio*  $\xi_i$

$$c_i = \alpha \omega_i^2 + \beta = 2 \xi_i \omega_i$$

- $\xi_i$  represents a percentage of critical damping,  
e.g. 2-5% for metallic materials →  $0.02 \leq \xi_i \leq 0.05$