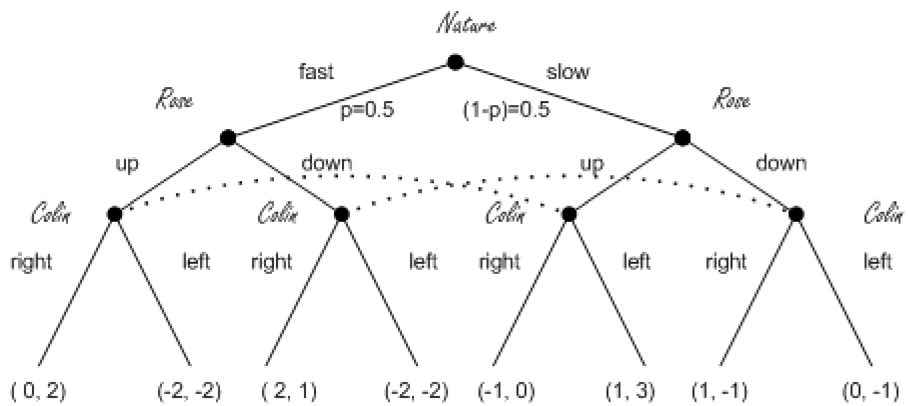
Supplementary Problem Set

Problem 1.

a) Solve the following game tree in pure strategies.
b) Did Colin learn anything useful from Rose's choice of strategy?



Answer Problem 1.

Rose sees nature. Therefore her strategies are

1. Ignore nature. Always go up.
2. Ignore nature. Always go down.
3. Respond to nature. If nature goes fast, go up. Otherwise go down.
4. Respond to nature. If nature goes slow, go up. Otherwise go down.

Colin does not see nature but does see Rose's actions. Therefore his strategies are

1. Ignore Rose. Always go right.
2. Ignore Rose. Always go left.
3. Respond to Rose. If Rose goes up, go right. Otherwise go left.
4. Respond to Rose. If Rose goes down, go right. Otherwise go left.

We can label these strategies a number of different ways. For now, we will adopt the numbers as given. If we want to be completely unambiguous, we write out the intersection of all strategies, resolving conditional strategies. In the table below, strategies in parentheses mean (do this if nature goes fast, do this if nature goes slow).

		Colin			
		1	2	3	4
Rose	1	Up, Right	Up, Left	Up, Right	Up, Left
	2	Down, Right	Down, Left	Down, Left	Down, Right
	3	(Up, Down), Right	(Up, Down), Left	(Up, Down), (Right, Left)	(Up, Down), (Left, Right)
	4	(Down, Up), Right	(Down, Up), Left	(Down, Up), (Left, Right)	(Down, Up), (Right, Left)

Note that all cells are subjected to expected utility since nature began the game. No player can eliminate randomness, although Rose can prepare for it. We write out the payoffs for all players using expected utility.

		Colin			
		1	2	3	4
Rose	1	(-0.5, 1)	(-0.5, 0.5)	(-0.5, 1)	(-0.5, 0.5)
	2	(0.5, 0)	(-1, -1.5)	(-1, -1.5)	(0.5, 0)
	3	(0.5, 0.5)	(-1, -1.5)	(0, 0.5)	(-0.5, -1.5)
	4	(0.5, 0.5)	(-0.5, 0.5)	(-1.5, -1)	(1.5, 2)

Now we eliminate dominated strategies.

		Colin			
		1	2	3	4
Rose	1	(-0.5, 1)	(-0.5, 0.5)	(-0.5, 1)	(-0.5, 0.5)
	2	(0.5, 0)	(-1, -1.5)	(-1, -1.5)	(0.5, 0)
	3	(0.5, 0.5)	(-1, -1.5)	(0, 0.5)	(-0.5, -1.5)
	4	(0.5, 0.5)	(-0.5, 0.5)	(-1.5, -1)	(1.5, 2)

The procedure goes like this.

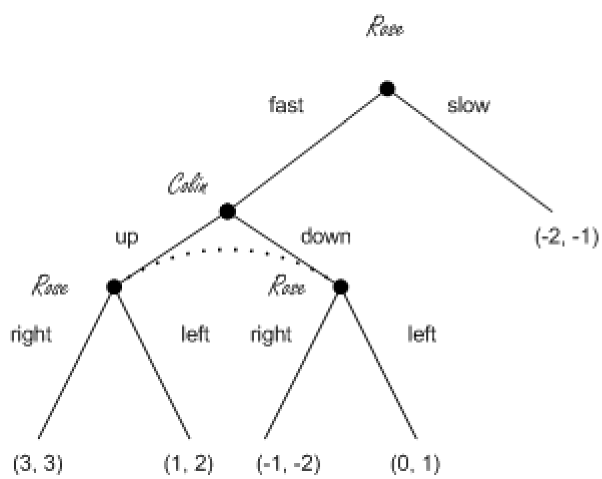
- Colin 2 is weakly dominated by Colin 1.
- Colin 3 is weakly dominated by Colin 1.
- Rose 1 is weakly dominated by Rose 4.
- Rose 2 is weakly dominated by Rose 4.
- Rose 3 is weakly dominated by Rose 4.
- Colin 1 is strongly dominated by Colin 4.

The outcome of the game is Rose 4, Colin 4, resulting in a payoff of (1, 1.5) – 1 to Rose, 1.5 to Colin. Note that we eliminated a lot of weakly dominated solutions, and didn't search for mixed strategies. There may be additional equilibria available, if for instance we were to solve it in Gambit. We would however still expect to find equilibrium {4,4} regardless of the presence of other equilibria.

Rose responded to Nature's signal, Colin responded to Rose's signal.

Problem 2.

a) Solve the following game tree, which in the latter part of the tree is a simultaneous move game. Note how Rose can choose to opt out of the game as a potential strategy.
b) Is there also a mixed strategy solution to this game?



Answer Problem 2.

The solution to the simultaneous move game is to first recognize that Colin will always go up, since up dominates down. If Colin goes up, then Rose will go right. Rose now has the choice of going fast, right and earning 3, or going slow and earning -2. Fast, right is to be preferred.

The resulting play of the game is {fast, right; up}. This results in a payoff of (3,3).

A mixed strategy solution is not possible in this game.

Begin with Colin. First, consider the possibility that Rose would mix in her slow strategy. This never dominates moving fast, so it will not be an active strategy. We can eliminate this strategy from consideration. Now, Colin can create indifference between Rose R and Rose L, as shown below.

$3p - 1 + p = p$
$p = 1/3$

This shows that there is a probability p which makes Rose different.

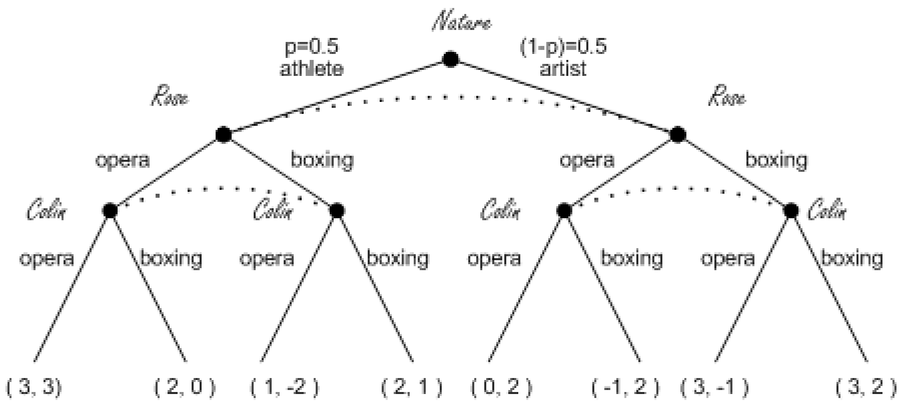
Now we turn to Rose. There is no possibility for Rose to make Colin indifferent. As before we can see that Rose will not play her slow move. Thus, she attempts to mix R and L.

$3r + 2 - 2r = -2r + 1 - r$
$3r + 1 = -r$
$4r = -1.$

Colin will always defect to U if Rose attempts to mix. There is no mixed strategy possible, since Rose cannot find a payoff equalizing strategy for Colin.

Problem 3.

a) This game has the second moving player seeing the state of nature, but not the first-moving player. How should this be treated when solving?
b) What are the strategy sets?
c) Find the pure strategy Nash equilibrium of the following game based upon eliminating strongly dominated strategies.



Solution to Problem 3.

Although we haven't seen this exact information set before, we treat it like all the others we have seen. We enumerate strategies based upon the information available to the player. The advantages of having better information will work themselves out in having available a more effective set of strategies for the player to select from.

Rose sees nothing – not Nature, and since she moves first, not Colin either. Her strategies are:

1. opera always
2. boxing always

Colin sees Nature, but he does not see Rose's move. His strategies are therefore.

1. Ignore nature, and always go to opera.
2. Ignore nature, and always go to boxing.
3. Utilize nature. If nature says athlete, go boxing. Otherwise, go opera.
4. Utilize nature. If nature says athlete, go opera. Otherwise go boxing.

We now draw out the resulting (2x4) payoff matrix. The parentheses in Colin's strategy set indicates what he will do if nature goes (athlete, artist) respectively.

		Colin			
		1	2	3	4
Rose	1	Opera, Opera	Opera, Boxing	Opera, (Boxing, Opera)	Opera, (Opera, Boxing)
	2	Boxing, Opera	Boxing, Boxing	Boxing, (Boxing, Opera)	Boxing (Opera, Boxing)

Now we work out the payoff matrices, using the principle of expected utility. Recall that Rose can neither plan for nature, nor eliminate the uncertainty of nature. Colin can plan for nature, but he still cannot eliminate it.

		Colin			
		1	2	3	4
Rose	1	(1.5,2.5)	(0.5, 1)	(1,1)	(1, 1.5)
	2	(2, -1.5)	(2.5, 1.5)	(2.5, 0)	(2, 0)

		Colin			
		1	2	3	4
Rose	1	~~(1.5,2.5)~~	~~(0.5, 1)~~	~~(1,1)~~	~~(1, 1.5)~~
	2	~~(2, -1.5)~~	(2.5, 1.5)	~~(2.5, 0)~~	~~(2, 0)~~

The procedure goes like this. Rose strategy 2 (boxing) is clearly superior to Rose strategy 1 (opera). Colin should not expect her to play anything different. Colin may select among his available strategies against Rose strategy 2 to achieve the highest payoff. The resulting outcome is Rose 2, Colin 2.

Thus, both players go to boxing regardless, and earn an expected utility of (2.5, 1.5), payoffs to Rose and Colin respectively.