Thermodynamica 1

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college 10 – boek hoofdstuk 6

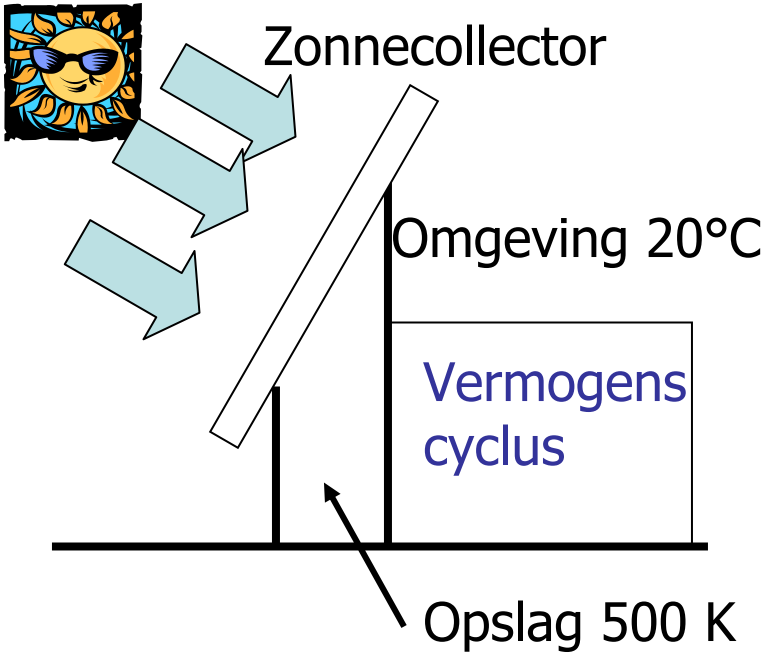
Thermodynamica 1

- Tweede hoofdwet
- Thermisch rendement
- Kelvin temperatuur schaal
- Carnot Kring process → de ideale thermodynamische cyclus

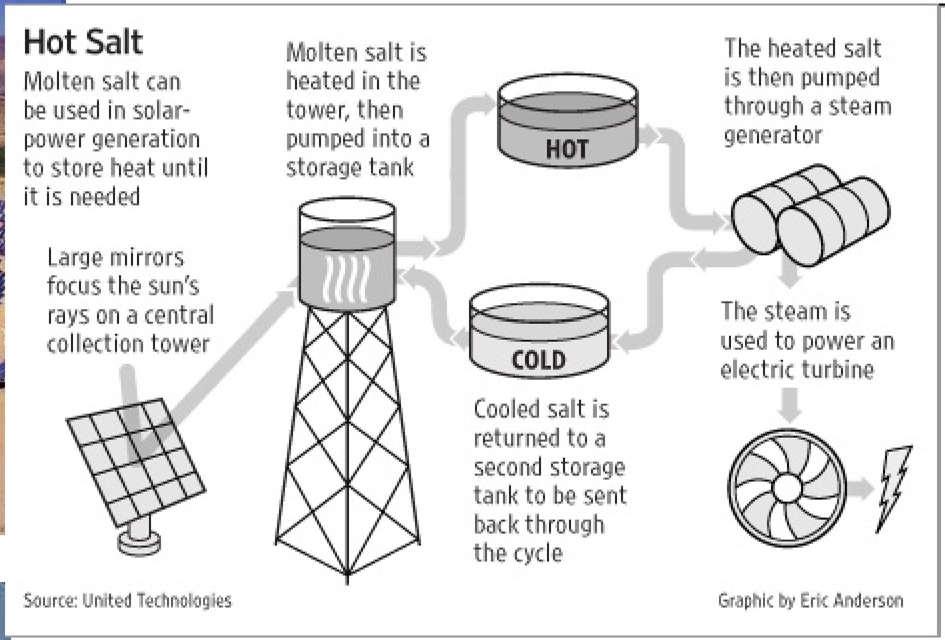
Sadi Carnot

Concentrated solar power (CSP)

Voorbeeld: opgave 5.33



Concentrated Solar Power



Hot Salt

Molten salt can be used in solar-power generation to store heat until it is needed

Large mirrors focus the sun's rays on a central collection tower

Molten salt is heated in the tower, then pumped into a storage tank

The heated salt is then pumped through a steam generator

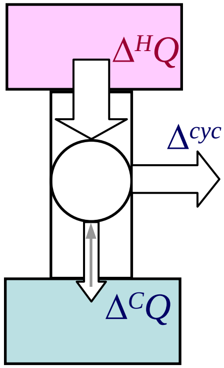
Cooled salt is returned to a second storage tank to be sent back through the cycle

The steam is used to power an electric turbine

Source: United Technologies

Graphic by Eric Anderson

Recap: 2nd law and corollaries for cycle-processes



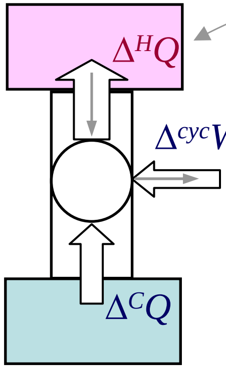
negative value

$\Delta^{cyc}W = \Delta^H Q + \Delta^C Q$

$\Delta^C Q = 0$ not feasible (Kelvin-Planck)

power cycle

↑ arrow shows convention of direction
$\Delta^C Q$ is negative, with
$\Delta^C Q = -Q^C_{out}$



negative values

$\Delta^{cyc}W = \Delta^H Q + \Delta^C Q$

$\Delta^{cyc}W = 0$ not feasible (Clausius)

refrigeration / heat pump cycle

Analysis of (Carnot) power cycles revealed

⇒ even a reversible (i.e. idealized) power cycle has an efficiency of $\eta^{rev} < 1$

⇒ all reversible Carnot-like power cycles operating between the same temperature of two thermal reservoirs have the same thermal efficiency η^{rev}

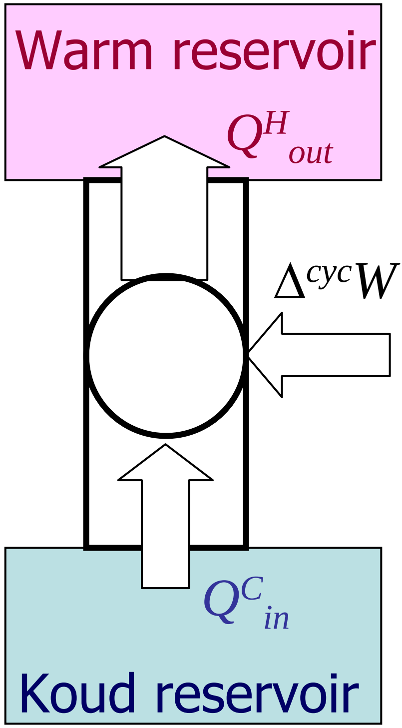
- irrespective of type of process-steps in power cycle processes
- irrespective of working fluid

$$\eta^{Carnot} = \underbrace{\frac{\Delta^{cyc}W}{\Delta^H Q} = 1-\frac{|\Delta^C Q|}{\Delta^H Q}}_{\text{thermal efficiency } \eta^{th} \text{ of any power cycle}} = \underbrace{1-\frac{T_C}{T_H}}_{\text{for Carnot cycle}}$$

⇒ an irreversible (i.e. real) power cycle has yet a lower efficiency $\eta^{real} < \eta^{rev} < 1$

Koel- & Warmtepompkringprocessen

Koel- & Warmtepompkringprocessen



Warm reservoir

Q^H_{out}

$\Delta^{cyc}W = Q^H_{out} - Q^C_{in}$

Q^C_{in}

Koud reservoir

Warmtepomp (WP) Warmtefactor (COP):

$$\gamma = COP_{WP} = \frac{Q^H_{out}}{\Delta^{cyc}W} \qquad \gamma = \frac{Q^H_{out}}{Q^H_{out} - Q^C_{in}}$$

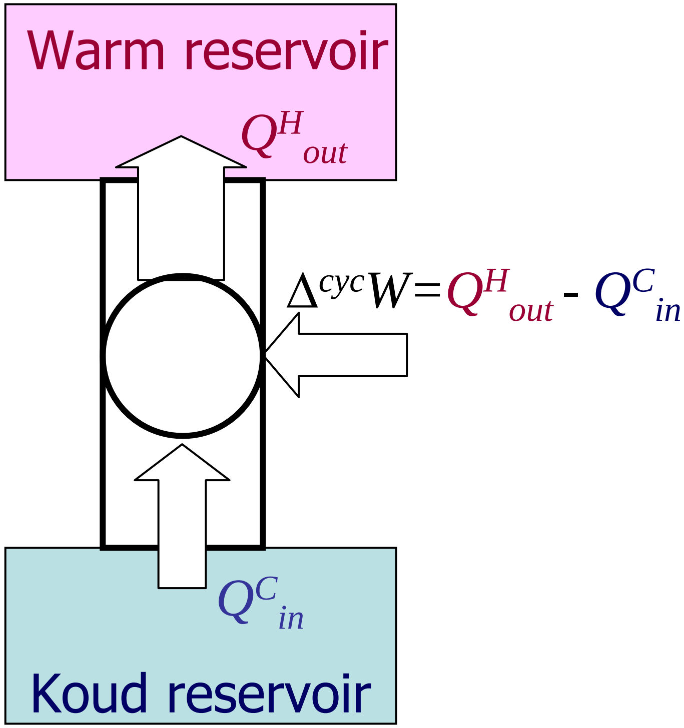
"coefficient of performance"

Koelmachine (KM) Koudefactor (COP):

$$\beta = COP_{KM} = \frac{Q^C_{in}}{\Delta^{cyc}W} \qquad \beta = \frac{Q^C_{in}}{Q^H_{out} - Q^C_{in}}$$

- Hoewel het processchema voor koel- en warmtepompkringprocessen identiek is, verschillen de *COP*'s
- $COP_{WP} \geq 1$ Uit de definitie volgt: $\gamma = \beta + 1$
- Een reversibel kringproces heeft de hoogst mogelijke waarde van *COP*

Maximum rendement van koel- & WP-kringprocessen



Warm reservoir

$Q^H{}_{out}$

$\Delta^{cyc}W = Q^H{}_{out} - Q^C{}_{in}$

$Q^C{}_{in}$

Koud reservoir

voor Carnot-proces

$$\left(\frac{Q^C_{in}}{Q^H_{out}}\right)_{Carnot} = \frac{T_C}{T_H}$$

Warmtepomp (WP) Warmtefactor (COP)

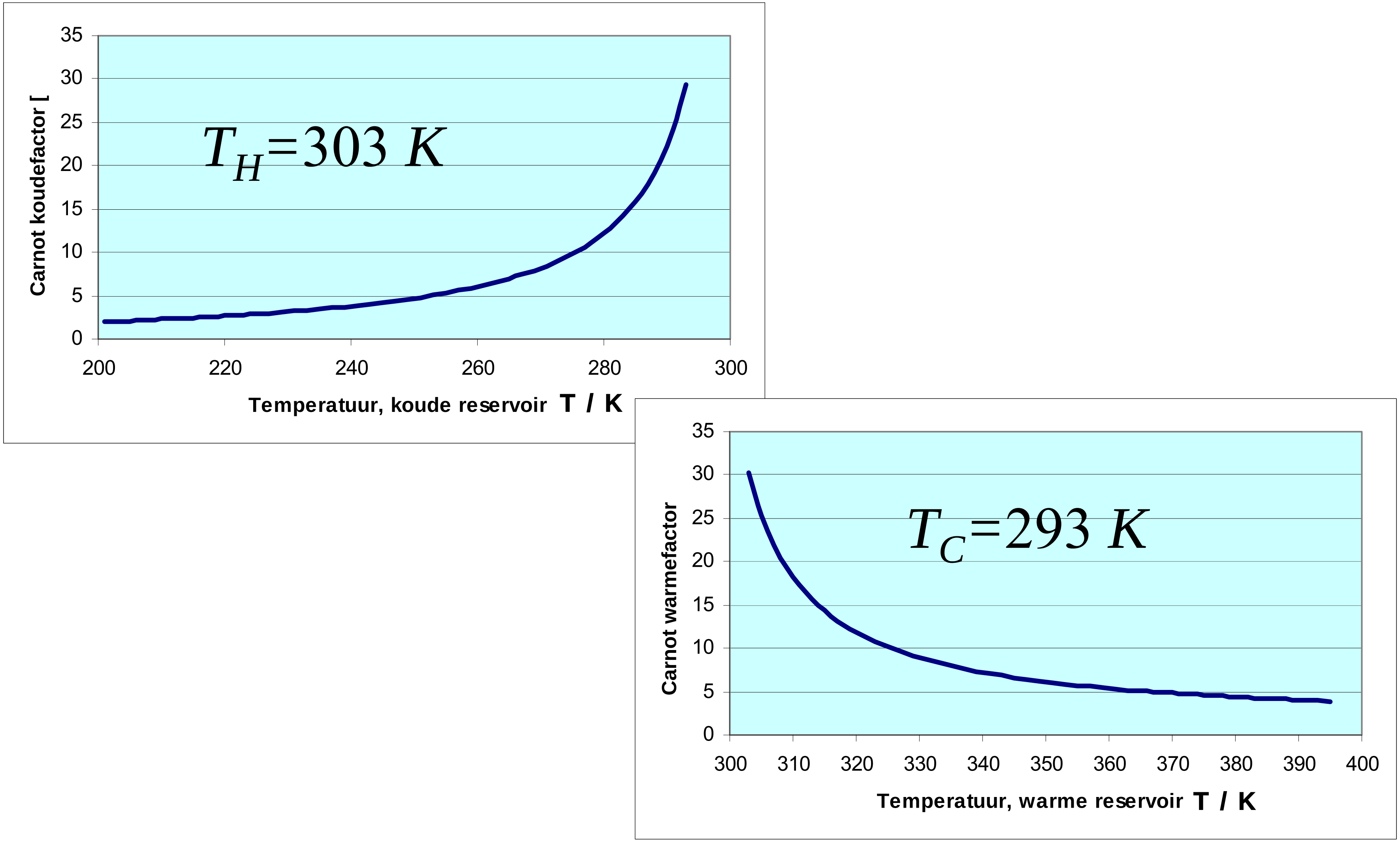
$$COP_{WP} = \frac{Q^H_{out}}{Q^H_{out} - Q^C_{in}} \qquad COP_{WP,Carnot} = \frac{T_H}{T_H - T_C}$$

Koelmachine (KM) Koudefactor (COP)

$$COP_{KM} = \frac{Q^C_{in}}{Q^H_{out} - Q^C_{in}} \qquad COP_{KM,Carnot} = \frac{T_C}{T_H - T_C}$$

Het maximale rendement vereist een proces met isotherme warmteoverdracht ("Carnot-like" proces)

Maximum rendement van koel- & WP-kringprocessen



Voorbeeld 1

De omgeving van een vrieskast heeft een temperatuur van $27°\mathrm{C}$. Om de vrieskast op $-40°\mathrm{C}$ te houden moet een energiestroom van $1.25\ kW$ afgevoerd worden

Bepaal

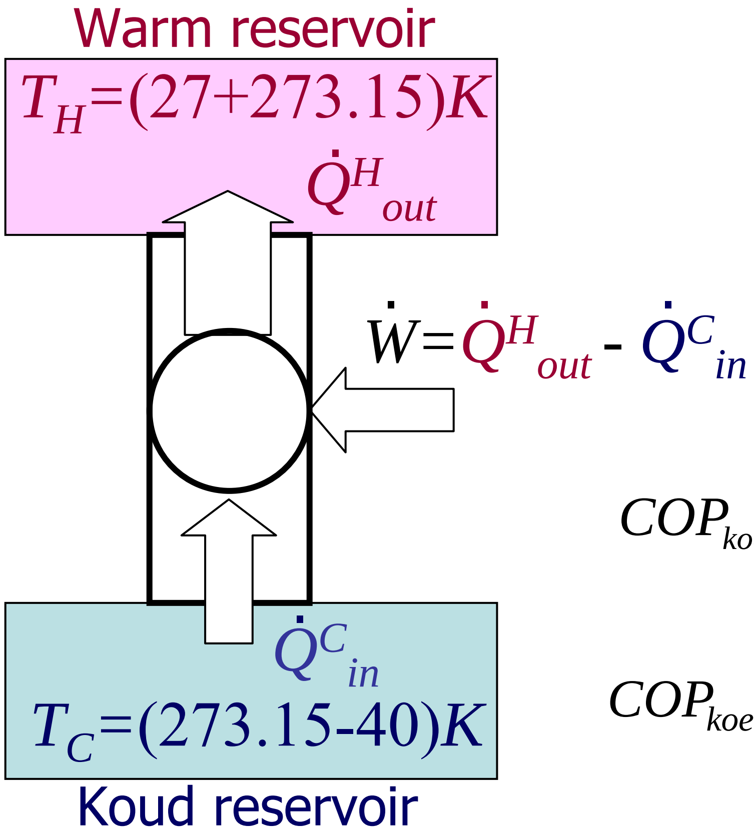
(1) De maximale koudefactor (COP).

(2) Het minimum theoretisch gevraagd aandrijfvermogen, in kW, voor een koelkringproces dat onder deze omstandigheden werkt

COP = 3.5

W=0.35kW

Voorbeeld 1 (uitwerking)



Warm reservoir

$T_H=(27+273.15)K$

$\dot{Q}^H{}_{out}$

$\dot{W}=\dot{Q}^H{}_{out} - \dot{Q}^C{}_{in}$

$\dot{Q}^C{}_{in}$

$T_C=(273.15\text{-}40)K$

Koud reservoir

$$COP_{koelmachine} = \frac{T_C}{T_H - T_C} = \frac{233.15K}{67.0K} = 3.48$$

$$COP_{koelmachine} = \frac{\dot{Q}^C_{in}}{\dot{Q}^H_{out} - \dot{Q}^C_{in}}$$

$$\dot{W} = \dot{Q}^H_{out} - \dot{Q}^C_{in} = \frac{\dot{Q}^C_{in}}{COP_{koelmachine}} = \frac{1.25kW}{3.48} = 0.359kW$$

Warmtepomp of CV ketel

- CV ketel op gas ongeveer 25 kW
- Rendement 95%
- Stookwaarde gas 32MJ/m^3
- Verbruik /h =2.7 m^3 = (2 Euro)

- Warmtepomp 25 kW
- T_h=22 graden celsius, Q_h=25 kW
- T_l= 4 graden celsius Q_l= 23.5kW
- Theoretische arbeid W=1.5kW (COP = 16)
- Rendement 25% Benodigd vermogen 6.0kW (COP = 16*.25=4)
- Verbruik/h=6kWh = (1.5 Euro)

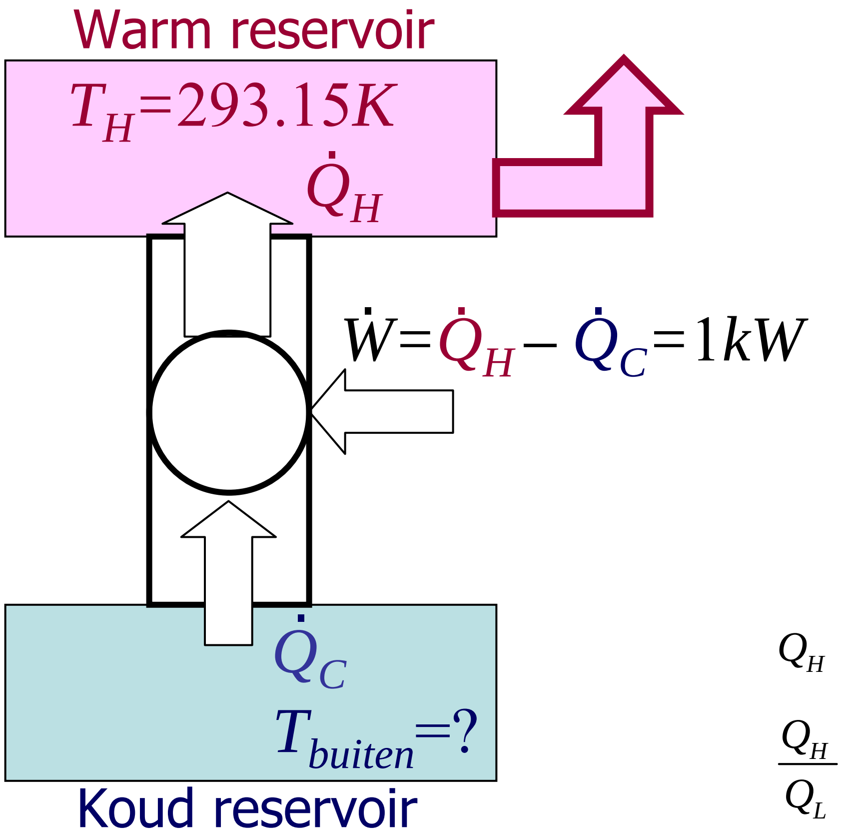
  - Aanschafprijs warmtepomp is hoger, een COP van 4 is realiseerbaar
  - Warmtepomp neemt meer ruimte in
  - T_l=15 graden celsius ➔ 0.56 Euro

Aardwarmte

Voorbeeld 2

- Een ideale Carnot-warmtepomp heeft een aandrijfvermogen van 1 kW nodig om een woning op 20°C te houden
- Het warmtetransport door wanden en dak is gelijk aan 3000 kJ/h per graad temperatuurverschil tussen binnen en buiten (warmtebron)
- Bepaal de laagste buitentemperatuur waarvoor de warmtepomp nog voldoet
- Opmerking: deze buitentemperatuur is ook de temperatuur van het koude reservoir

Voorbeeld2 (uitwerking)



$$T_H = 20^o C$$

$$W = -1kW$$

$$\left.\begin{aligned} Q_H &= \frac{3000}{3600}(T_H - T_L) \\ \frac{Q_H}{Q_L} &= \frac{T_H}{T_L} \\ Q_H &= Q_L + W \end{aligned}\right\rangle Q_H\left(1-\frac{T_H}{T_L}\right) = W \Rightarrow Q_H = W\left(1-\frac{T_H}{T_L}\right)^{-1}$$

$$W\left(1-\frac{T_H}{T_L}\right)^{-1} = \frac{3000}{3600}(T_H - T_L) => -1\left(1-\frac{293}{T_L}\right)^{-1} = \frac{3}{3.6}\left(293 - T_L\right)$$

$$T_L \approx 275K$$

Entropy

& the 2nd law as a balance equation

Analyzing the Carnot power cycle – the entropy

- From the 1st and 2nd law we know that

$\Delta^C Q = -Q^C_{out}$ negative value according to regular convention (ΔQ in the direction of system)

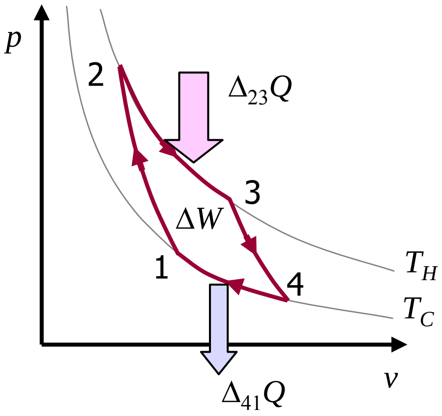
$$\eta^{rev} = 1 - \frac{|\Delta^C Q|}{\Delta^H Q} = 1 - \frac{T^C}{T^H}$$

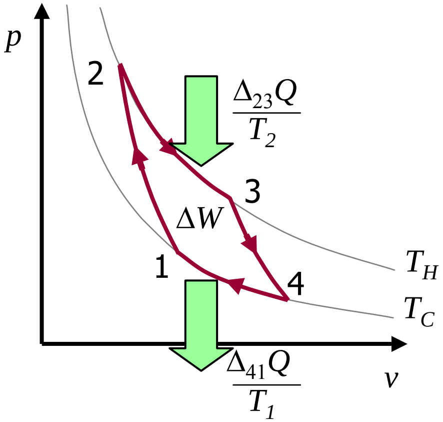
Carnot

- from

$$\frac{|\Delta^C Q|}{\Delta^H Q} = \frac{T^C}{T^H} \quad \Rightarrow \quad \frac{|\Delta^C Q|}{T^C} = \frac{\Delta^H Q}{T^H} \quad \Rightarrow \quad \frac{\Delta^C Q}{T^C} + \frac{\Delta^H Q}{T^H} = 0$$

- it's clear, that the sum of the ratio $\Delta Q/T$ is preserved in a reversible Carnot cycle





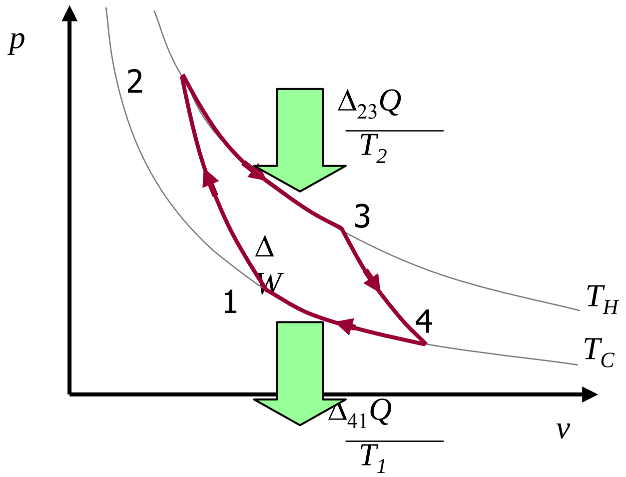
$\Delta Q/T$ passes unchanged through reversible cycles

Toelichting

$$\Delta_{23}Q = mRT_H \ln(V_3/V_2)$$

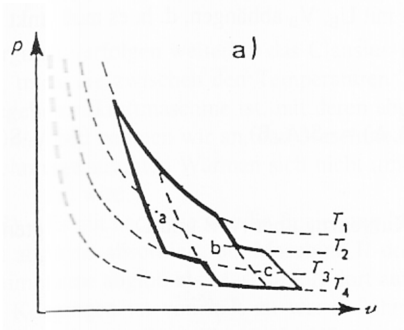
$$\Delta_{14}Q = mRT_C \ln(V_4/V_1)$$

$$V_3/V_2 = V_4/V_1 \Rightarrow \frac{\Delta_{23}Q}{T_H} = \frac{\Delta_{14}Q}{T_C}$$



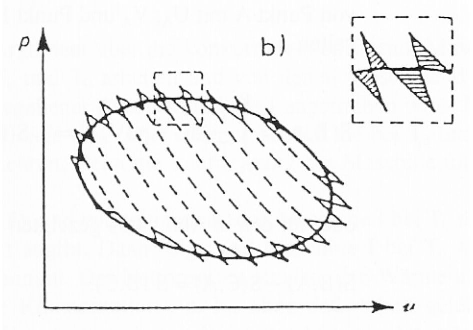
Generalizing to arbitrary reversible cycles – the entropy

- We can consider a reversible cycle with multiple Carnot-steps (i.e. multiple adiabatic and isothermal steps)



analysis of such cycles gives $\Rightarrow \quad \sum_i \frac{\Delta^i Q}{T^i} = 0 \qquad (\#)$

- We can then consider arbitrary reversible cycles. We can always substitute them by an equivalent cycle composed of infinitesimal Carnot-steps



from (#) with $\Delta \to 0 \quad \Rightarrow \quad \oint \frac{\delta Q}{T} = 0$

$\Rightarrow$ the ratio $\Delta Q/T$ (or $\delta Q/T$) is preserved in any reversible cycle. Clausius called this ratio the **entropy** S, with

$$dS = \frac{\delta Q}{T}$$

unit of entropy S is $[J/K]$ and $\oint dS = \oint \frac{\delta Q}{T} = 0$

2nd law for reversible processes in closed system

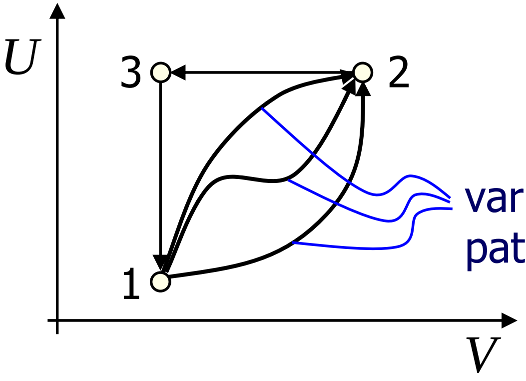
The entropy – a state property (toestandsgrootheid)

With the first law $dU = \delta Q - pdV$

we can rewrite $$\oint dS = \oint \frac{\delta Q}{T} = 0$$

as $$\oint dS = \oint \frac{1}{T}\left(dU + pdV\right) = 0$$

We analyze an arbitrary cycle process 1→2→3→1



various process paths for 1→2

$$\oint dS = \int_1^2 dS + \int_2^3 dS + \int_3^1 dS = 0$$

since this equation holds for any process path 1→2, we conclude that $\int dS$ is not path-dependent ⇒

$$\int_1^2 dS = S_2 - S_1 = S\left(T_2, v_2\right) - S\left(T_2, v_2\right)$$

⇒ the entropy is a **state property**. It is fully determined at a thermodynamic state point, say (T,v) (or for mixtures (T,v,x_i)).

The entropy - variables

Like for any other state property, the entropy S is fully defined, for a set of variables that fully specify a state, such combinations can be $\mathrm{S(v,T)}$, $\mathrm{S(p,T)}$, $\mathrm{S(p,v)}$, $\mathrm{S(h,T)}$, … (for pure components, otherwise the composition x_i also needs to be specified)

Often times, the entropy is expressed as $S(v,T)$, or $S(p,T)$

The second law of Thermodynamics

- one can write the 2nd law for closed systems in differential & in integral form

$$dS = \frac{\delta Q}{T} + \delta\sigma \qquad \delta\sigma \geq 0 \qquad \text{(closed system)}$$

$$\Delta_{12}S = \int_1^2 \frac{\delta Q}{T} + \underbrace{\int_1^2 \delta\sigma}_{\Delta_{12}\sigma} \qquad \Delta_{12}\sigma \geq 0 \qquad \text{(closed system)}$$

$\delta\sigma = 0, \quad \Delta_{12}\sigma = 0$ entropy production for reversible processes

$\delta\sigma > 0, \quad \Delta_{12}\sigma > 0$ entropy production of non-reversible processes (due to irreversibilities)

- the entropy S is a state property. The entropy production σ is a process quantity, NOT a state property

- The entropy production $T_0 \cdot \sigma$ is the "lost work" of the system, i.e. the non-utilized work potential
- For a close-to-ideal process (close to reversible), σ approaches zero (but in practice does not reach zero)

Cycle process with irreversibilities

Let's compare a Carnot cycle with a "Carnot-like" cycle where irreversibilities prevail. Both cycles will have the same heat transfer $\Delta^{H}Q$ from the warm reservoir

The first law for the Carnot cycle

$$\Delta W^{Carnot} = \Delta^{H} Q + \Delta^{C} Q^{Carnot} \qquad (1)$$

& "Carnot-like" cycle with irreversibilities

$$\Delta W^{irrev} = \Delta^{H} Q + \Delta^{C} Q^{irrev} \qquad (2)$$

The second law for a Carnot cycle, is

$$\frac{\Delta^{C} Q^{Carnot}}{T^{C}} + \frac{\Delta^{H} Q}{T^{H}} = 0 \qquad (3)$$

& for the irreversible cycle, it is

$$\frac{\Delta^{C} Q^{irrev}}{T^{C}} + \frac{\Delta^{H} Q}{T^{H}} + \Delta^{cyc}\sigma = 0 \qquad (4)$$

(2) in (1) $\Delta W^{Carnot} - \Delta W^{irrev} = \Delta^{C} Q^{Carnot} - \Delta^{C} Q^{irrev}$

(2) in (1) $\Delta^{C} Q^{Carnot} - \Delta^{C} Q^{irrev} = T_{C} \sigma^{cyc}$

$\Rightarrow$ $\boxed{\Delta W^{Carnot} - \Delta W^{irrev} = T_{C} \cdot \Delta^{cyc}\sigma}$ and $\eta^{irrev} = \eta^{Carnot} \cdot \eta_{\sigma}$ with

$$\eta_{\sigma} = \frac{\Delta W^{irrev}}{\Delta W^{Carnot}} = 1 - \frac{T_{C} \cdot \Delta^{cyc}\sigma}{\Delta W^{Carnot}}$$

$\Rightarrow$ the **entropy production** (times a reference temperature) is the "**lost work**" (i.e. non-utilized work potential) due to irreversibilities.

The third law of Thermodynamics

- the entropy of matter at $T=0K$ is zero independent of pressure or the type of crystalline phase

$$S(T=0K,P) = S(T=0K) = 0\,J/K$$

- The third law is no definition, but a finding
- It was determined experimentally (Nernst) and supplemented theoretically by Statistical Thermodynamics (Planck)

the temperature $T=0K$ can not be reached experimentally, however. In fact that is a result of the 2nd law saying: the efficiency of a refrigeration system approaches zero when T_C approaches zero

Working with the entropy and the 2nd law

Combining the 1st and 2nd law – the $T{\cdot}dS$ equations

The first law for closed systems reads

$$dU = \underbrace{\delta Q_{int.rev}}_{= TdS} - \underbrace{\delta W_{int.rev}}_{= pdV}$$

(2nd law)

$$\Rightarrow \quad \boxed{dU = TdS - pdV \qquad \text{the Gibbs equation (for pure components)}}$$

with $H = U + pV \quad \Rightarrow \quad dH = dU + d(pV)$

$$= dU + pdV + Vdp$$

$$\Rightarrow \quad \boxed{dH = TdS + Vdp}$$

the equations follow from considering a closed system. But they relate only state properties of the involved material. It does not matter if the material is in an open or closed system and the equations are not restricted to closed systems.

these equations are not applied to process steps, they relate material properties. Molecular motion of materials is perpedual (reversible), so that the assumption “int.rev.” is justified!

Entropy change of an ideal gas

For an ideal gas, $u(T,V)$ and $h(T,P)$ are functions of T only; $u(T)$ and $h(T)$

$$\Rightarrow \qquad du = c_v^{ig}\, dT \qquad \text{and} \qquad c_v^{ig} = c_v^{ig}(T)$$

$$\Rightarrow \qquad dh = c_p^{ig}\, dT \qquad \text{and} \qquad c_p^{ig} = c_p^{ig}(T)$$

and $\qquad Pv = RT$

$$du = Tds - pdv \qquad \Rightarrow \qquad ds = \frac{du}{T} + \underbrace{\cancel{\frac{p}{T}}}_{= R/v} dv$$

(ideal gas)

$$\Rightarrow \qquad ds = \frac{c_v^{ig}(T)}{T} dT + R \cdot d\ln(v)$$

$$\Rightarrow \qquad s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} \frac{c_v^{ig}(T)}{T} dT + R \cdot \ln\left(\frac{v_2}{v_1}\right)$$

Entropy change of an ideal gas

For an ideal gas, $u(T,V)$ and $h(T,P)$ are functions of T only; $u(T)$ and $h(T)$

$\Rightarrow \quad du = c_v^{ig}\,dT$ and $c_v^{ig} = c_v^{ig}(T)$

$\Rightarrow \quad dh = c_p^{ig}\,dT$ and $c_p^{ig} = c_p^{ig}(T)$

and $\quad Pv = RT$

$$dh = Tds + vdp \quad \Rightarrow \quad ds = \frac{dh}{T} - \underbrace{\frac{v}{T}}_{=R/p} dp$$

(ideal gas)

$$\Rightarrow \quad ds = \frac{c_p^{ig}(T)}{T}\,dT - R \cdot d\ln(p)$$

$$\Rightarrow \quad s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} \frac{c_p^{ig}(T)}{T}\,dT - R \cdot \ln\left(\frac{p_2}{p_1}\right)$$

Voorbeeld: opgave 6.33

- Lucht wordt gecomprimeerd van een toestand met druk 0.1 MPa en temperatuur 27°C naar een toestand met druk 0.5 MPa en temperatuur 177 °C
- Kan dit proces adiabatisch verlopen?
- In geval het antwoord positief is, bereken de arbeid per kg lucht, in kJ/kg Anders, bepaal de richting van de warmteoverdracht

$$s(T_2, p_2) - s(T_1, p_1) = s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1}$$

$$T_1 = 300.15K \qquad Tabel\_A22 \;\Rightarrow\; s^o(T_1 = 300.15K) = 1.70253\frac{kJ}{kg\,K}$$

$$T_2 = 450.15K \qquad Tabel\_A22 \;\Rightarrow\; s^o(T_2 = 450.15K) = 2.11195\frac{kJ}{kg\,K}$$

$$s(T_2, p_2) - s(T_1, p_1) = (2.11195 - 1.70253)\frac{kJ}{kg\,K} - \frac{8.314\dfrac{kJ}{kmol\,K}}{28.97\dfrac{kg}{kmol}}\ln\left(\frac{0.5MPa}{0.1MPa}\right)$$

$$= -0.05247\frac{kJ}{kg\,K}$$

Er wordt warmte afgevoerd

waarom ?

Entropy of incompressible liquids & solids

$$du = Tds - pdv \quad \Rightarrow \quad ds = \frac{du}{T} + \frac{p}{T}\underbrace{\cancel{dv}}_{=0}$$

$$du = \underbrace{\cancel{\left(\frac{\partial u}{\partial T}\right)_v}}_{=c_v(T,v)=c_v(T)\cong c_p(T)} dT + \left(\frac{\partial u}{\partial v}\right)_T \overbrace{\cancel{dv}}^{=0} \quad \Rightarrow \quad du \cong c_p(T)dT$$

incompressible

$$\Rightarrow \quad ds = \frac{c_p(T)}{T}dT$$

$$\Rightarrow \quad s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} \frac{c_p}{T}dT$$

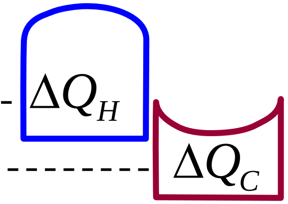
$$\Rightarrow \quad s(T_2, p_2) - s(T_1, p_1) = s^0(T_2) - s^0(T_1)$$

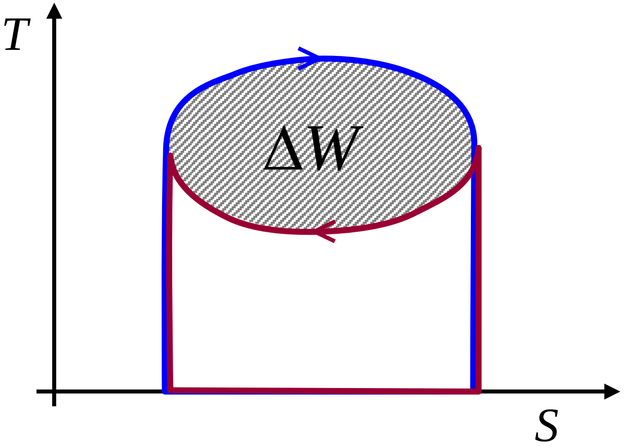
where $\{p_1, p_2\}$ is within the range, where substance is assumed incompressible

T-S Diagram

- we have seen, that the entropy is a state variable – a given state-point (T,V) or (T,P) or (P,V) for pure components one-to-one defines the entropy S at this state
- the entropy can be calculated from the Gibbs equation

- Given this, reversible cycle processes can be evaluated in T-S diagrams. The advantage of such diagrams are
  - that $\Delta Q_H = \int TdS$ holds for the upper part of a cycle process
  - and $\Delta Q_C = \int TdS$ holds for the lower part of a cycle process
  - the work of the cycle is obtained from $\Delta W = \Delta Q_H - \left|\Delta Q_C\right|$
  - an adiabatic reversible process is, because of $\delta Q = TdS = 0\,kJ$ an isentropic process and is represented as a vertical line



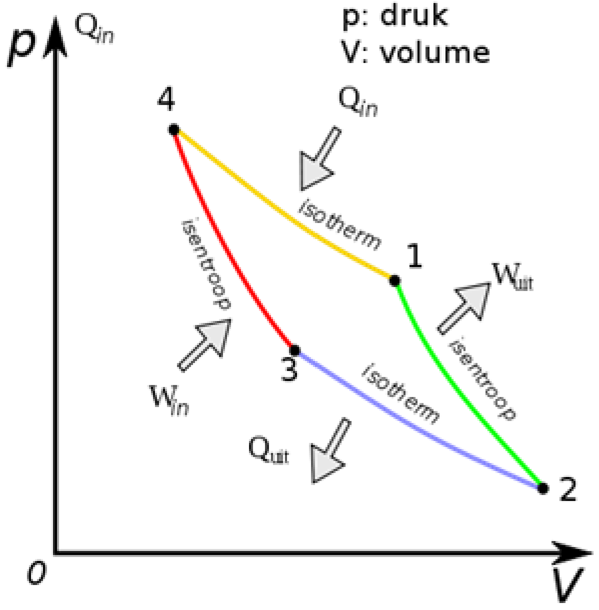


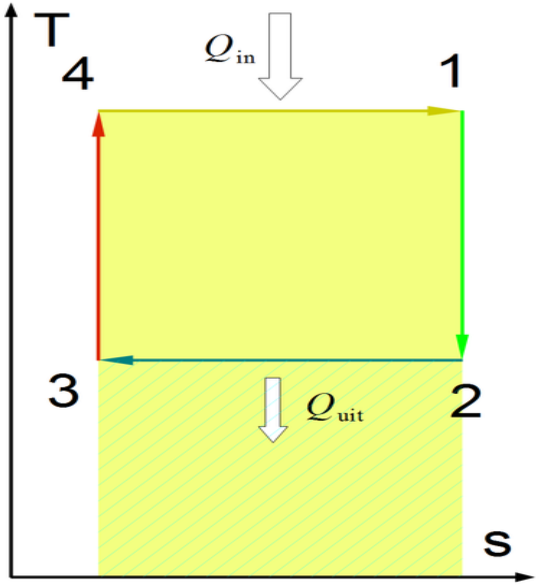
T-S versus P-V diagram

- Arbeidsdiagram p versus V
- Warmtediagram T versus S

$$\delta W = pdV$$

$$\delta Q = TdS$$





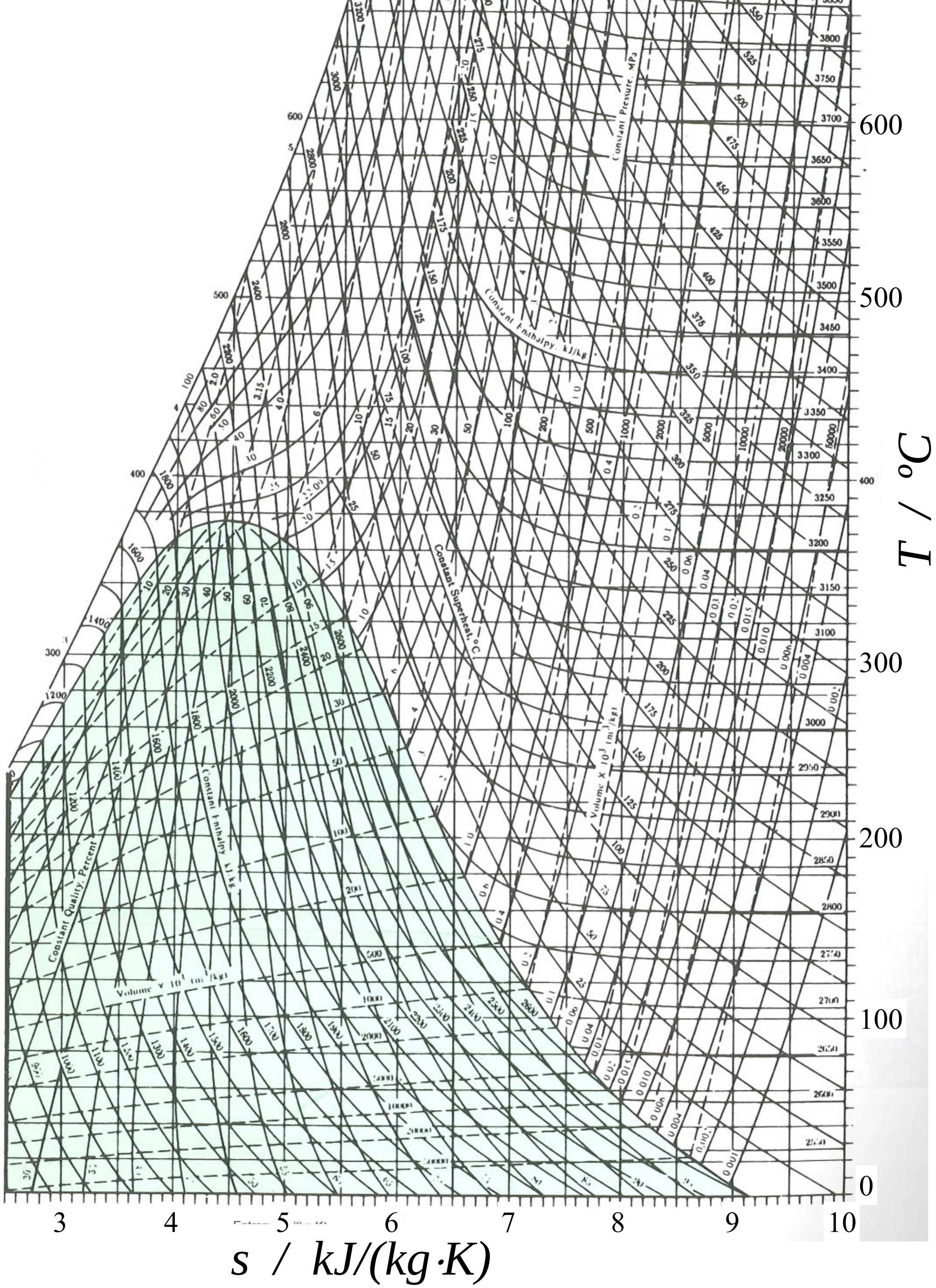
Adiabatisch reversibel proces is isentroop, dus S=constant

Oppervlakken zijn precies even groot omdat

$$Q_{cyc} = W_{cyc}$$

$$\int TdS = \int pdV$$

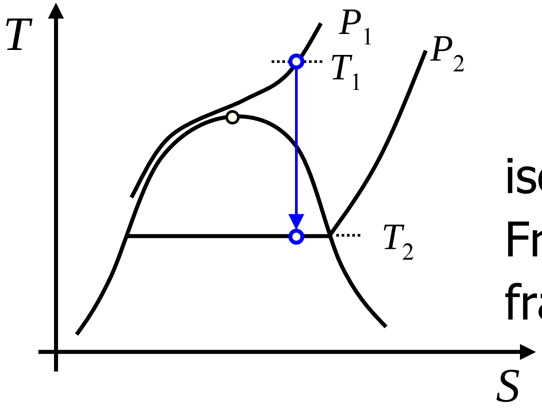
T-S Diagram


T / °C
600
500
300
200
100
0
3
4
5
6
7
8
9
10
s / kJ/(kg·K)
Constant Pressure, MPa
Constant Enthalpy, kJ/kg
Constant Superheat, °C
Volume × 10³ (m³/kg)
Constant Quality, Percent
Constant Enthalpy, kJ/kg
Volume × 10³ (m³/kg)

Adiabatic reversible processes ⇒ Isentropic processes

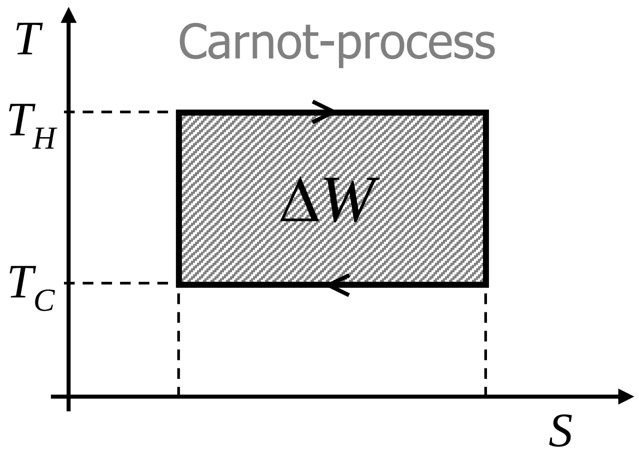
- isentropic = ‘with constant entropy S’
- second law for adiabatic reversible processes $dS = \underbrace{\cancel{\frac{\delta Q}{T}}}_{=0} + \underbrace{\cancel{\delta\sigma}}_{=0} \Rightarrow S=const.$

adiabatic reversible



isentropic expansion from pressure p_1 to p_2. From T-s-diagram, one obtains T_2 and vapor fraction x^{vap} (“quality”) $s = (1 - x^{vap})s_f + x^{vap}s_g$

Carnot process in a *T*-*S* diagram



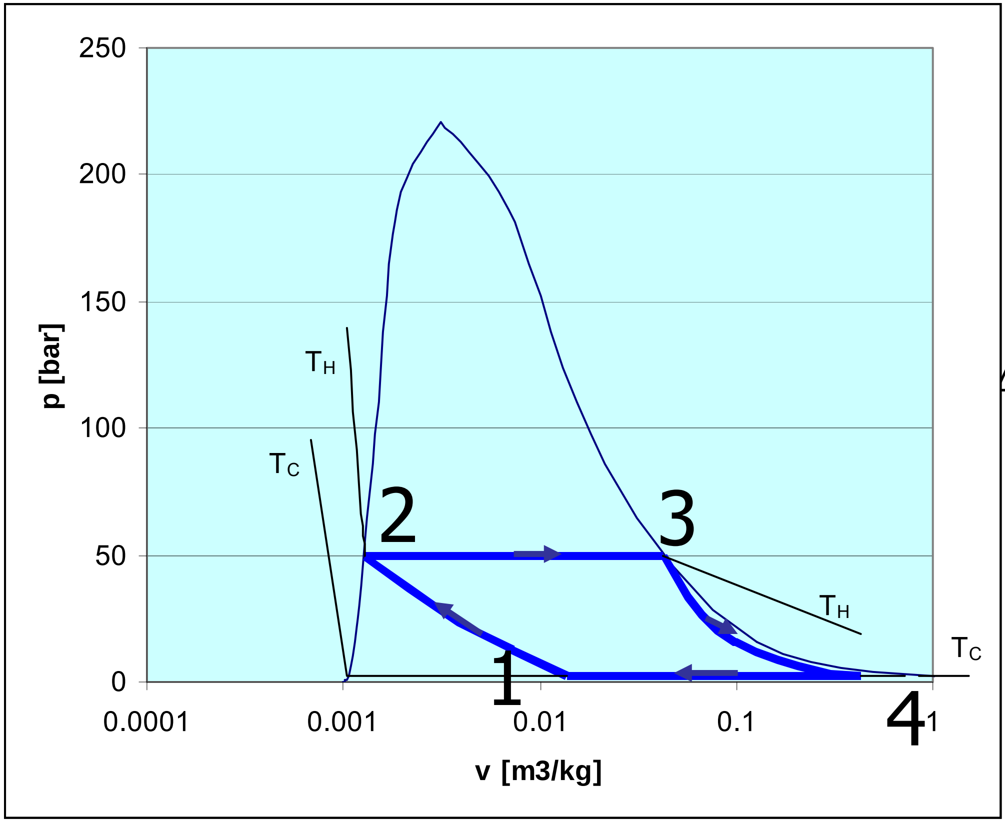
The Carnot process is composed of 2 isothermal and 2 adiabatic process steps

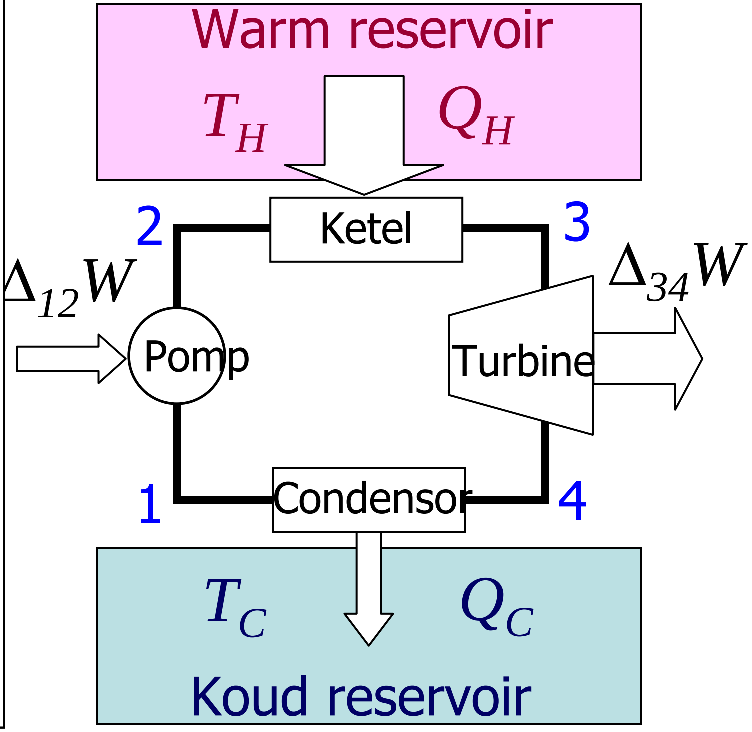
In a T-S diagram they are represented by two horizontal and 2 vertical process steps, respectively

It is graphically clear, that for a given T_H and given T_C, no cycle process can have a higher thermal efficiency than the Carnot process

Two other cycle processes will turn out to also have the Carnot efficiency – both, of course, have 2 isothermal process steps

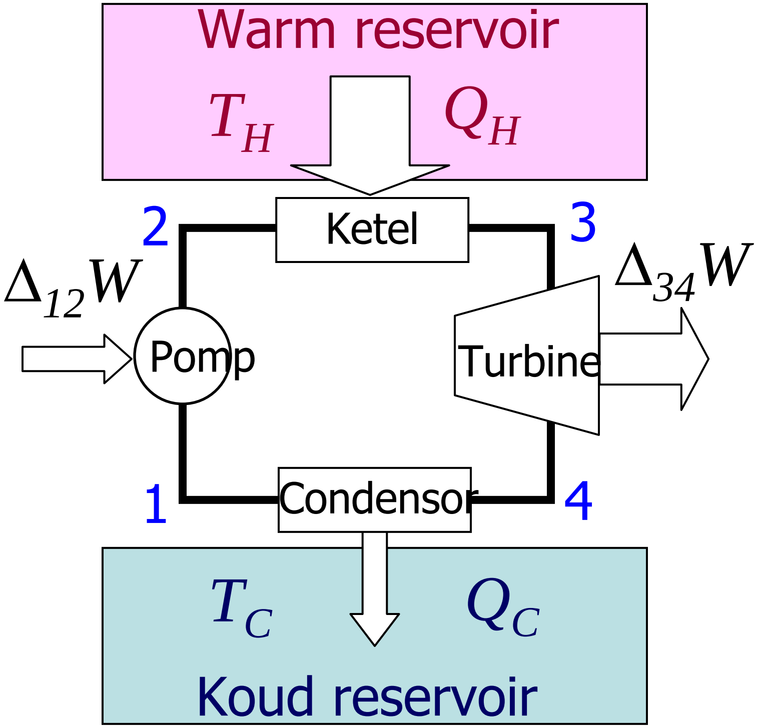
Carnot waterdamp-vermogenskringproces





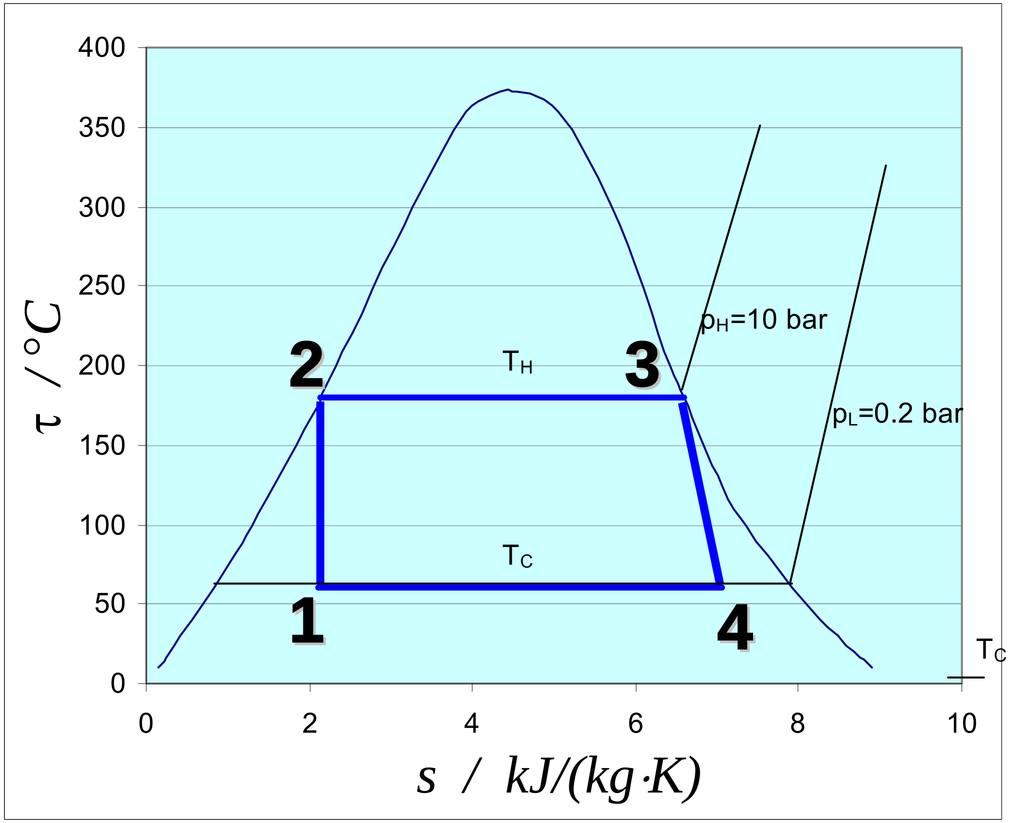
Voorbeeld: opgave 6.24

- De figuur is een schema van een waterdamp vermogenskringproces waarin water door de vier componenten stroomt. Het water stroomt door ketel en condensor met constante druk en stroomt adiabatisch door turbine en pomp.
- Schets het kringproces in een T-s diagram.;
- Bepaal het thermisch rendement van een Carnot vermogenskringproces dat tussen dezelfde minimum en maximum temperaturen opereert.



2 verzadigd vloeistof met 10 bar
3 verzadigd damp met 10 bar
4 x=88%; 0.2 bar
1 x=18%; 0.2 bar.

Voorbeeld: opgave 6.24



Tabel A-3

2 τ=179.9°C; s=2.1387 kJ/kgK

3 τ=179.9°C; s=6.5863 kJ/kgK

4 τ=60.06°C;
s=(0.88*7.9085 + (1–0.88)*
0.8320) kJ/kgK =7.059 kJ/kgK

1 τ=60.06°C;
s=(0.18*7.9085 + (1–0.18)*
0.8320) kJ/kgK =2.106 kJ/kgK

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{60.06 + 273}{179.9 + 273} = 0.27$$

$$\frac{Q_C}{Q_H} = \frac{T_H \Delta S}{T_C \Delta S} = .$$

Aanwijzigingen voor zelfstudie

- H6.1 – 6.4 en 6.5.1 – 6.5.4 zijn behandeld. Goed doorlezen.

- Maak opgaven 6.1, 6.7, 6.9
- Maak enkele van de opgaven 6.18 – 6.25
- Maak opgave 6.28, 6.29, 6.35 en 6.30 + 6.31 (er is een tikfout in opgave 6.31, bedoeld is … reconsider the data of problem 6.30…)
- Maak opgaven 6.38