wb1224 - Thermodynamics 2
Lecture 12 – Exergy Analysis 1

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Content
Lecture 12 - overview

- Definition and evaluation of exergy

- Examples

  ➢ Improving the use of energy in a chemical process

  ➢ Geothermally driven cooling system

- Control volume exergy analysis: exergy balance

Definition of exergy

Generalizing from the previous
example on the Available Energy

- **DEFINITION:** EXERGY is the maximum theoretical work that can be obtained from a system as it interacts with the environment and reaches equilibrium

- Environment: if the system is in equilibrium with it ➜ NO work

- Environment in practical terms: at a certain distance from the system the environment is characterized by T_0 and P_0

- T_0 and P_0 do not change because of the system – env. interaction

Evaluation of exergy (1)

- From its definition: closed volume, system combined with surrounding, maximum work

- Volume is large enough: heat transfer between system and environment does not affect the latter

- System's volume can vary but not the total volume (sum)

- Interaction: initial state and final state
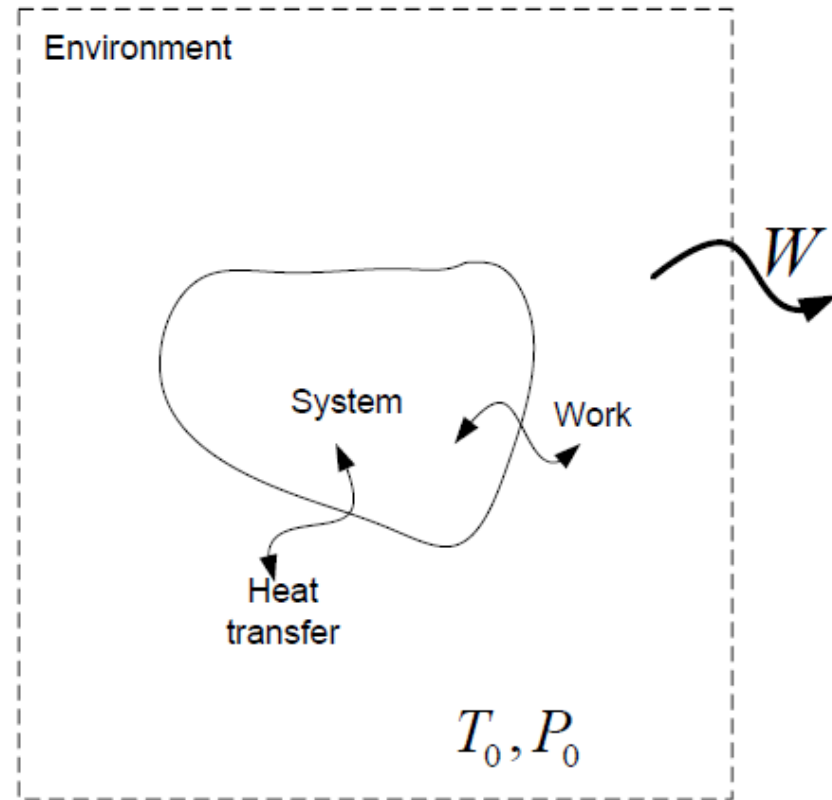
Evaluation of exergy (2)

- First law for combined system

- Energy of the system

- Variation of the energy of the total CV

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Evaluation of exergy (3)

- Variation of the total energy of the environment

- Variation of the total energy of the system
  (equilibrium in the final state ➜ only internal energy)

- Variation of the total energy of the CV

Evaluation of exergy (4)

- Variation of internal energy of the environment ΔU_{env}
  (from Gibbs equation $du = Tds - Pdv$)

- Variation of total energy for the control volume becomes

- Work for the control volume ($W_{tot} = - \Delta E_{tot}$)

- Change in volume of the environment

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Evaluation of exergy (5)

- W_{tot} becomes

- Entropy balance for the control volume

- Entropy production

- Entropy change of the system

- Entropy change of the environment

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Evaluation of exergy (6)

- Work is therefore

- Definition of exergy ➔ $\mathscr{P}_s = 0$

- Depends only on initial and final state

- Irreversibilities $T_0\mathscr{P}_s$ depends on the process (path from initial to final)

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Summary

$$\Delta E_{\mathrm{TOT}} = \cancel{\varphi}_{\mathrm{TOT}} - W_{\mathrm{TOT}}$$

$$E_{\mathrm{SYS}} = U_{\mathrm{SYS}} + E_{\mathrm{SYS}}^{\mathrm{KN}} + E_{\mathrm{SYS}}^{\mathrm{PT}}$$

convention (calculus): $\Delta E \equiv E_{\mathrm{final}} - E_{\mathrm{initial}}$

$$\Delta E_{\mathrm{TOT}} = \Delta E_{\mathrm{ENV}} + \Delta E_{\mathrm{SYS}}$$

$$\Delta E_{\mathrm{ENV}} = \Delta U_{\mathrm{ENV}} \quad \left(\Delta E_{\mathrm{ENV}}^{\mathrm{KN}} = 0; \Delta E_{\mathrm{ENV}}^{\mathrm{PT}} = 0 \right)$$

$$\Delta E_{\mathrm{SYS}} = U_0 - E_{\mathrm{SYS}}$$

$$\Delta E_{\mathrm{TOT}} = \Delta U_{\mathrm{ENV}} + \left(U_0 - E_{\mathrm{SYS}} \right)$$

$$\Delta U_{\mathrm{ENV}} = T_0 \Delta S_{\mathrm{ENV}} - P_0 \Delta V_{\mathrm{ENV}}$$

$$\Delta E_{\mathrm{TOT}} = \left(U_0 - E_{\mathrm{SYS}} \right) + T_0 \Delta S_{\mathrm{ENV}} - P_0 \Delta V_{\mathrm{ENV}}$$

$$W_{\mathrm{TOT}} = -\Delta E_{\mathrm{TOT}} = \left(E_{\mathrm{SYS}} - U_0 \right) - \left(T_0 \Delta S_{\mathrm{ENV}} - P_0 \Delta V_{\mathrm{ENV}} \right)$$

$$\Delta V_{\mathrm{ENV}} = -\left(V_{\mathrm{SYS}} - V_0 \right)$$

$$W_{\mathrm{TOT}} = \left(E_{\mathrm{SYS}} - U_0 \right) + P_0 \left(V_{\mathrm{SYS}} - V_0 \right) - T_0 \Delta S_{\mathrm{ENV}}$$

$$\Delta S_{\mathrm{TOT}} = \mathscr{P}_s$$

$$\mathscr{P}_s = \Delta S_{\mathrm{SYS}} + \Delta S_{\mathrm{ENV}}$$

$$\Delta S_{\mathrm{SYS}} = \left(S_0 - S_{\mathrm{SYS}} \right)$$

$$\mathscr{P}_s = \left(S_0 - S_{\mathrm{SYS}} \right) + \Delta S_{\mathrm{ENV}}$$

$$\Delta S_{\mathrm{ENV}} = \mathscr{P}_s - \left(S_0 - S_{\mathrm{SYS}} \right)$$

$$W_{\mathrm{TOT}} = \left(E_{\mathrm{SYS}} - U_0 \right) + P_0 \left(V_{\mathrm{SYS}} - V_0 \right)$$

$$- T_0 \left(S_{\mathrm{SYS}} - S_0 \right) - T_0 \mathscr{P}_s$$

$$\boxed{\mathcal{E}_{SYS} = \left(E_{\mathrm{SYS}} - U_0 \right) + P_0 \left(V_{\mathrm{SYS}} - V_0 \right) - T_0 \left(S_{\mathrm{SYS}} - S_0 \right)}$$

Forms of exergy

- Specific exergy function (not a function of state)

$$e = \left(e - u_0\right) + P_0\left(v - v_0\right) - T_0\left(s - s_0\right)$$

- Where

$$e = u + \frac{v^2}{2} + gz$$

- Physical exergy, potential exergy, kinetic exergy

$$e_{\text{SYS}}^{\text{PH}} = \left(u - u_0\right) + P_0\left(v - v_0\right) - T_0\left(s - s_0\right); \quad e_{\text{SYS}}^{\text{KN}} = \frac{v^2}{2}; \quad e_{\text{SYS}}^{\text{PT}} = gz$$

Example 1: improving the use of energy in a chemical process

- A chemical process needs 2 kg/sec steam at 1 MPa and 320 °C

- Nearby power plant: steam generator (2 MPa, 600 °C)

- First solution: spill the steam using a throttle valve and cool it down with a heat exchanger...not that smart!
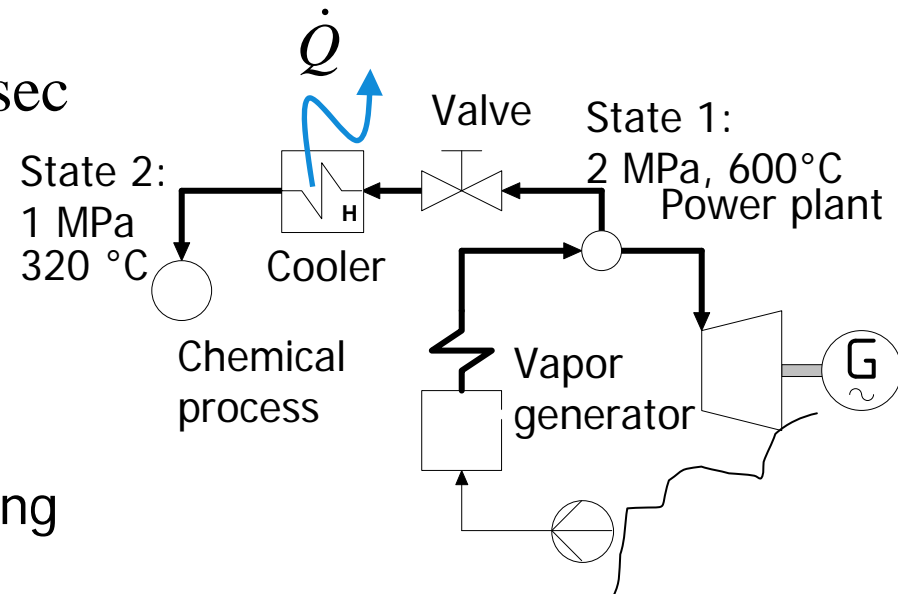
$$T_0 = 20 \text{ °C} = 293.15 \text{ K}$$
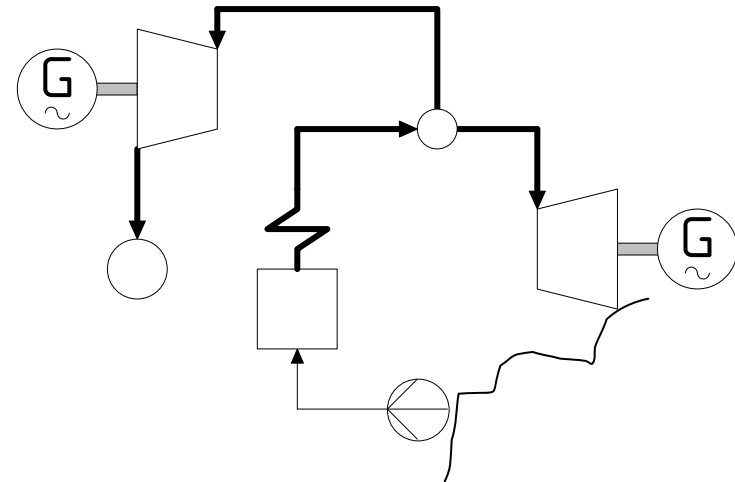
How much power could be obtained?

Enthalpy and entropy of state 1 and state 2:

Taking $T_0 = 20\ °C = 293.15\ K$, exergy of state 1 and state 2:

Maximum power

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Better solution

- Turbine is 75% efficient

- At 0.1 €/kWh and utilization of 70%

- Value of electricity <u>per year</u> is 420 k€

- Decision: maintenance cost and security of supply

Example 2: geothermally driven cooling system

- Remote desert area: geothermal well makes 50 kg/h saturated steam at $150 °C$.

- $T_{env} = 45 °C$.

- Cooling for homes at $23 °C$, steam from system is re-injected as condensate at 1 bar
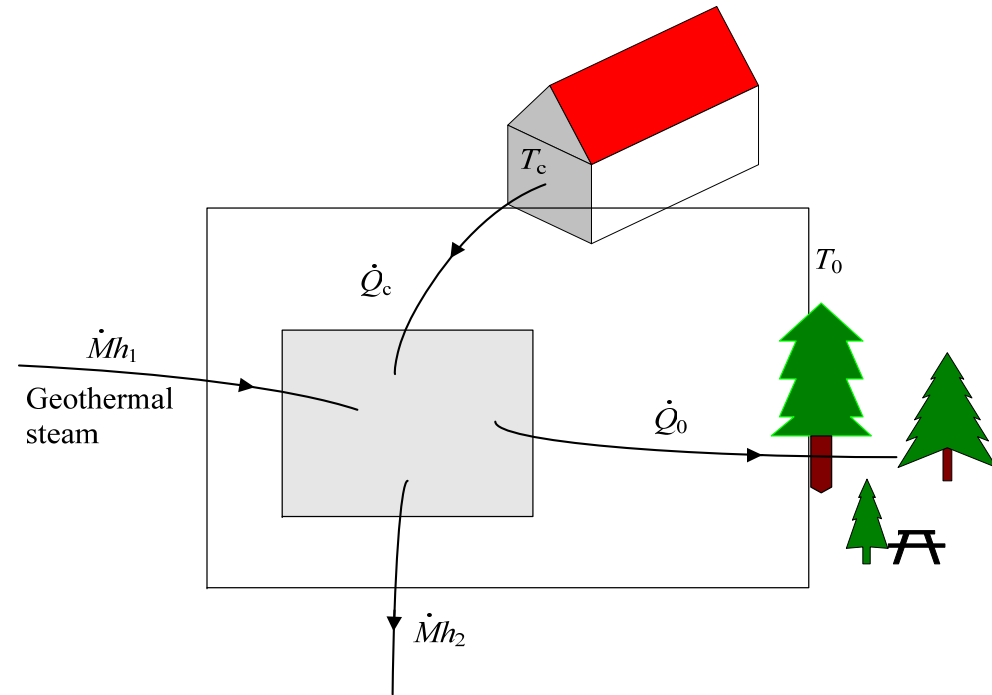
- Maximum cooling rate?

TUDelft
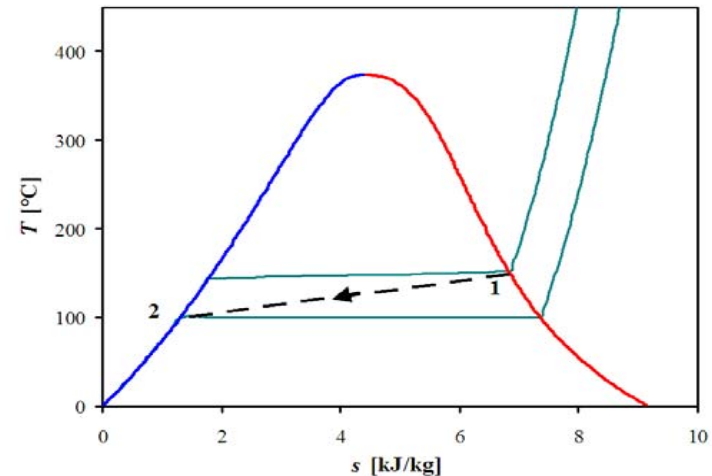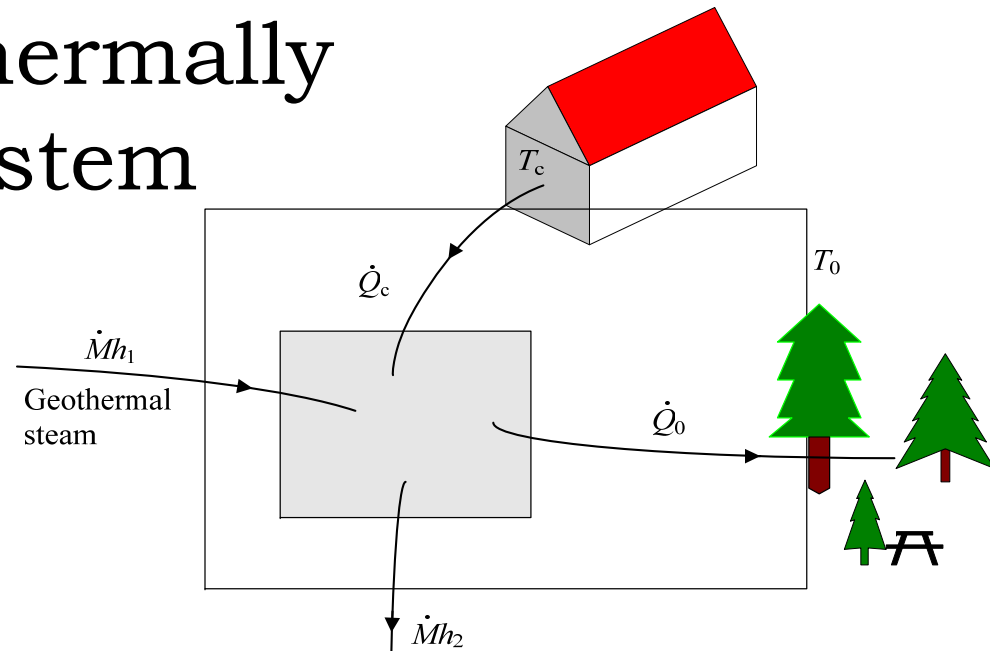
Problem setup

- Control volume definition

- Assumptions:

  ➢ Steady flow, steady state

  ➢ One-dimensional flow at the inlet and outlet

  ➢ Neglect potential and kinetic energy changes of the flow
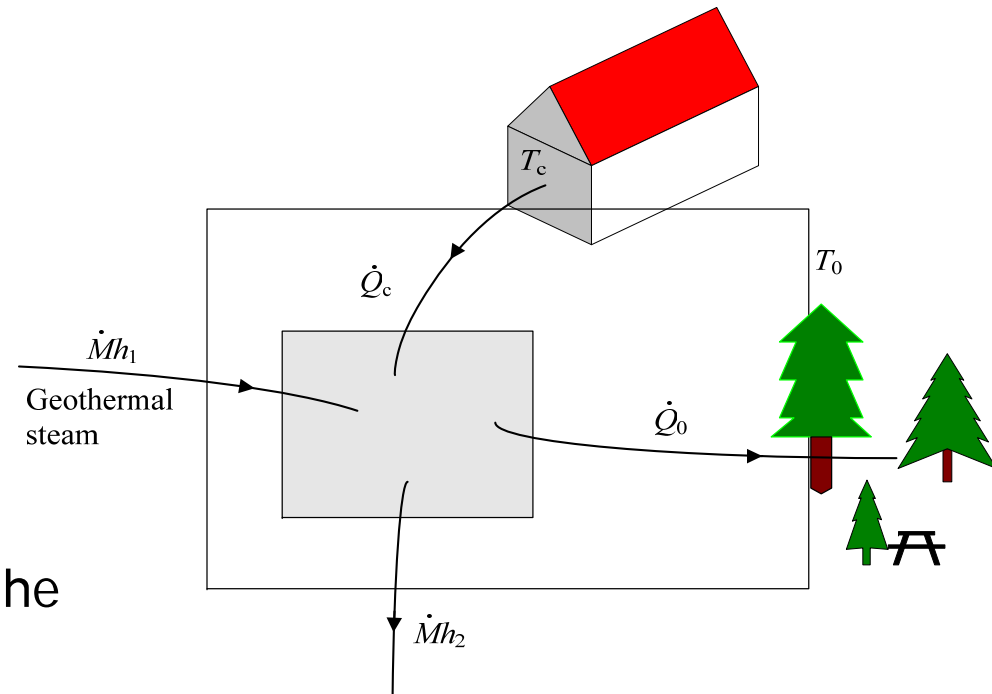
Exergy analysis

- Energy balance

- Entropy balance

- Cooling rate, maximum cooling rate

Calculation of the value

- State 1 ($h, s, \boldsymbol{e} = h - T_0\,s$):

- State 2:

- Maximum cooling rate

- Realistic cooling rate (50% of maximum)

Note: maximum performance does not depend on the process!

# Control volume exergy analysis

- General case (unsteady, time variant)

- CV exchanging mechanical power and transferring heat with the environment $(T_b > T_0, ; T_b = \text{const.})$

- Apply first and second law of thermodynamics

$$\dot{M}_1$$
$$(e + Pv)_1$$
$$s_1$$

$$T_b$$

$$\dot{W}$$

$$\dot{Q}_b$$

$$\dot{M}_2$$
$$(e + Pv)_2$$
$$s_2$$

$\widetilde{T}U$Delft
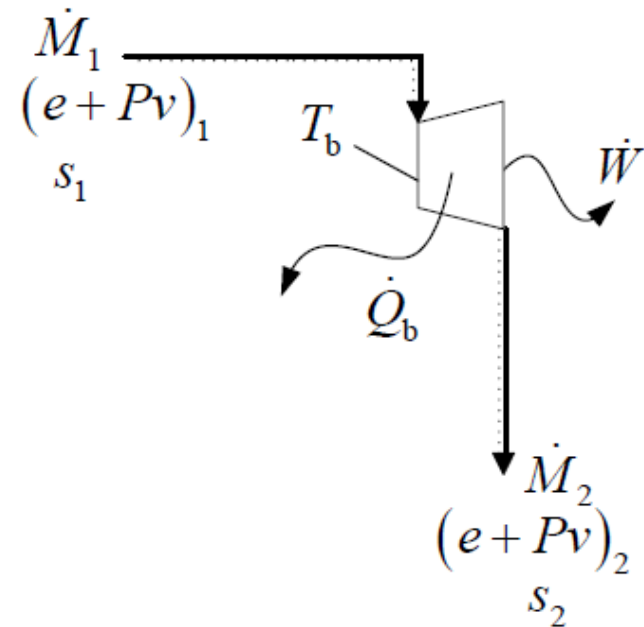
Application of first and second law of thermodynamics to CV

$$\underbrace{\frac{dE_{CV}}{dt}}_{\substack{\text{rate of energy} \\ \text{accumulation}}} = \underbrace{(e+Pv)_1 \dot{M}_1}_{\text{rate of energy input}} - \underbrace{\left[(e+Pv)_2 \dot{M}_2 + \dot{W} + \dot{Q}\right]}_{\text{rate of energy output}}$$

$$\underbrace{\frac{dS_{CV}}{dt}}_{\substack{\text{rate of entropy} \\ \text{accumulation}}} = \underbrace{\dot{M}_1 s_1}_{\substack{\text{rate of entropy} \\ \text{input}}} - \underbrace{\left(\dot{M}_2 s_2 + \frac{\dot{Q}}{T_b}\right)}_{\substack{\text{rate of entropy} \\ \text{output}}} + \underbrace{\dot{\mathscr{P}}_s}_{\substack{\text{rate of entropy} \\ \text{production}}}$$
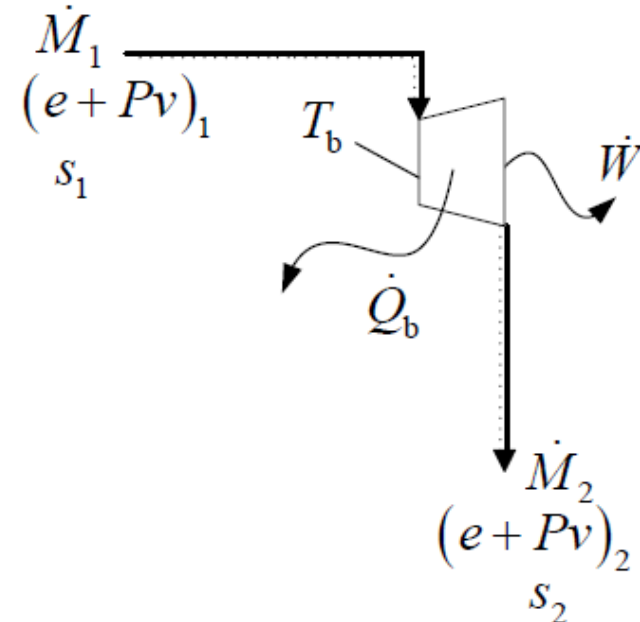
Subtract entropy balance multiplied by T_0 from the energy balance

$$\frac{dE_{CV}}{dt} - T_0 \frac{dS_{CV}}{dt} = (e+Pv)_1 \dot{M}_1 - \left[(e+Pv)_2 \dot{M}_2 + \dot{W} + \dot{Q}\right]$$

$$- T_0 \left[\dot{M}_1 s_1 - \left(\dot{M}_2 s_2 + \frac{\dot{Q}}{T_b}\right) + \dot{\mathscr{P}}_s\right]$$

Rearrange terms and use exergy definition

Definition of exergy of a CV

$$\mathcal{E} \equiv \left(E - U_0 \right) + P_0 \left(V - V_0 \right) - T_0 S$$

Differentiation wtr time

$$\frac{d\mathcal{E}}{dt} - P_0 \frac{dV}{dt} = \frac{dE}{dt} - T_0 \frac{dS}{dt}$$

Therefore

$$\frac{dE}{dt} - T_0 \frac{dS}{dt} = \frac{d\mathcal{E}}{dt} - P_0 \frac{dV}{dt}$$

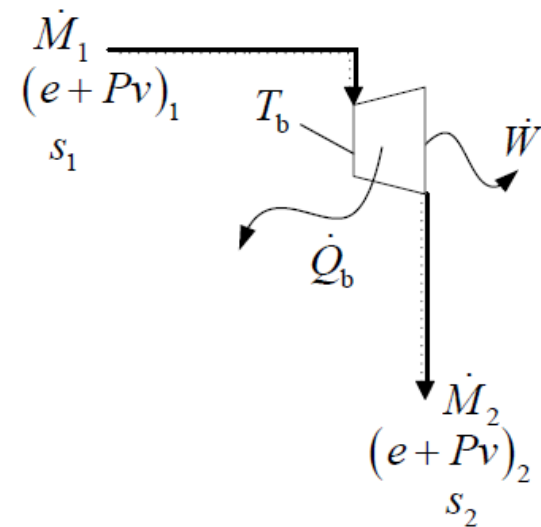$\dfrac{dV}{dt}$ only if the boundary moves in time (piston engine)

$$\frac{d\mathcal{E}_{\mathrm{CV}}}{dt} = \dot{M}_1\left[\left(e+Pv\right)_1 - T_0 s_1\right] - \dot{M}_2\left[\left(e+Pv\right)_1 - T_0 s_1\right]$$

Total specific energy: internal +kinetic + potential

$$-\left(1-\frac{T_0}{T_{\mathrm{b}}}\right)\dot{Q} - \left(\dot{W} - P_0\frac{dV}{dT}\right) - T_0\dot{\mathscr{P}}_s$$

Anticipation: <u>specific flow exergy</u>. It is the maximum potential of performing work by a fluid flow of 1 kg/s

$$e_f \equiv \left(h - h_0\right) + \frac{\boldsymbol{u}^2}{2} + gz - T_0\left(s - s_0\right)$$

Introducing the specific exergy flow

from

$$h \equiv u + Pv \;\; ; \;\; e = u + \frac{\boldsymbol{\omega}^2}{2} + gz \;\; ; \;\; \boldsymbol{e}_f \equiv \left(h - h_0\right) + \frac{\boldsymbol{\omega}^2}{2} + gz - T_0\left(s - s_0\right)$$

we have

$$\left(e + Pv\right) - T_0 s = \boldsymbol{e}_f - \left(h_0 - T_0 s_0\right)$$

$$\frac{d\boldsymbol{\mathcal{E}}_{\mathrm{CV}}}{dt} = \dot{M}_1\left[\boldsymbol{e}_{f,1} - \left(h_0 - T_0 s_0\right)\right] - \dot{M}_2\left[\boldsymbol{e}_{f,2} - \left(h_0 - T_0 s_0\right)\right]$$

$$- \left(1 - \frac{T_0}{T_{\mathrm{b}}}\right)\dot{Q} - \left(\dot{W} - P_0\frac{dV}{dT}\right) - T_0\dot{\boldsymbol{\mathscr{P}}}_s$$

Interpretation of exergy balance for CV

$$\frac{d\mathcal{E}_{\mathrm{CV}}}{dt} = \dot{M}_1 \left[\boldsymbol{e}_{f,1} - \left(h_0 - T_0 s_0 \right) \right] - \dot{M}_2 \left[\boldsymbol{e}_{f,2} - \left(h_0 - T_0 s_0 \right) \right]$$

$$- \left(1 - \frac{T_0}{T_{\mathrm{b}}} \right) \dot{Q} - \left(\dot{W} - P_0 \frac{dV}{dT} \right) - T_0 \dot{\mathcal{P}}_s$$

Maximum amount of mech. power that could be extracted from $\dot{Q}$

➜ Exergy transfer associated with heat transfer

Exergy transfer associated with work.

(consider max work if no mass flows, no heat transfer and reversible process)

$$\dot{\mathcal{E}}_{\mathrm{d}} \equiv T_0 \dot{\mathcal{P}}_s$$

Rate of exergy destruction

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Flow exergy $\dot{\mathcal{E}}_f \equiv \dot{M} e_f = \dot{M}\left[\left(h - h_0\right) + \dfrac{\boldsymbol{u}^2}{2} + gz - T_0\left(s - s_0\right)\right]$
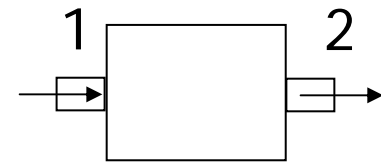
- Exergy per unit time associated with a mass flow of a simple compressible substance

- Interpretation: assume steady-state, no heart transfer. The maximum amount of work that the system can convert occurs for a reversible process, therefore

$$\dot{M}_1 = \dot{M}_2 = \dot{M}$$

$$\dot{\mathscr{P}}_s = 0$$

$$0 = \dot{\mathcal{E}}_{f,1} - \dot{\mathcal{E}}_{f,2} - \dot{W}_{\text{MAX}}$$

Max mech. power?

1   2

- Energy associated with mass transfer: $e + Pv$

Specific flow exergy is exergy associated with mass transfer and flow work

Rate of exergy destruction

- Closely related to entropy production $\dot{\mathcal{E}}_d \equiv T_0 \dot{\mathcal{P}}_s$

- The exergy of a closed system always decreases (II law)

- Represents dissipation of mechanical power, due to the increase of disorder of molecules that always accompanies energy conversion

- Objective of energy systems design: minimize exergy destruction

Energy, entropy, exergy: example

- Steady-state, constant T at the interfaces

- Energy is transferred unaltered

- Because of the temperature difference, some exergy is lost (destroyed)

- Entropy is created
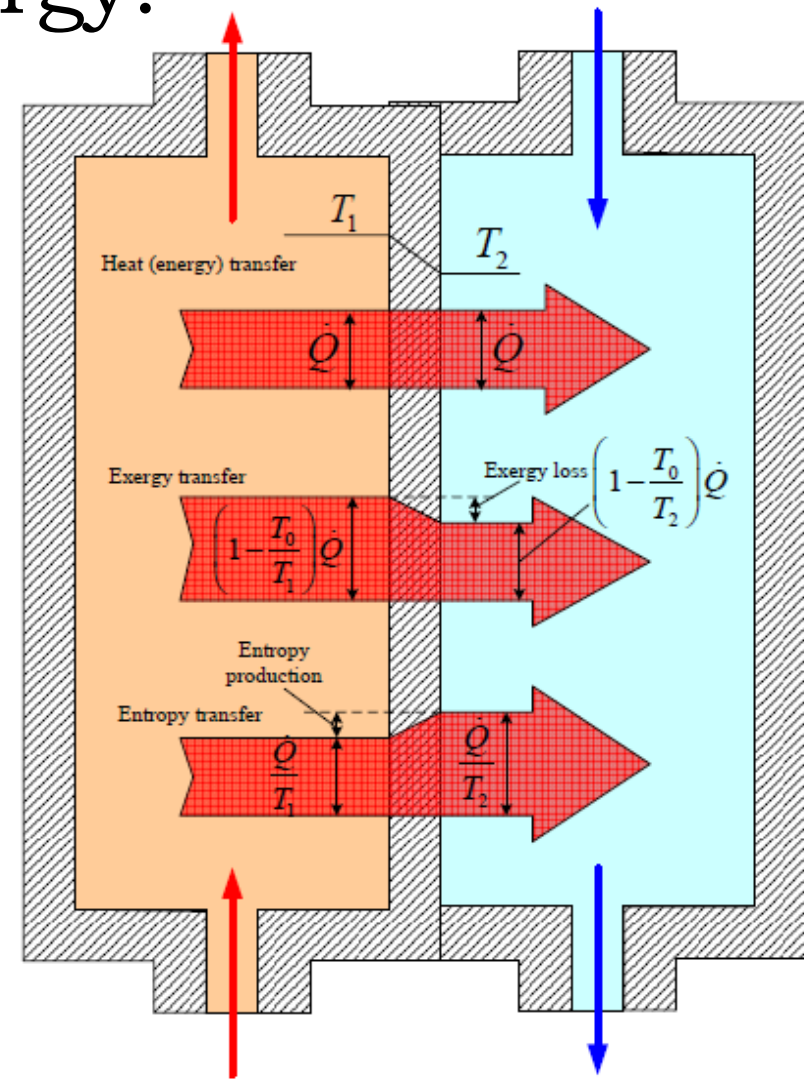
Steady-state exergy balance
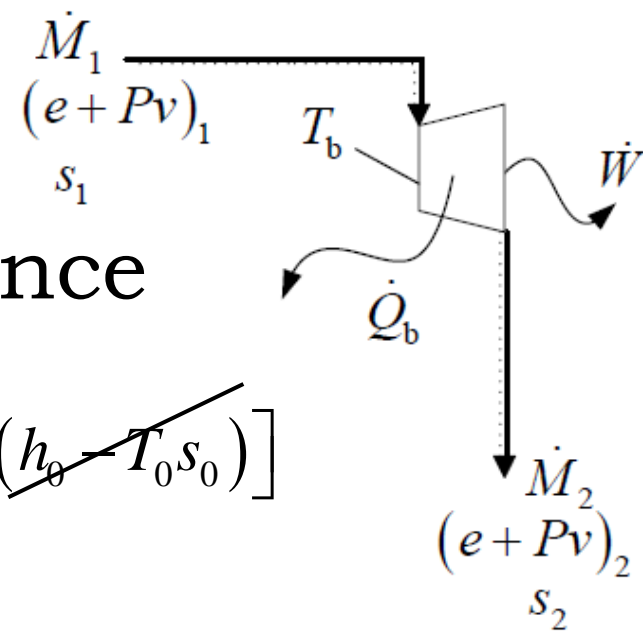
$$\frac{d\mathcal{E}_{cv}}{dt} = \dot{M}_1 \left[e_{f,1} - \left(h_0 - T_0 s_0 \right) \right] - \dot{M}_2 \left[e_{f,2} - \left(h_0 - T_0 s_0 \right) \right]$$

$$- \left(1 - \frac{T_0}{T_b} \right) \dot{Q} - \left(\dot{W} - P_0 \frac{dV}{dT} \right) - T_0 \dot{\mathcal{S}}_s$$

$$\dot{M}_1 = \dot{M}_2 = \dot{M}$$

$$\dot{M} \left(e_{f,1} - e_{f,2} \right) = \left(1 - \frac{T_0}{T_b} \right) \dot{Q} + \dot{W} + \dot{\mathcal{E}}_d$$

$$e_{f,2} - e_{f,1} = \left(h_2 - h_1 \right) - T_0 \left(s_2 - s_1 \right) + \left(\frac{u_2^2}{2} - \frac{u_1^2}{2} \right) + g \left(z_2 - z_1 \right)$$