wb1224 - Thermodynamics 2

Lecture 13 – II-Law efficiency

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Content

Lecture 13 - overview

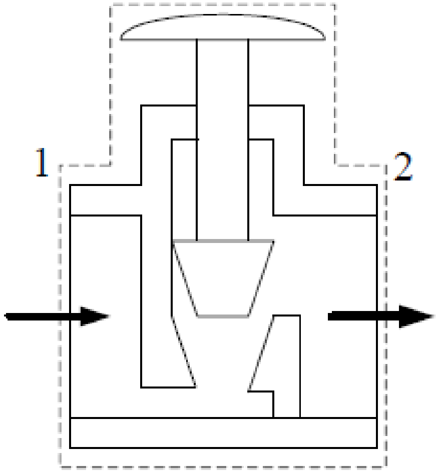
- Example of exergy balance: valve, HX, compressor
- Exergetic efficiency
- Examples of II-law efficiency: domestic heating. HX, turbine, mixer
- Example: exergy analysis of a simple Organic Rankine Cycle power plant

Exergy balance for a throttling valve

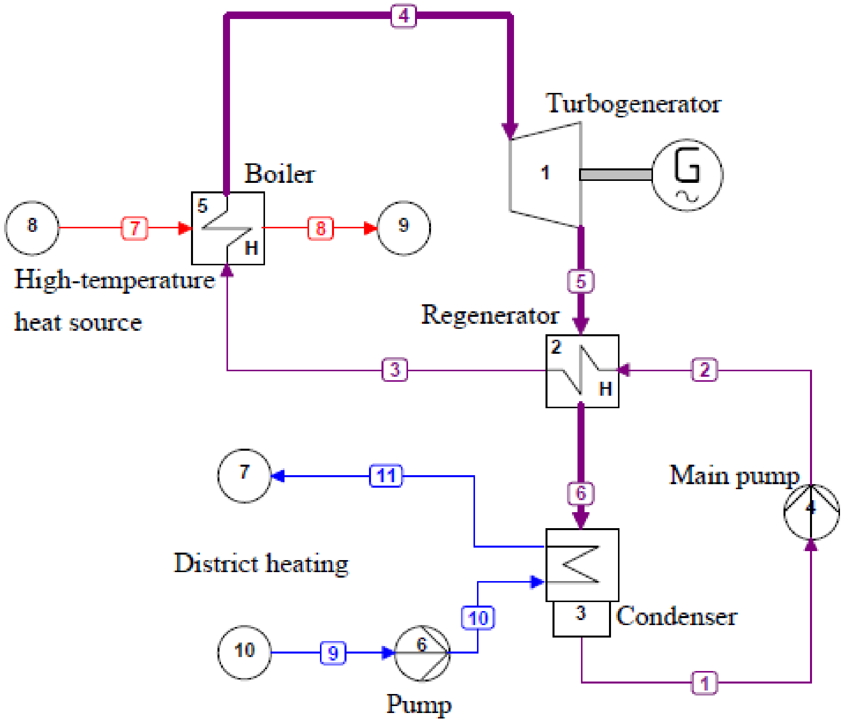
$$\dot{M}\left(e_{f,2}-e_{f,1}\right)=\left(1-\frac{T_0}{T_b}\right)\cancelto{0}{\dot{Q}}-\cancel{\dot{W}}-\dot{\mathcal{E}}_d$$

- All the exergy is wasted
- Difference with a turbine: molecular motion
- Data: CO_2

$$\boxed{\begin{aligned} \dot{M} &= 0.5\ \text{kg/s} \\ T_1 &= 100\ °\text{C} \\ P_1 &= 90\ \text{bar} \\ P_2 &= 50\ \text{bar} \\ T_0 &= 25\ °\text{C} \\ P_0 &= 1.013\ \text{bar} \end{aligned}}$$



Exergy balance for a heat exchanger



- Calculate
  - The rate of exergy destruction
  - Change in flow exergy for both streams
- Perfect insulation
- Data
  - Working fluid: Siloxane MDM

$T_0 = 10\ ^\circ\mathrm{C}$

$P_0 = 1.013\ \mathrm{bar}$

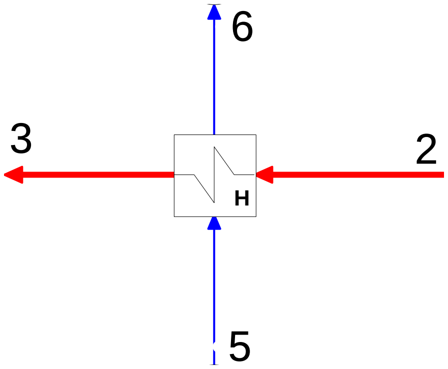
$\dot{M} = 13.5\ \mathrm{kg/s}$

$P_2 = 28.0\ \mathrm{bar}$ $P_5 = 0.8\ \mathrm{bar}$

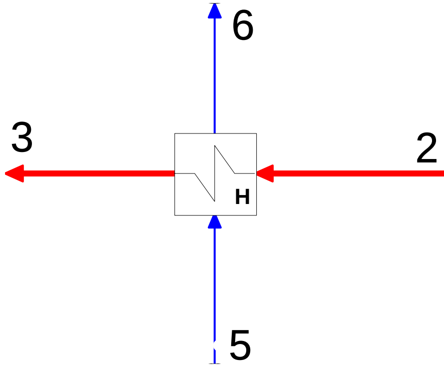
$T_2 = 146.2\ ^\circ\mathrm{C}$ $T_5 = 183.2\ ^\circ\mathrm{C}$

$P_3 = 27.0\ \mathrm{bar}$ $P_6 = 0.8\ \mathrm{bar}$

$T_3 = 161.0\ ^\circ\mathrm{C}$



$$\boxed{\underbrace{\dot{\mathcal{E}}_{f,2}+\dot{\mathcal{E}}_{f,5}}_{\text{rate of exergy input}} = \underbrace{\dot{\mathcal{E}}_{f,2}+\dot{\mathcal{E}}_{f,5}+\left(1-\frac{T_0}{T_\mathrm{b}}\right)\cancel{\dot{Q}}}_{\text{rate of exergy output}} + \underbrace{\dot{\mathcal{E}}_d}_{\text{rate of exergy destruction}}}$$



Comments on results

- Flow exergy increase of the liquid stream is lower than the decrease of the vapor stream
- Exergy destruction (entropy production)➔ heat transfer and flow friction
- Indicator of thermodynamic loss due to heat transfer ➔ temperature difference between hot and cold stream
- Flow friction ➔ Pressure drop in the pipes

Exergy balance for a compressor

- Calculate the rate of exergy destruction
- Constant temperature at the interface with the environment
- Data
  - Working fluid: air

$T_0 = 25\ °\mathrm{C}$ $\quad P_0 = 1.013\ \mathrm{bar}$

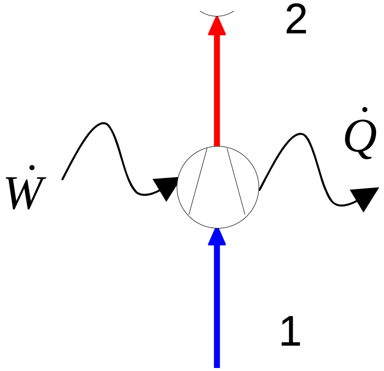
$\dot{M} = 20\ \mathrm{kg/s}$

$\dot{Q} = 200\ \mathrm{kW}$ $\quad T_b = 135\ °\mathrm{C}$

$P_1 = 1.013\ \mathrm{bar}$ $\quad P_2 = 4.0\ \mathrm{bar}$

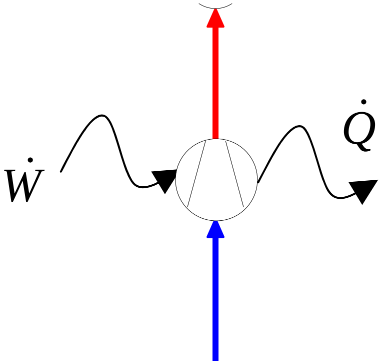
$T_1 = 25\ °\mathrm{C}$ $\quad T_2 = 234.5\ °\mathrm{C}$

$v_1 = 200\ \mathrm{m/s}$ $\quad v_2 = 100\ \mathrm{m/s}$



$$\dot{M}\left(\boldsymbol{e}_{f,2} - \boldsymbol{e}_{f,1}\right) = \dot{W} - \left(1 - \frac{T_0}{T_\mathrm{b}}\right)\dot{Q} - \dot{\mathcal{E}}_d$$

$$\dot{\mathcal{E}}_d = \dot{W} - \dot{M}\left(\boldsymbol{e}_{f,2} - \boldsymbol{e}_{f,1}\right) - \left(1 - \frac{T_0}{T_\mathrm{b}}\right)\dot{Q}$$



Comment on results

Net rate of flow exergy out	3.09 MW	72.6 %
Mechanical power in (exergy rate with mech. power)	4.25 MW	100 %
Thermal power out (exergy rate with heat transfer)	0.05 MW	1.3 %
Rate of exergy destruction (flow friction)	1.11 MW	26.1 %

- Most of the exergy associated with input mechanical power goes into flow exergy
- Some exergy is destroyed within the flow by friction
- Exergy accompanying heat transfer is small

Causes of exergy losses

- Thermo-mechanical systems
- Exergy loss is related to entropy production
- Flow friction: mechanical energy of a fluid in motion is dissipated into heat because of viscosity
- Heat transfer, mixing, chemical reactions

Exergetic efficiency (1)

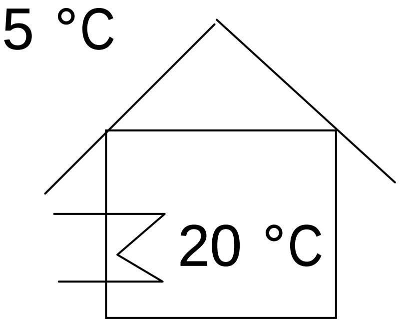
- Efficiency: comparison with an ideal process ➔ reference definition!
- For a thermal power station: Carnot cycle
- For a refrigerator: inverse Carnot cycle
- II-law efficiency (exergetic efficiency): $\boxed{\eta_{\mathrm{II}} = \frac{\eta_{\mathrm{I}}}{\eta_{\max}} \quad ; \quad \eta_{\mathrm{II}} = \frac{\mathrm{COP}}{\mathrm{COP}_{\max}}}$
- If no mech. power⇔thermal power➔reference is process with $\dot{\mathcal{E}}_{\mathrm{d}} = 0$ (e.g. reactor, gasifier)

Exergetic efficiency (2)

- Better index for thermodynamic quality with respect to η_{I}
- With η_{I} one could assume that 100 % available energy $\Rightarrow W_{\mathrm{mech.}}$
- In general

$$\boxed{\eta_{\mathrm{II}} \equiv \frac{\text{Rate of exergy obtained}}{\text{Rate of exergy provided}}}$$

Example: domestic heating (1)

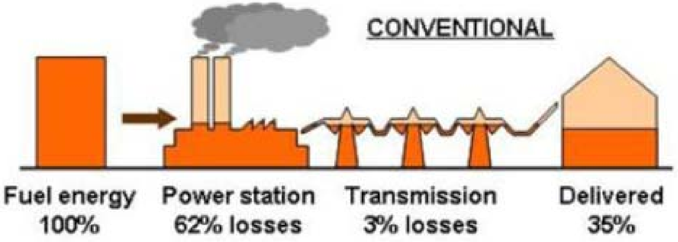


- Comparison between an electric heater and a heat pump
- Outside temperature is 5 °C and thermo-stated temp. 20 °C
- $\mathrm{COP}_{\max} \equiv \mathrm{COP}_{\text{inverse Carnot}} = \dfrac{1}{1-\dfrac{T_0}{T_{\text{set}}}} = \dfrac{1}{1-\dfrac{278.15\ \mathrm{K}}{293.15\ \mathrm{K}}} = 19.5$
- For the electrical heater, neglecting minor losses $\mathrm{COP} = \dfrac{\dot{Q}_{\text{useful}}}{\dot{W}_{\text{needed}}} = 1$

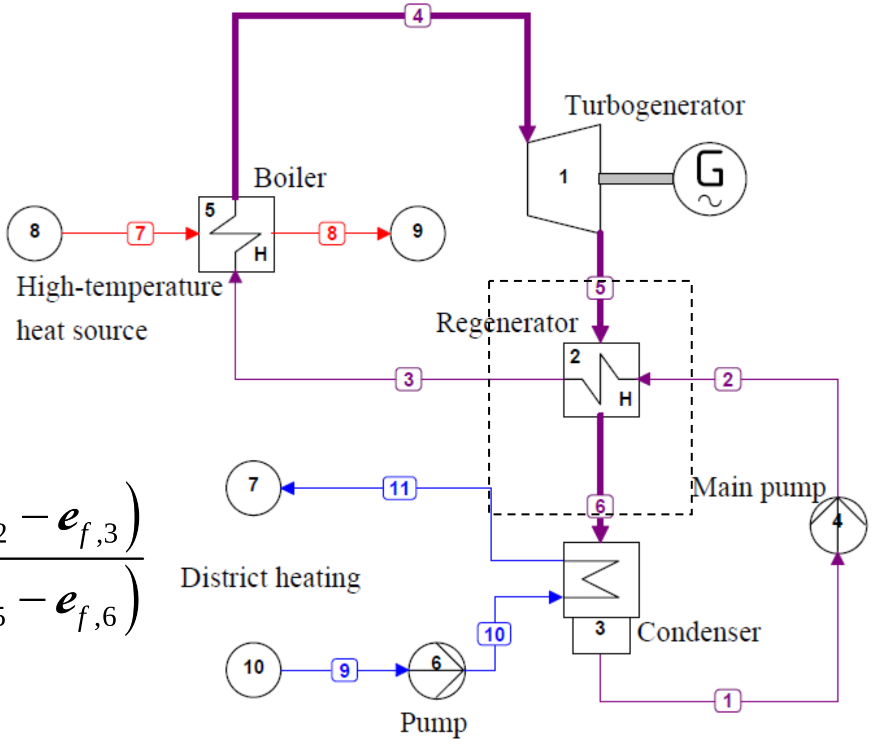
$$\mathrm{COP}_{\text{II,electrical heater}} = \frac{\mathrm{COP}}{\mathrm{COP}_{\max}} = \frac{1}{19.5} = 0.051 = 5.1\%$$

Example: domestic heating (2)

- Typical first-law efficiency of a heat pump for these operating conditions is $\mathrm{COP} = 4.7$
- $$\mathrm{COP}_{\mathrm{II,heat\ pump}} = \frac{\mathrm{COP}}{\mathrm{COP}_{\max}} = \frac{4.67}{19.5} = 0.24 = 24.0\%$$
- Comparison with natural-gas boiler ⇨ the whole chain starting from the fuel must be considered
- Best exergetic solution?

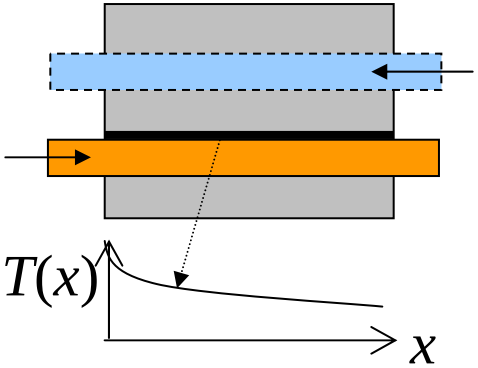


Example: η_{II} of heat exchanger (1)



- Assumptions:
  - Kinetic and potential energy change of streams are negligible
  - Perfectly insulated
  - Steady state
- $$\eta_{\text{II}} = \frac{\text{Rate of exergy obtained}}{\text{Rate of exergy provided}} = \frac{\dot{M}_2\left(e_{f,2} - e_{f,3}\right)}{\dot{M}_5\left(e_{f,5} - e_{f,6}\right)}$$
  $$= \frac{\dot{M}_2\left[\left(h_2 - h_3\right) - T_0\left(s_2 - s_3\right)\right]}{\dot{M}_5\left[\left(h_5 - h_6\right) - T_0\left(s_5 - s_6\right)\right]}$$
- The useful effect is the increase of temperature of the liquid

Equivalent thermodynamic temperature (1)



- Consider one stream of the heat exchanger, variable temperature along pipes ⇨ steady state exergy balance

$$\dot{M}\left(\boldsymbol{e}_{f,\mathrm{in}}-\boldsymbol{e}_{f,\mathrm{out}}\right)=\int_{x_{\mathrm{in}}}^{x_{\mathrm{out}}}\left(1-\frac{T_0}{T(x)}\right)\cdot\dot{q}(x)\,dx$$

- $\dot{q}(x)$ is the heat flux per unit length
- The only source of exergy destruction is flow friction and this is negligible for most applications $\left(\dot{\mathcal{E}}_{\mathrm{d}}\approx 0\right)$
- We can introduce the equiv. thermod. temp. and obtain

$$\boxed{\dot{M}\left(\boldsymbol{e}_{f,\mathrm{in}}-\boldsymbol{e}_{f,\mathrm{out}}\right)=\left(1-\frac{T_0}{\overline{T}}\right)\dot{Q}}$$

Equivalent thermodynamic temperature (2)

- The equivalent thermodynamic temperature is defined as the constant temperature at which the heat transfer should occur in order to generate the same amount of exergy transfer.
- We note that in $\dot{M}\left(e_{f,\text{in}} - e_{f,\text{out}}\right) = \left(1 - \frac{T_0}{\overline{T}}\right)\dot{Q}$ $\quad \dot{Q} = \dot{M}\left(h_{\text{out}} - h_{\text{in}}\right)$
- Therefore we can write

$$\dot{M}\left[\left(h_{\text{out}} - h_{\text{in}}\right) - T_0\left(s_{\text{out}} - s_{\text{in}}\right)\right] = \left(1 - \frac{T_0}{\overline{T}}\right)\dot{M}\left(h_{\text{out}} - h_{\text{in}}\right)$$

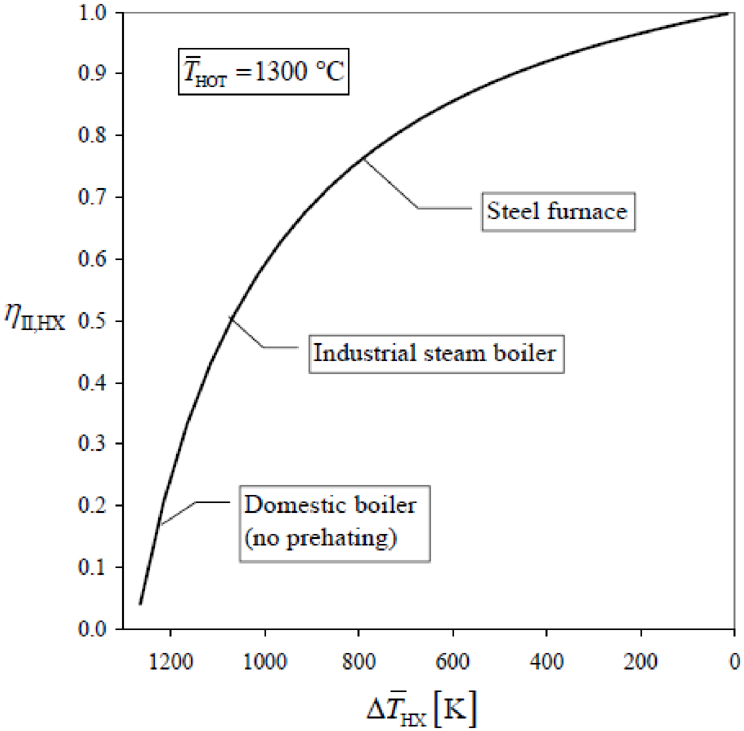
- The equivalent thermodynamic temperature is thus

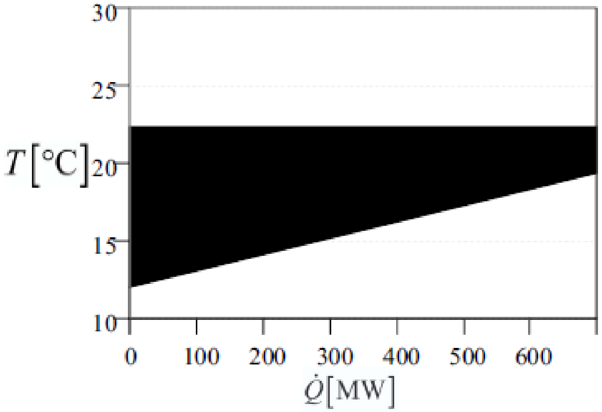
$$\boxed{\overline{T} = \frac{s_{\text{out}} - s_{\text{in}}}{h_{\text{out}} - h_{\text{in}}}}$$

Exergetic efficiency of HX with $\bar{T}$

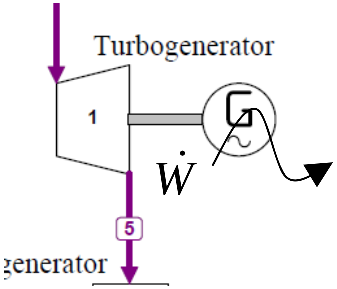
$$\eta_{\mathrm{II,HX}} = \frac{\dot{M}\left(e_{f,\mathrm{in}} - e_{f,\mathrm{out}}\right)_{\mathrm{HOT}}}{\dot{M}\left(e_{f,\mathrm{in}} - e_{f,\mathrm{out}}\right)_{\mathrm{COLD}}} = \frac{\left(1-\dfrac{T_0}{\bar{T}_{\mathrm{HOT}}}\right)}{\left(1-\dfrac{T_0}{\bar{T}_{\mathrm{COLD}}}\right)}$$

The thermodynamic quality of heat transfer depends on average temperature difference





Example: turbine



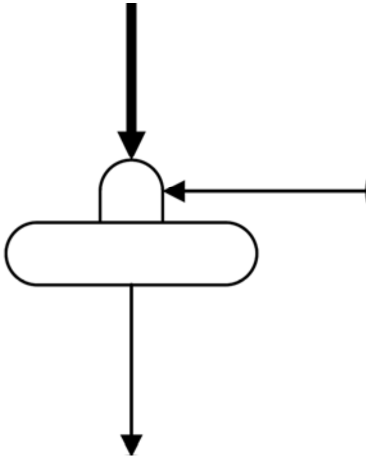
- Assumptions:
  - Kinetic and potential energy change of streams are negligible
  - Perfectly insulated
  - Steady state

$$\dot{M}\left(e_{f,\text{in}} - e_{f,\text{out}}\right) = \dot{W} + \dot{\mathcal{E}}_d$$

$$\eta_{\text{II,turb}} = \frac{\dot{W}}{\dot{M}\left(e_{f,\text{in}} - e_{f,\text{out}}\right)}$$

Exergetic efficiency takes into account the temperature and pressure at which energy is made available (comparison of 2 turbines operating at different T and P)

Example: mixer (dearator)



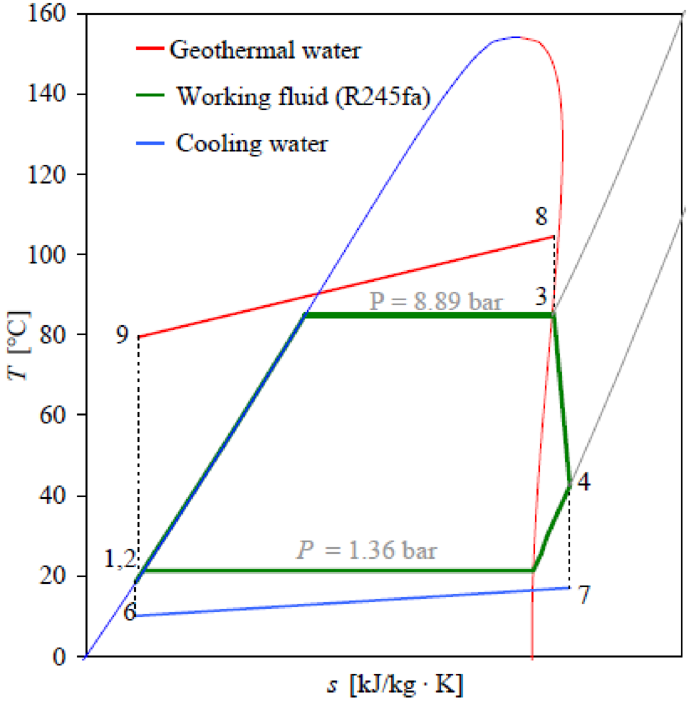
- Common in many processes
- Power plant: dearator (or contact heat exchanger)
- Same assumptions

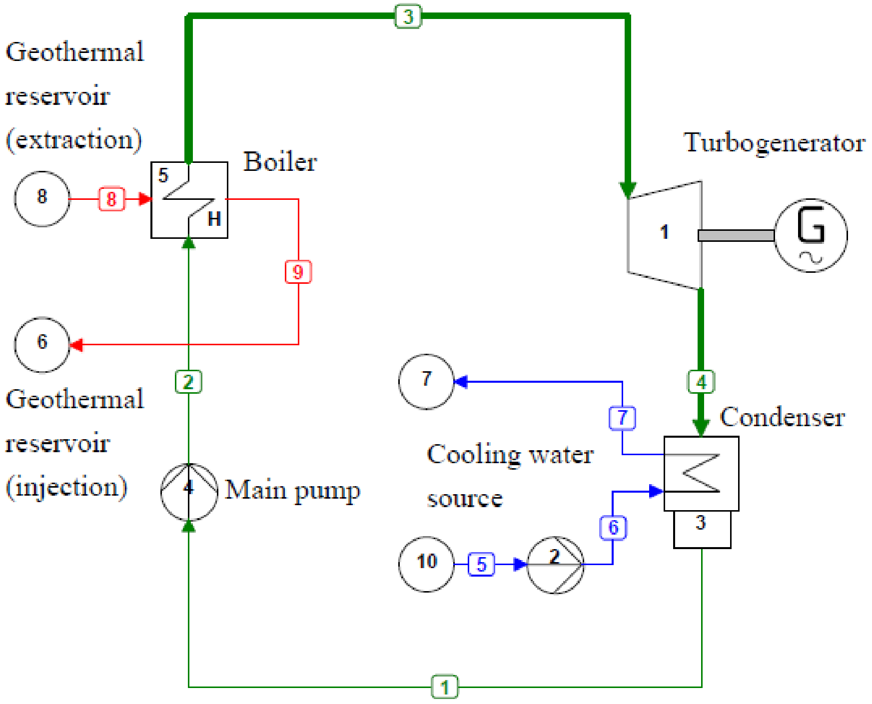
$$\begin{cases} \dot{M}_1 e_{f,1} + \dot{M}_2 e_{f,2} = \dot{M}_3 e_{f,3} + \dot{\mathcal{E}}_d \\ \dot{M}_1 + \dot{M}_2 = \dot{M}_3 \end{cases} \Rightarrow \dot{M}_2\left(e_{f,2} - e_{f,3}\right) = \dot{M}_1\left(e_{f,3} - e_{f,1}\right) + \dot{\mathcal{E}}_\mathrm{d}$$

$$\boxed{\eta_{\mathrm{II,mixer}} = \frac{\dot{M}_1\left(e_{f,3} - e_{f,1}\right)}{\dot{M}_2\left(e_{f,2} - e_{f,3}\right)}}$$

Exergetic efficiency shows how much of the exergy of the hot stream is transferred to the cold stream

Exergy analysis of a simple ORC power plant (1)





Calculate exergy losses of all the ORC components

Exergy analysis of a simple ORC power plant (2)

Data

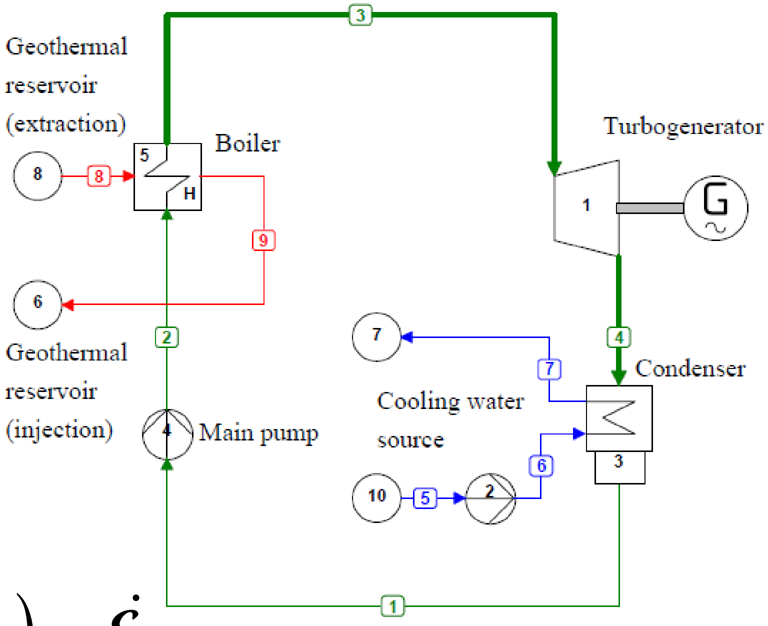
Boiler pressure	$P_{\text{boiler}} = 8.89\ \text{bar}$
Turbine inlet temperature	$T_{\text{in,turb}} = 85.6\ °\text{C}$
Condensation pressure	$P_{\text{cond}} = 1.36\ \text{bar}$
Isentropic efficiency of the turbine	$\eta_{\text{turb}} = 0.80$
Isentropic efficiency of the feeding pump	$\eta_{\text{pump}} = 0.35$
Min. temperature difference in the boiler	$\Delta T_{\text{boiler}} = 4\ °\text{C}$
Min. temperature difference in the condenser	$\Delta T_{\text{cond}} = 4\ °\text{C}$
Mass flow of geothermal water	$\dot{M} = 22.5\ \text{kg/s}$

Exergy analysis of a simple ORC power plant (3)

State no.	**Model**	**Fluid**	***M*** [kg/s]	***P*** [bar]	***T*** [°C]	***h*** [kJ/kg]	***s*** [kJ/kg.K]
1	RefProp	R245fa	9.913	1.36	22.69	229.4	1.1032
2	RefProp	R245fa	9.913	8.89	23.74	231	1.1067
3	RefProp	R245fa	9.913	8.89	85.60	465.9	1.7852
4	RefProp	R245fa	9.913	1.36	40.89	438.3	1.8075
5	IF97	Water	70.746	1.013	10.00	42.09	0.1510
6	IF97	Water	70.746	1.513	10.01	42.17	0.1511
7	IF97	Water	70.746	1.013	17.01	71.43	0.2533
8	IF97	Water	22.5	6.00	104.00	436.3	1.3514
9	IF97	Water	22.5	6.00	79.40	332.8	1.0678

Component	**Power** [kW]
Boiler	2328.0
Turbine	273.4
Pump	18.7
Condenser	2070.4

Exergy analysis of a simple ORC power plant (4)



Exergy balances of the components

- Boiler $\dot{M}\left(e_{f,8}-e_{f,9}\right)+\dot{M}\left(e_{f,2}-e_{f,3}\right)=\dot{\mathcal{E}}_{\mathrm{d}}$
- Turbine $\dot{M}\left(e_{f,3}-e_{f,4}\right)=\dot{W}_{\mathrm{turb}}+\dot{\mathcal{E}}_{\mathrm{d}}$
- Condenser $\dot{M}\left(e_{f,4}-e_{f,1}\right)+\dot{M}\left(e_{f,6}-e_{f,7}\right)=\dot{\mathcal{E}}_{\mathrm{d}}$
- Pump $\dot{M}\left(e_{f,2}-e_{f,1}\right)=-\dot{W}_{\mathrm{pump}}+\dot{\mathcal{E}}_{\mathrm{d}}$

With $e_{f,\mathrm{in}}-e_{f,\mathrm{out}}=\left(h_{\mathrm{out}}-h_{\mathrm{in}}\right)-T_0\left(s_{\mathrm{out}}-s_{\mathrm{in}}\right)$

And $T_0=10\ ^{\circ}\mathrm{C}\ ;\ P_0=1.013\ \mathrm{bar}$

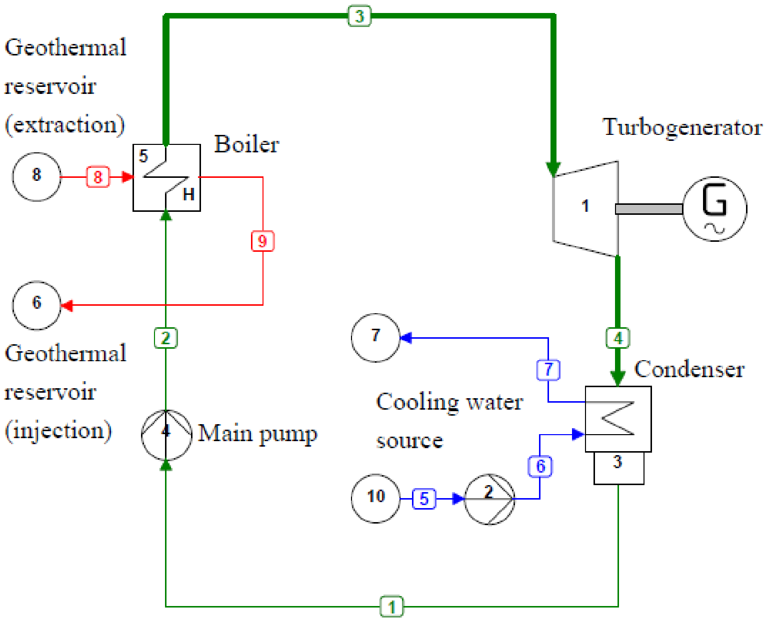
Exergy analysis of a simple ORC power plant (5)

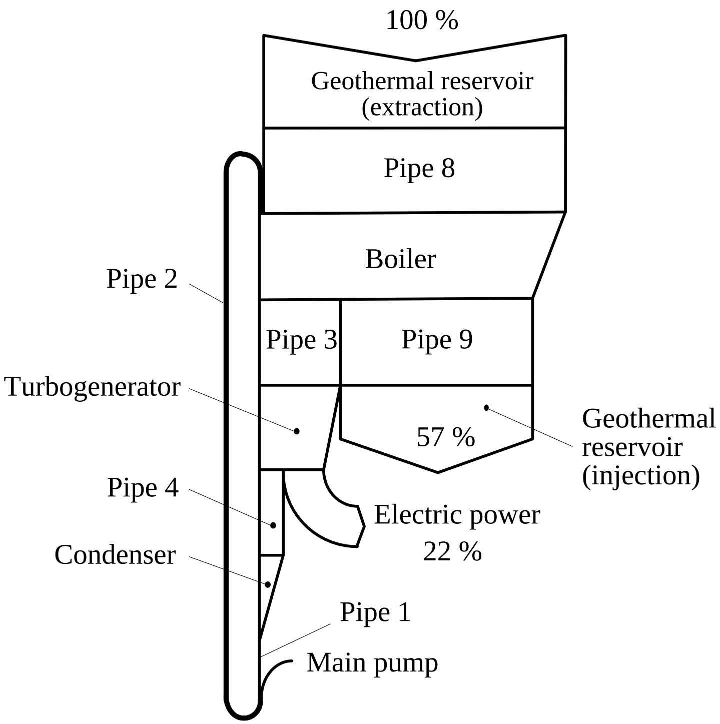
Component	**Exergy loss**	**Exergy Eff.**
	[kW]	[%]
Boiler	97.69	81.3
Turbine	62.61	81.4
Pump	12.70	32.2
Condenser	70.77	24.4

Solving exergy balance for exergy losses

- Boiler is largest
- Condenser and turbine similar
- Boiler: add heat transfer surface, but additional cost
- Turbine fluid dynamics performance: R&D

Grassman diagram





Concluding remarks

- Comparison combined cycle plant (η_I=59%) and geothermal ORC plant (η_I=10%)
- Better comparing η_II: $\eta_\mathrm{II,CCP} = 0.52$ and $\eta_\mathrm{II,ORC} = 0.58$
- Exergoeconmics
- Life-cycle analysis
- Arbitrariness: definition of exergy obtained and provided, definition of the system, conditions