THERMODYNAMICA 2
(WB1224)

8 april 2011
14.00 - 17.00 u.

AANWIJZINGEN

Het tentamen bestaat uit drie open vragen op 10 bladzijden.

Het tentamen is een **GESLOTEN BOEK** tentamen. Dit betekent dat tijdens het tentamen uitsluitend het uitgereikte formuleblad mag worden geraadpleegd. Gebruik van aantekeningen of andere hulpmiddelen (zoals een eigen formuleblad) is *niet* toegestaan. Gebruik van een (niet programmeerbaar) rekenapparaat is wel toegestaan.

Elke opgave moet op een afzonderlijk blad worden ingeleverd.

Zorg er voor dat het ingeleverde werk (elk blad afzonderlijk) is voorzien van naam en studienummer.

Zorg ervoor dat bij het inleveren alle bladen in elkaar zijn gevouwen.

Gebruik bij het beantwoorden van de vragen een zwarte of blauwe pen. Het gebruik van een rode pen en/of potlood is niet toegestaan.

Geef bij het beantwoorden van de vragen zo duidelijk mogelijk aan hoe het antwoord is verkregen: laat zien welke relaties zijn gebruikt en waar deze relaties op zijn gebaseerd.

Vermeld bij de uitkomsten de eenheden en kijk of de gevonden waarden realistisch zijn. Zo niet, geef aan waarom.

OPGAVE 1

Stikstof komt een turbine binnen bij 100bar en 300K en komt de turbine uit bij 40bar en 245K. Er vindt geen warmteuitwisseling plaats met de omgeving. Bereken de geleverde arbeid in kJ/kg en de entropieproductie in kJ/(kg K) met behulp van de volgende gegevens:

- Stikstof kan niet worden beschouwd als een ideaal gas. Voor de afwijking van ideaal gasgedrag kunt u gebruik maken van de bijgevoegde “departure” grafieken.

- Voor stikstof zijn de volgende gegevens bekend: T_c=126K, p_c=33.9 bar, molmassa 28 gram/mol. Bij lage druk is de c_p van stikstof gelijk aan 28 J mol^{-1} K^{-1} en deze waarde is niet afhankelijk van de temperatuur.

OPGAVE 2

1 mol gasvormig argon ondergaat een volumeverandering van volume V_1 naar volume V_2, die reversibel en adiabatisch wordt uitgevoerd. Van dit proces willen we graag de temperatuursverandering $\Delta T=T_2-T_1$ weten. Argon kan niet worden beschouwd als een ideaal gas.

(a) Laat zien dat uit de 2^e hoofdwet van de thermodynamica volgt dat dit een proces is bij constante entropie.

(b) Naar welke partiële afgeleide moeten we op zoek gaan op de temperatuursverandering te kunnen berekenen?

Het verband tussen druk p, molair volume v en temperatuur T van gasvormig argon kan worden beschreven met de toestandsvergelijking van van der Waals

$$\left(p+\frac{a}{v^2}\right)(v-b)=RT$$

Hierin geldt dat $b<v$.

(c) Laat zien dat de soortelijke warmte bij constante volume (c_v) van gasvormig argon niet afhangt van het volume.

(d) Leid een formule af voor de temperatuursverandering $\Delta T=T_2-T_1$ van bovengenoemd proces, indien verondersteld kan worden dat de soortelijke warmte van argon bij constant volume een lineaire functie is van de temperatuur, dat wil zeggen $c_v=a+bT$ waarin a en b constanten zijn. Hint: denk aan de min-1 regel.

OPGAVE 3

Given the configuration for a solar power plant based on the regenerated supercritical CO_2 Brayton cycle as in figure 1 and the following data:

Working fluid	CO_2 (molar mass: 44 g/mol)
Compressor inlet temperature	37 °C
Compressor inlet pressure	77 bar
Compression (expansion ratio)	2.3
Compressor isentropic efficiency	0.82
Turbine inlet temperature	650 °C
Turbine isentropic efficiency	0.87
Regenerator Temperature difference – hot streams (T_5- T_3)	40 °C
Regenerator Temperature difference – cold streams (T_6- T_2)	30 °C
Net Electrical Power output	$\dot{W} = 25\ \text{MW}_\text{e}$

Environment for exergy calculations $T_0 = 20\ °\text{C}$; $P_0 = 1.013$ bar

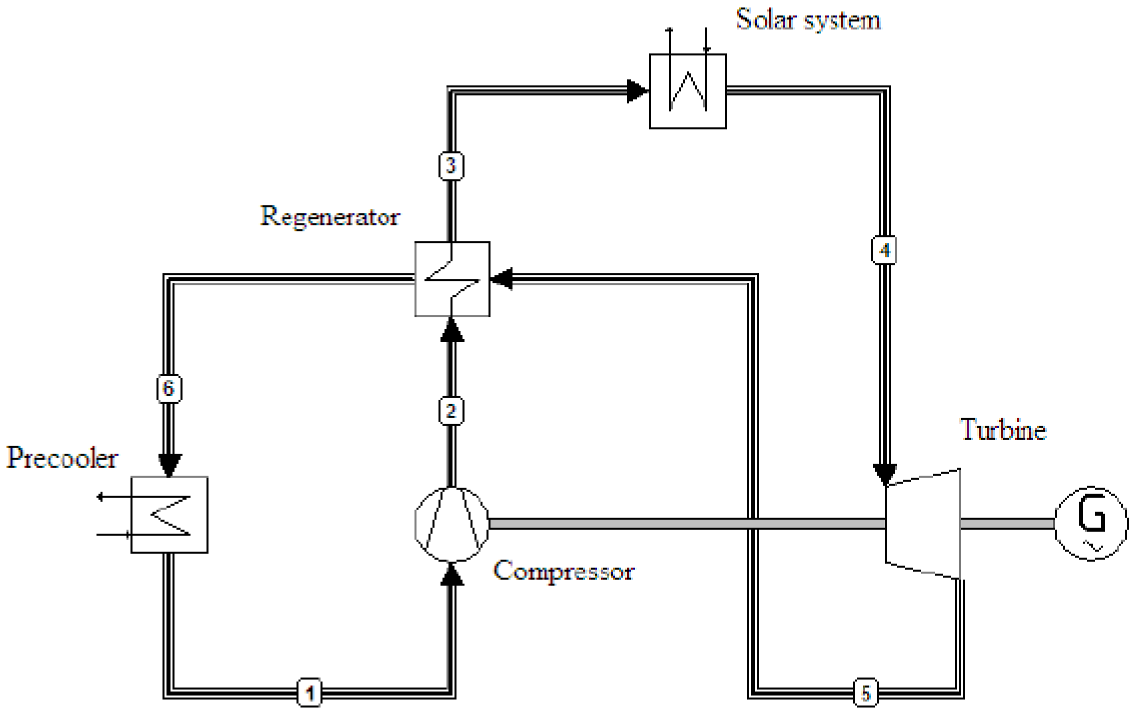


Figure 1: process flow diagram of the solar power plant

Questions:

(a) In a *T*-*s* diagram of CO_2, draw the saturated vapor-liquid line and the thermodynamic cycle, indicating the processes forming the cycle (compression, regenerator-heating, solar heating, expansion, regenerator-cooling, cooling). Note that the critical values for pressure and temperature of CO_2 are $P_{\mathrm{cr}} = 73.77$ bar, $T_{\mathrm{cr}} = 30.98$ °C

(b) Calculate the values of *P*, *T*, *h*, *s* for all the state points of the thermodynamic cycle (1, 2, 3, 4, 5, 6) using the polytropic ideal gas thermodynamic model (constant isobaric specific heat $\bar{C}_P$ along each thermodynamic process) with:

$$\bar{C}_{P,1\text{-}2} = 3{,}98\ \mathrm{kJ/(kg\cdot K)} \qquad \bar{C}_{v,1\text{-}2} = 0{,}98\ \mathrm{kJ/(kg\cdot K)}$$

$$\bar{C}_{P,2-3-4} = 1{,}62\ \mathrm{kJ/(kg\cdot K)} \qquad \bar{C}_{v,2-3-4} = 0{,}97\ \mathrm{kJ/(kg\cdot K)}$$

$$\bar{C}_{P,4-5} = 1{,}23\ \mathrm{kJ/(kg\cdot K)} \qquad \bar{C}_{v,4-5} = 1{,}01\ \mathrm{kJ/(kg\cdot K)}$$

$$\bar{C}_{P,5-6-1} = 2{,}66\ \mathrm{kJ/(kg\cdot K)} \qquad \bar{C}_{v,5-6-1} = 0{,}95\ \mathrm{kJ/(kg\cdot K)}$$

(c) Using the values previously calculated with the polytropic ideal gas thermodynamic model, make a table like the one below and fill in the missing values.

State	CO_2 properties			
	P	*T*	*h*	*s*
1				
2				
3				
4				
5				
6				

(d) Calculate the mechanical power consumption of the compressor $\dot{W}_{\mathrm{comp}}$, the thermal power exchanged in the regenerator $\dot{Q}_{\mathrm{reg}}$, the thermal power input $\dot{Q}_{\mathrm{solar\ input}}$, the mechanical power produced by the turbine $\dot{W}_{\mathrm{turb}}$ and the thermal efficiency (I-law efficiency) of the system using the data from the table you produced in question c.

(e) Calculate the exergy losses occurring in the regenerator and in the turbine using the thermodynamic data from the table you produced in question c.

Formuleblad Thermodynamica 2

Ideale Gaswet $pV = nRT$ en dus $pv = RT$

Constantes $R = 8.314472\ J mol^{-1} K^{-1}$ $N_{AV} = 6.02214 \bullet 10^{23}$ moleculen/mol

1$^{\text{ste}}$ HW gesloten system $dU = \delta Q - \delta W$

Reversibele arbeid $W = \int_{V_1}^{V_2} p dV$

1$^{\text{ste}}$ HW open systeem $\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} - \dot{m}_2 \left(h_2 + \frac{c_2^2}{2} + g z_2 \right) + \dot{m}_1 \left(h_1 + \frac{c_1^2}{2} + g z_1 \right)$. Hierin wordt de uitgaande stroom aangeduid met “2” en de ingaande stroom met “1”.

2$^{\text{de}}$ HW gesloten systeem $dS = \frac{\delta Q}{T} + \delta S$ met $\delta S \geq 0$

2$^{\text{de}}$ HW open system $\frac{dS_{CV}}{dt} = \frac{\dot{Q}_{CV}}{T^{trans}} - \dot{m}_2 s_2 + \dot{m}_1 s_1 + \dot{\sigma}_{CV}$ met $\dot{\sigma}_{CV} \geq 0$ waarbij het warmtetransport plaatsvindt bij temperatuur T^{trans}

Thermodynamische potentialen $H = U + pV$ $A = F = U - TS$ $G = U - TS + pV$

Veranderingen van *U, H, F, G*

$dU = TdS - pdV + \mu dN$ $dH = TdS + Vdp + \mu dN$

$dF = -SdT - pdV + \mu dN$ $dG = -SdT + Vdp + \mu dN$

Partiële afgeleiden voor *f(x,y)*

$df = \left(\frac{\partial f}{\partial x} \right)_y dx + \left(\frac{\partial f}{\partial y} \right)_x dy = Mdx + Ndy$ dus $M = \left(\frac{\partial f}{\partial x} \right)_y$ en $N = \left(\frac{\partial f}{\partial y} \right)_x$

Maxwell relatie voor *f(x,y)*

Indien $df = Mdx + Ndy$ dan $\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$ oftewel $\frac{\partial}{\partial x} \left[\left(\frac{\partial f}{\partial y} \right)_x \right]_y = \frac{\partial}{\partial y} \left[\left(\frac{\partial f}{\partial x} \right)_y \right]_x$

Integreren

$$f(x_2, y_1) = f(x_1, y_1) + \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial x} \right)_y dx$$

Min 1 regel

$$\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial z}{\partial x} \right)_y \left(\frac{\partial y}{\partial z} \right)_x = -1$$

Definitie warmtecapaciteit

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} = T\left(\frac{\partial S}{\partial T}\right)_{V,N} \quad \text{en} \quad C_p = \left(\frac{\partial H}{\partial T}\right)_{P,N} = T\left(\frac{\partial S}{\partial T}\right)_{P,N}$$

Departure Gibbs energy

$$\ln\frac{f}{p} = \ln\varphi = \frac{\mu - \mu^{IG}}{RT} = \int_0^p \frac{Z(p)-1}{p}\,dp$$

Clapeyron equation

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{s_g - s_f}{v_g - v_f} = \frac{h_g - h_f}{T\left(v_g - v_f\right)}$$

Clausius-Clapeyron equation

$$\left(\frac{d\ln P}{dT}\right)_{sat} = \frac{h_g - h_f}{RT^2}$$

Carnot efficiency

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H}$$

Thermal power cycle efficiency (I-law efficiency)

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

Isentropic process for an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \quad \text{met} \quad \gamma = \frac{C_P}{C_v}$$

Entropy change of an ideal gas with temperature and pressure

$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

Isentropic efficiencies for compression and expansion

$$\eta_{is,exp} = \frac{h_1 - h_2}{h_1 - h_{2,is}}, \quad \eta_{is,compr} = \frac{h_{2,is} - h_1}{h_2 - h_1}$$

COP (coefficient of performance)

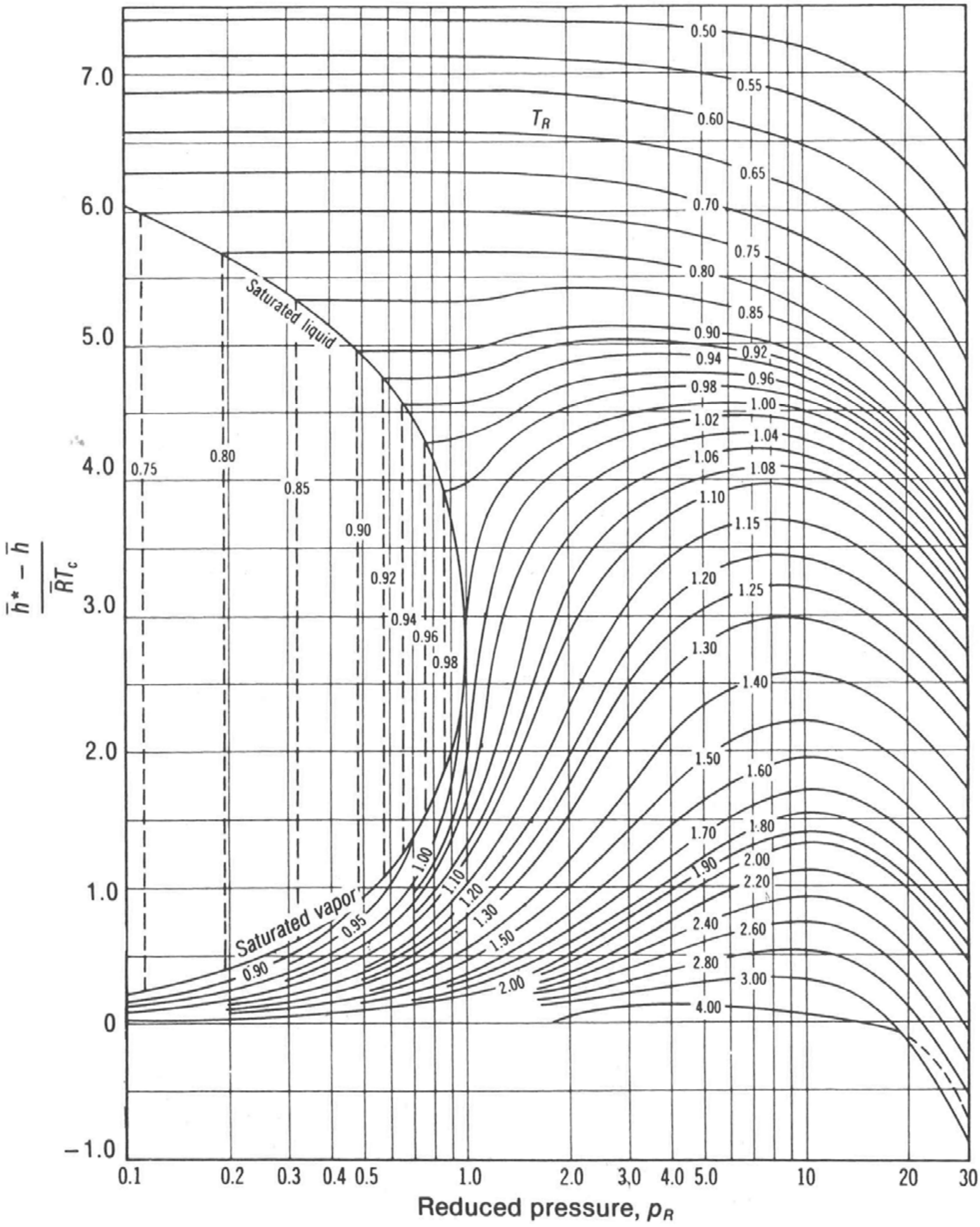
$$COP_{cooling} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}, \; COP_{heating} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

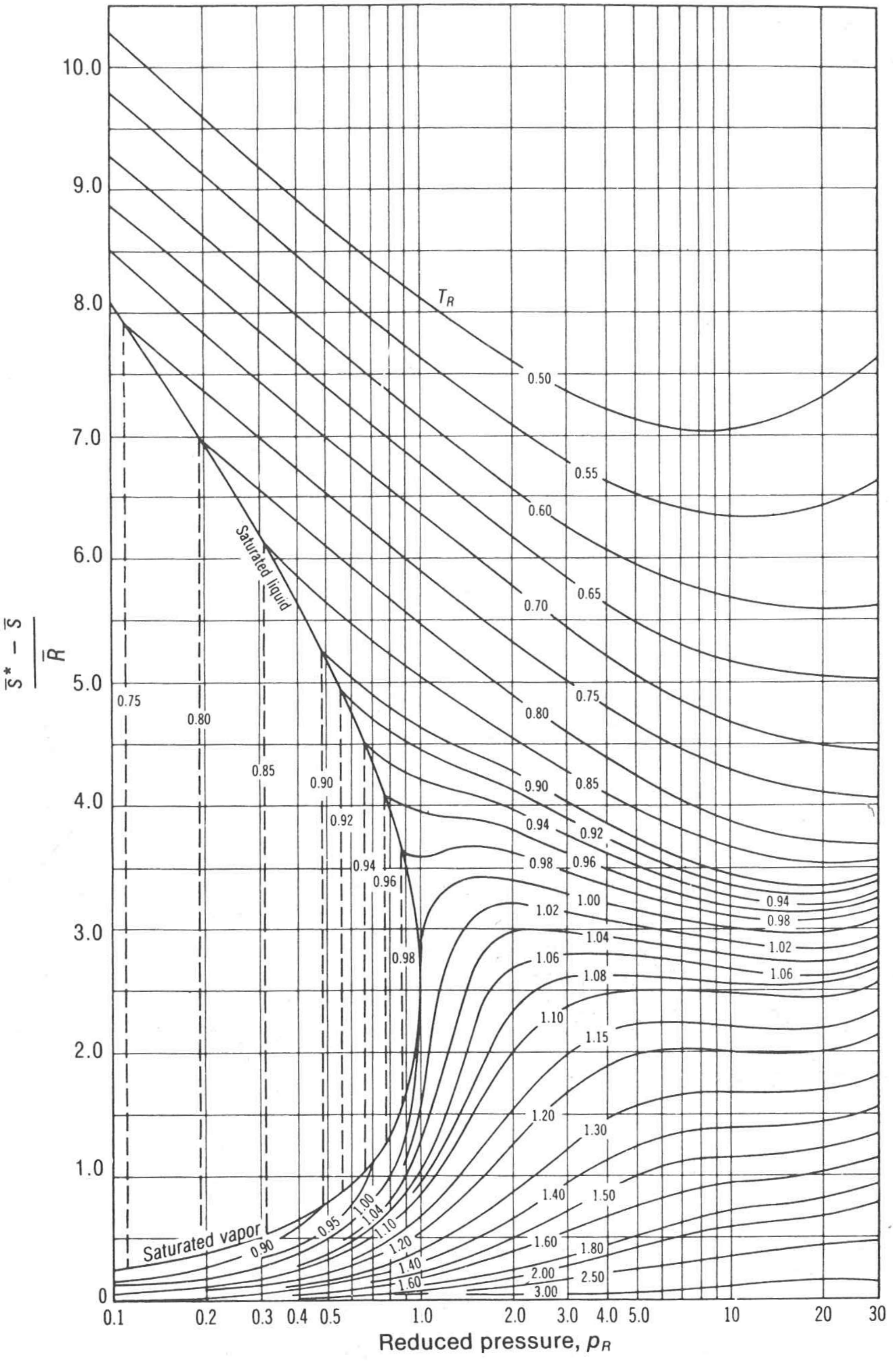
Exergy: specific flow exergy and balances

$$ex_f = h - h_0 + \frac{v^2}{2} + gz - T_0\left(s - s_0\right)$$

$$\frac{dEx}{dt} = \dot{E}x_{f,1} - \dot{E}x_{f,2} + (h_0 - T_0 s_0)(\dot{M}_1 - \dot{M}_2) + \left(1 - \frac{T_0}{T_b}\right)\dot{Q} + \dot{W} + \dot{E}x_d$$

$$\dot{M}\left(ex_{f,1} - ex_{f,2}\right) = -\left(1 - \frac{T_0}{T_b}\right)\dot{Q} + \dot{W} + \dot{E}x_d$$


T_R
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.92
0.94
0.96
0.98
1.00
1.02
1.04
1.06
1.08
1.10
1.15
1.20
1.25
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.00
2.20
2.40
2.60
2.80
3.00
4.00
Saturated liquid
Saturated vapor
0.75
0.80
0.85
0.90
0.92
0.94
0.96
0.98
0.90
0.95
1.00
1.10
1.20
1.30
1.50
2.00
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
−1.0
$\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}$
0.1
0.2
0.3
0.4
0.5
1.0
2.0
3.0
4.0
5.0
10
20
30
Reduced pressure, p_R


$\frac{\bar{s}^* - \bar{s}}{\bar{R}}$
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
0.1
0.2
0.3
0.4
0.5
1.0
2.0
3.0
4.0
5.0
10
20
30
Reduced pressure, p_R
T_R
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.92
0.94
0.96
0.98
1.00
1.02
1.04
1.06
1.08
1.10
1.15
1.20
1.30
1.40
1.50
1.60
1.80
2.00
2.50
3.00
0.75
0.80
0.85
0.90
0.92
0.94
0.96
0.98
0.94
0.98
1.02
1.06
Saturated liquid
Saturated vapor
0.90
0.95
1.00
1.04
1.10
1.20
1.40
1.60

Opgave 1

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad \text{en} \quad \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

$$h_1^{ig} - h_2^{ig} = -\int_{T_1}^{T_2} \left(\frac{\partial h}{\partial T}\right)_{p,N} dT = -\int_{T_1}^{T_2} c_p \, dT = 28 \times (300-245)\frac{J}{mol} = 1540 \frac{J}{mol}$$

$$T_{R_1} = \frac{300}{126} = 2.38 \quad P_{R_1} = \frac{100}{33.9} = 2.95 \quad T_{R_2} = \frac{245}{126} = 1.94 \quad P_{R_2} = \frac{40}{33.9} = 1.18$$

uit de tabellen

$$\left(\frac{h^{ig} - h}{RT_c}\right)_1 = 0.5 \quad \left(\frac{h^{ig} - h}{RT_c}\right)_2 = 0.31 \quad \left(\frac{s^{ig} - s}{R}\right)_1 = 0.21$$

$$\left(\frac{s^{ig} - s}{R}\right)_2 = 0.14$$

$$\text{dus} \quad \frac{\dot{W}_{cv}}{\dot{m}} = 1540 \frac{J}{mol} - \left(8.31 \frac{J}{mol\,K}\right) \cdot (126\,K)(0.5 - 0.31) = 1341 \frac{J}{mol} \,\hat{=}\, 47 \frac{kJ}{kg}$$

$$s_2^{ig} - s_1^{ig} = \int_{T_1}^{T_2} \left(\frac{\partial s}{\partial T}\right)_{p,N} dT + \int_{p_1}^{p_2} \left(\frac{\partial s}{\partial p}\right)_{T,N} dp = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1}$$

(lage druk; ig benadering)

$$= 28 \ln\frac{245}{300} - 8.31 \ln\frac{40}{100} = 1.94 \frac{J}{mol\,K}$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = 1.94 \frac{J}{mol\,K} - 8.31 \frac{J}{mol\,K}(0.14 - 0.21) = 2.5 \frac{J}{mol\,K} \,\hat{=}\, 89 \frac{J}{kg}$$

Opgave 2

a) $dS = \frac{\delta Q}{T} + \delta S$; reversibel dus $\delta S = 0$, adiabatisch dus $\delta Q = 0$ $\Big\}\ dS = 0$

b) $\left(\frac{\partial T}{\partial V}\right)_{S,N}$

c) $$\left(\frac{\partial c_V}{\partial V}\right)_{T,N} = \left(\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_{V,N}\right)_{T,N} = \left(\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_{T,N}\right)_{V,N}$$

$$= \frac{\partial}{\partial T}\left[-p + T\left(\frac{\partial p}{\partial T}\right)_{V,N}\right] = T\left(\frac{\partial^2 p}{\partial T^2}\right)_{V,N}$$

$\left(\frac{\partial p}{\partial T}\right)_{V,N} = \frac{nR}{V-b}$; $\left(\frac{\partial^2 p}{\partial T^2}\right)_{V,N} = 0$ dus $\left(\frac{\partial c_V}{\partial V}\right)_{T,N} = 0$

d) $\left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial S}{\partial T}\right)_V \left(\frac{\partial V}{\partial S}\right)_T = -1$ dus

$$\left(\frac{\partial T}{\partial V}\right)_S \frac{c_V}{T} \left(\frac{\partial V}{\partial S}\right)_T = -1 \Rightarrow \left(\frac{\partial T}{\partial V}\right)_S = \frac{-T}{c_V}\left(\frac{\partial S}{\partial V}\right)_T$$

$$= \frac{-T}{c_V}\left(\frac{\partial p}{\partial T}\right)_V$$

(Maxwell uit
$dA = -SdT - pdV$)

2d, vervolg

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{T}{c_V}\left(\frac{\partial p}{\partial T}\right)_V = \frac{-T}{c_V}\,\frac{R}{V-b}$$

$$= \frac{-T}{a+bT}\,\frac{R}{V-b}$$

$$\Rightarrow \int_{T_1}^{T_2} -dT\left[\frac{a+bT}{T}\right] = \int_{V_1}^{V_2}\frac{R}{V-b}\,dV$$

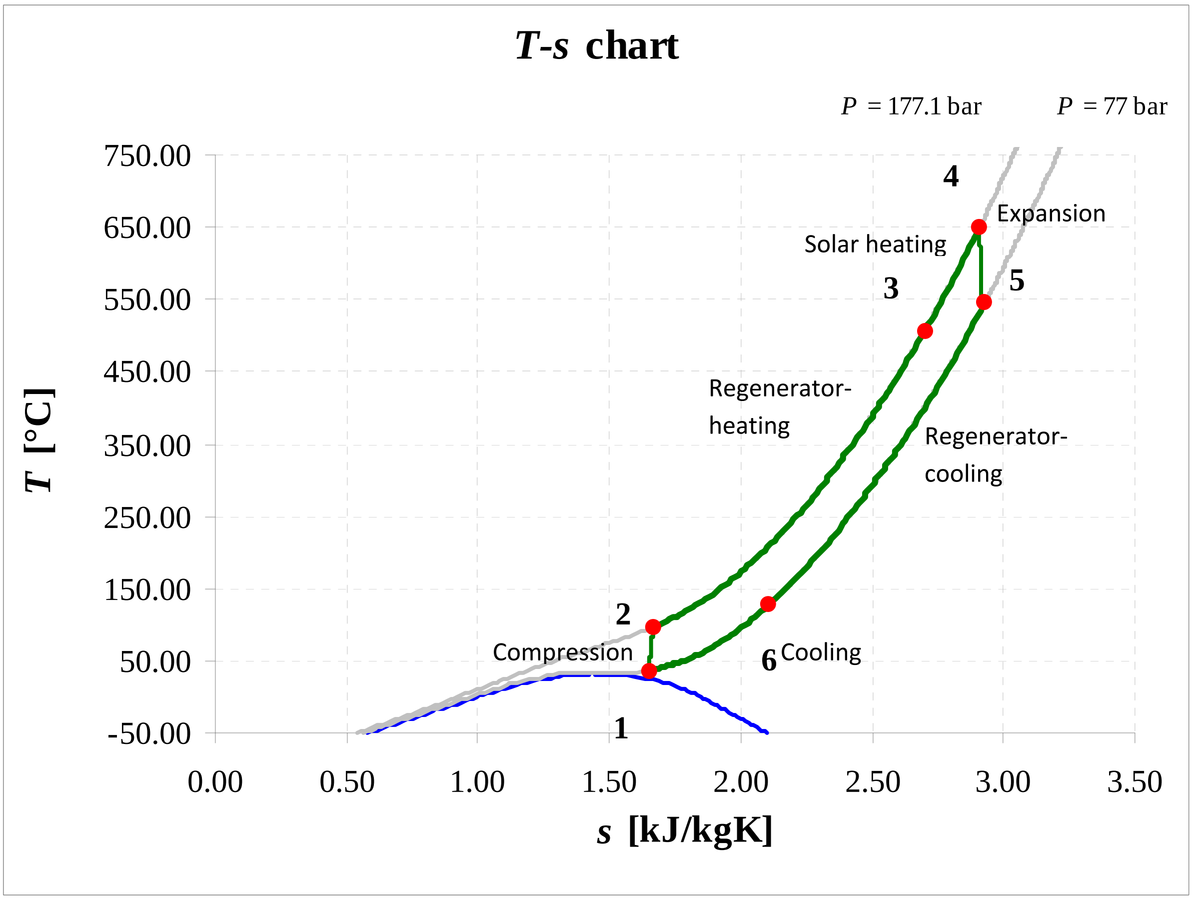
$$-\Big[a\ln T + bT\Big]_{T_1}^{T_2} = R\Big[\ln(V-b)\Big]_{V_1}^{V_2}$$

$$\boxed{-a\ln\frac{T_2}{T_1} + b(T_1-T_2) = R\ln\frac{V_2-b}{V_1-b}}$$

Solution of Exercise #3

A very similar exercise has been worked out during lecture and is available on **slide 30 of Part 10.** In that case an accurate model for CO_2 has been used. In this case the polytropic ideal gas model must be employed for simplicity. The polytropic ideal gas model is given by $Pv = RT$ and C_P (or C_v) = const. (within a temperature interval). The polytropic ideal gas model leads to large errors in the calculation of thermodynamic properties of fluids, if applied to states in the vicinity of the vapor-liquid critical point, which is the case here. This is maybe a bit tricky to realize, even if it has been explained during lectures. Anyway, the ability to calculate the states of the Brayton cycle is tested here.

a)



b) Given the average values of C_P and C_v, the average value of the polytropic exponent γ and the average value of $\zeta = (\gamma - 1) / \gamma$ for each of the processes can be calculated from C_P / C_v:

$$\overline{\gamma}_{1-2} = \frac{\overline{C}_{P,1-2}}{\overline{C}_{v,1-2}} = \frac{3.98}{0.98} = 4.04 \qquad \Rightarrow \frac{\overline{\gamma}_{1-2} - 1}{\overline{\gamma}_{1-2}} = \zeta_{1-2} = \frac{4.04 - 1}{4.04} = 0.75$$

$$\overline{\gamma}_{2-3-4} = \frac{\overline{C}_{P,2-3-4}}{\overline{C}_{v,2-3-4}} = \frac{1.62}{0.97} = 1.67 \qquad \Rightarrow \frac{\overline{\gamma}_{2-3-4} - 1}{\overline{\gamma}_{2-3-4}} = \zeta_{2-3-4} = \frac{1.67 - 1}{1.67} = 0.40$$

$$\overline{\gamma}_{4-5} = \frac{\overline{C}_{P,4-5}}{\overline{C}_{v,4-5}} = \frac{1.23}{1.01} = 1.22 \qquad \Rightarrow \frac{\overline{\gamma}_{4-5} - 1}{\overline{\gamma}_{4-5}} = \zeta_{4-5} = \frac{1.22 - 1}{1.22} = 0.18$$

$$\overline{\gamma}_{5-6-1} = \frac{\overline{C}_{P,5-6-1}}{\overline{C}_{v,5-6-1}} = \frac{2.66}{0.95} = 2.79 \qquad \Rightarrow \frac{\overline{\gamma}_{5-6-1} - 1}{\overline{\gamma}_{5-6-1}} = \zeta_{5-6-1} = \frac{2.79 - 1}{2.79} = 0.64$$

State 1: compressor inlet

$P_1 = 77$ bar (given)
$T_1 = 37$ °C (given)
Assume state 1 is as the reference state for h and s respectively
$h_1 = 0$ kJ/kg
$s_1 = 0$ kJ/kgK

State 2: compressor outlet

$P_2 = P_1 * \beta = 77 * 2.3 = 177.1$ bar
Isentropic state at compressor outlet
$T_{2,\mathrm{is}} = T_1(P_2 / P_1)^{\zeta 1\text{-}2} = T_1(\beta)^{\zeta 1\text{-}2} = (37+273.15)*2.3^{0.75} - 273.15 = 306.1$ °C (= 579.3 K)
$h_{2,\mathrm{is}} - h_1 = C_{P,1\text{-}2}\,(T_{2,\mathrm{is}} - T_1) = 3.98*(306.1 - 37) = 1071.0$ kJ/kg

Use definition of isentropic efficiency to calculate the state at compressor outlet
$h_2 - h_1 = (h_{2,\mathrm{is}} - h_1) / \eta_{s,\mathrm{compr}} = 1071.0 / 0.82 = 1306.1$ kJ/kg
$h_2 = 1306.1 + h_1 = 1306.1 + 0 = 1306.1$ kJ/kg
$T_2 = (h_2 - h_1) / C_{P,1\text{-}2} + T_1 = 1306.1 / 3.98 + 37 = 365.2$ °C
$s_2 - s_1 = C_{P,1\text{-}2} \ln(T_2 / T_1) - (C_{P,1\text{-}2} - C_{v,1\text{-}2})\ln(P_2 / P_1) = C_{p,1\text{-}2} \ln(T_2 / T_1) - (C_{P,1\text{-}2} - C_{v,1\text{-}2})\ln(\beta) =$
$3.98\ln([365.2 + 273.15] / [37+273.15]) - (3.98-0.98)\ln 2.3 = 0.374$ kJ/kgK
$s_2 = 0.374 + 0 = 0.374$ kJ/kgK

State 4: turbine inlet

$P_4 = P_2 = 177.1$ bar
$T_4 = 650$ °C (given)
$h_4 - h_2 = C_{p,2\text{-}3\text{-}4}(T_4 - T_2) = 1.62*(650 - 365.2) = 461.4$ kJ/kg
$h_4 = 461.4 + 1306.1 = 1767.5$ kJ/kg

State 5: turbine outlet

$P_5 = P_1 = 77$ bar
Isentropic state at turbine outlet
$T_{5,\mathrm{is}} = T_4(P_5 / P_4)^{\zeta 4\text{-}5} = T_4(\beta)^{-\zeta 4\text{-}5} = (650+273.15)*2.3^{-0.18} - 273.15 = 521.5$ °C
$h_4 - h_{5,\mathrm{is}} = C_{P,4\text{-}5}(T_4 - T_{5,\mathrm{is}}) = 1.23*(650 - 521.5) = 158.1$ kJ/kg

Use definition of isentropic efficiency to calculate the state at turbine outlet
$h_4 - h_5 = (h_4 - h_{5,\mathrm{is}}) * \eta_{s,\mathrm{turb}} = 158.1 * 0.87 = 137.5$ kJ/kg
$h_5 = 1767.5 - 137.5 = 1630.0$ kJ/kg
$T_5 = T_4 - (h_4 - h_5) / C_{P,4\text{-}5} = 650 - 137.5 / 1.23 = 538.2$ °C
$s_5 - s_4 = C_{P,4\text{-}5} \ln(T_5 / T_4) - (C_{P,4\text{-}5} - C_{v,4\text{-}}5)\ln(P_5 / P_4) = C_{P,4\text{-}5}\ln(T_5 / T_4) + (C_{P,4\text{-}5} - C_{v,4\text{-}}5)\ln(\beta) =$
$1.23*\ln([538.2 + 273.15] / [650 + 273.15]) + (1.23 - 1.01)\ln 2.3 = 0.024$ kJ/kgK

State 6: regenerator

$P_6 = P_5 = P_1 = 77$ bar

$T_6 = T_2 + 30 = 365.2 + 30 = 395.2$ °C

$h_6 - h_1 = C_{P,5\text{-}6\text{-}1}\,(T_6 - T_1) = 2.66*(395.2 - 37) = 952.8$ kJ/kg

$h_6 = 952.8 + 0 = 952.8$ kJ/kg

$$s_6 - s_1 = C_{P,5\text{-}6\text{-}1}\ln(T_6/T_1) - (C_{P,5\text{-}6\text{-}1} - C_{v,5\text{-}6\text{-}1})\ln(P_6/P_1) = C_{P,5\text{-}6\text{-}1}\ln(T_6/T_1) - 0 = 2.66*\ln([395.2 + 273.15]/[37+273.15]) = 2.042 \text{ kJ/kgK}$$

$s_6 = 2.042 + 0 = 2.042$ kJ/kgK

State 3: regenerator

$P_3 = P_2 = P_4 = 177.1$ bar

$T_3 = T_5 - 40 = 538.2 - 40 = 498.2$ °C

$h_3 - h_4 = C_{P,2\text{-}3\text{-}4}(T_3 - T_4) = 1.62*(498.2 - 650) = -245.9$ kJ/kg

$h_3 = 1767.5 - 245.9 = 1521.6$ kJ/kg

$$s_3 - s_2 = C_{P,2\text{-}3\text{-}4}\ln(T_3/T_2) - (C_{P,2\text{-}3\text{-}4} - C_{v,2\text{-}3\text{-}4})\ln(P_3/P_2) = C_{p,2\text{-}3\text{-}4}\ln(T_3/T_2) - 0 = 1.62\ln([498.2 + 273.15]/[365.2 + 273.15]) = 0.307 \text{ kJ/kgK}$$

$s_3 = 0.374 + 0.307 = 0.681$ kJ/kgK

State 4: turbine inlet

$$s_4 - s_3 = C_{P,2\text{-}3\text{-}4}\ln(T_4/T_3) - (C_{P,2\text{-}3\text{-}4} - C_{v,2\text{-}3\text{-}4})\ln(P_4/P_3) = C_{P,2\text{-}3\text{-}4}\ln(T_4/T_3) - 0 = 1.62\ln([650 + 273.15]/[498.2 + 273.15]) = 0.291 \text{ kJ/kgK}$$

$s_4 = 0.291 + s_3 = 0.291 + 0.681 = 0.972$ kJ/kgK

$s_5 = 0.024 + s_4 = 0.024 + 0.972 = 0.996$ kJ/kgK

c)

State	P [bar]	T [°C]	h [kJ/kg]	s [kJ/kgK]
1	77	37	0	0.000
2	177.1	365	1306	0.374
3	177.1	498	1522	0.681
4	177.1	650	1767	0.972
5	77	538	1630	0.996
6	77	395	953	2.042

d) Specific work for the compressor:

$w_{compr} = h_2 - h_1 = 1306.1$ kJ/kg

Specific work for the turbine

$w_{turb} = h_4 - h_5 = 137.5$ kJ/kg $\ll w_{compr}$ (!)

We realize that, as a result of using the inapplicable polytropic ideal gas model for the compression, the calculated power consumed by the compressor is larger than that produced by the turbine, therefore the net power output calculated under this assumption is negative, and so is the efficiency of the cycle! As a result, it is not possible to obtain 25 MW out of this system in case the polytropic ideal gas model is used.

Specific thermal power exchanged in the regenerator
$q_{reg} = h_6 – h_5 = h_3 – h_2 = 1521.6 – 1306.1 = 215.5$ kJ/kg

Specific thermal power input
$q_{solar} = h_4 – h_3 = 245.9$ kJ/kg

Thermal efficiency:

η_I = specific energy obtained / specific energy provided × 100%
$= (w_{turb} - w_{compr}) / q_{solar} \times 100\%$
$= (137.5 - 1306.1) / 245.9 \times 100\% = -472\%$!

Due to the fact that it is not possible to obtain 25 MW from the system, it is also not possible to calculate the mass flow rate from:

$W'_{net} = W'_{turb} – W'_{compr} = M' (h_4 – h_5) – M' (h_2 – h_1) = M' ([h_4 – h_5] – [h_2 – h_1])$
$M' = W'_{net} / ([h_4 – h_5] – [h_2 – h_1])$

Therefore it is not possible to determine the mechanical power consumption of the compressor, the power output of the turbine and the thermal power exchanged in the regenerator.
Power consumption compressor:

$W'_{compr} = M' (h_2 – h_1)$

Thermal power exchanged in regenerator:

$Q'_{reg} = M' (h_3 – h_2) = M' (h_6 – h_5)$

Thermal power input:

$Q'_{solar} = M' (h_4 – h_3)$

Mechanical power produced by turbine:

$W'_{turb} = M' (h_4 – h_5)$

e) Regenerator:

$Ex'_d = M' (e_{f2} – e_{f3}) + M' (e_{f5} – e_{f6})$
$= M' \{h_2 – h_3 – T_0(s_2 – s_3) + h_5 – h_6 – T_0(s_5 – s_6)\}$
$= -21.4 * \{1306.1 – 1521.6 – (20 + 273.15) * (0.374–0.681) +$
$1630.0 – 952.8 – (20 + 273.15) *(0.996- 2.042)\}$
$= 21.4 * (-125.5 + 983.8)$
$= 18.4 * 10^3$ kW = 18.4 MW

Or, because $h_2 - h_3 + h_5 - h_6 = 0$:

$Ex'_d = -M' T_0 \{ (s_2 - s_3) + (s_5 - s_6)\}$

Turbine:

$Ex'_d = M' (e_{f4} - e_{f5}) - W'_{turb}$

$= M' \{h_4 - h_5 - T_0(s_4 - s_5)\} - W'_{turb}$

$= -21.4 * \{137.5 - (20 + 273.15) (-0.024)\} - -2.9 * 10^3$

$= -3.1 * 10^3 + 2.9 * 10^3$

$= -0.2 * 10^3$ kW = -0.2 MW

Or, because $W'_{turb} = M' (h_4 - h_5)$:

$Ex'_d = -M' T_0 (s_4 - s_5)$