# AE4520: Advanced Structural Analysis

## Virtual Work

Mostafa Abdalla

# Learning Objectives

- State and Derive the principle of virtual work for a particle
- Understand the nature of internal constraint forces
  - Virtual work of internal constraint forces is zero
  - Virtual work of internal forces in deformable bodies *not* zero
- Derive and Understand the principle of virtual work for deformable bodies

# Summary

- Virtual displacements
  - Arbitrary
  - Infinitesimally small
  - Compatible with constraints (e.g., zero at supports)\
- For deformable bodies virtual displacements are *functions* of position. They change from one particle to another
  - We require them to be smooth (i.e., sufficiently differentaible)
- Virtual work is the work done by all forces (external and internal) on virtual displacements
- A body is in equilibrium **if and only if** virtual work is zero

# Statement of PVW

- Stress and Strain can be written as two 2D arrays.
  - Not physical vectors
  - Stress and Strain are tensors

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{pmatrix}, \quad \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{pmatrix}$$

- Principle of virtual work

$$W_{ex} + W_{in}$$

$$W_{in} = -\int_{body} \boldsymbol{\sigma}^t \, \delta\boldsymbol{\varepsilon} \, dV$$

$$W_{ex} = \int_{body} \mathbf{b}^t \delta\mathbf{u} \, dV + \int_{body} \mathbf{t}^t \delta\mathbf{u} \, dV$$