MODELLING

D MODELLING

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In empirical science existing reality is modelled. Central in this section stands the making of consistent verbal, mathematical and visual models and their relation to reality.

Modelling reality

There are many types of models, as Klaasen will explain in the first Chapter of this section. It is highly significant, that several types of models are in existence, but not several models of types.

Verbal models

The best described, most widely accepted form of consistency is formal logic. This also is on a higher level of abstraction a model (meta-language) of common language. Verbal models of architectural objects carry on their own level as an object-language the properties of this model. In the corresponding section de Jong adresses proposition and predicate logic, and their linguistic restrictions.

Mathematical models

De Jong elaborates different mathematical tools to be used in architectural, urban and technical design and evaluation. In the mathematical model of a design, connections may be read that enable evaluation of constructive or functional connectedness.

Visualisation and architecture

The language of the drawing is, due to its endless variation, less consistent than conversational language with her verifiable syntax, grammar and inherent logic. Considerable sensitivity as to context and interpretation of the drawing implies both her logical weakness and heuristic prowess at the same time. Yet, consistent and verifiable visual models can be made. Koutamanis gives examples.

The empirical cycle

On a higher level of abstraction the empirical cycle is also a model; according to many – including the author of that Chapter, Priemus – the only consistent model for scientific practice. It can be copied in any research project. That model is broadly accepted. It is based on the growth of knowledge to be generalised by well-defined testing. The time consuming shaping of a hypothesis, like with the architectural design, is in this respect ‘free’, not further modelled. The usual scientific approach pre-supposes in its turn consistency in discourse.

Forecasting and problem spotting

Mathematical models play an important rôle in forecasts and consequently in problem spotting that may give rise, for example, to the formulation of an architectural programme of requirements. Their conceptual framework is explained in the corresponding Chapter of de Jong and Priemus by way of large-scale examples.

Conclusion

A model demonstrates more relations than a concept or type, let alone an intuition: it is more consistent. However the model is not yet reality and should not be confused with it. Many relations – topographical, situational – will be lacking in the model. Incomplete models of a design may be made in order to make sector effect analyses and to test the design according to certain values and objectives of the relevant stakeholders (evaluations). Sometimes this requires more modelling than the design itself allows. A scale model is a model, if it allows evaluation like that; very realistically, like in a wind tunnel. A sketched-scale model rather has a function for further development of the design; also if it has not, as yet, the consistency of a model with its inter-subjective checking potential.

22 MODELLING REALITY

INA KLAASEN

In order to be able to inform, communicate and reflect on (future) reality we must imagine, articulate, calculate and simulate in models. In models reality is deliberately drastically reduced in a justifiable way. Models may be concrete, conceptual or formal. All two-dimensional architectural analyses, exploration studies and designs are conceptual models.

The functions of scientific models may be further classified in explorative, descriptive, explicative and projective functions, concerned with reality in the past and the present, and the probable and / or possible reality in the future. Insofar as architectural models are, contents-wise, made of words and numbers it must be feasible to translate them into spatial models (the so-called ‘medium switch’), or to contribute to the construction of spatial models. With the translation ‘backwards’ to (future) reality it is important to watch out for ‘model over-extension’.

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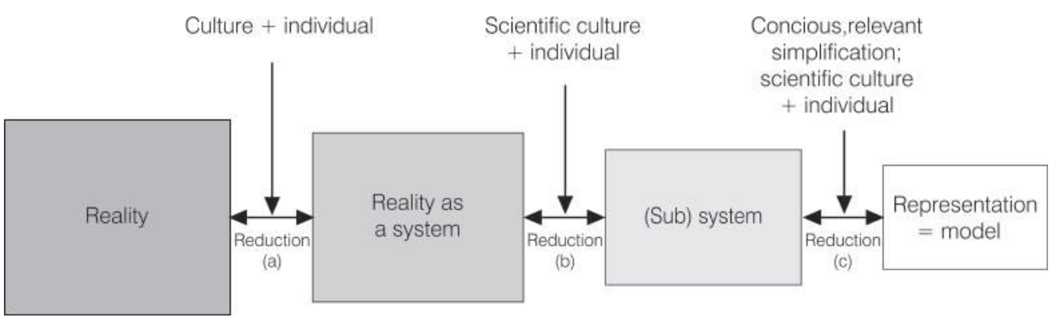
22.1 THE MODEL[a]

A simplified rendering of reality – present or future – is called a ‘model’ of that reality, provided that a structural relatedness exists with that reality and that the model is based on conscious interpretation of that reality (figure 155). Without models science is inconceivable.

In a model, reality is approached from a certain angle. Seen in one way, this angle is determined by (scientific) cultural backgrounds; often without conscious awareness. Seen another way, it is context-orientated; preferably explicitly. An urban architect makes a different model of a residential neighbourhood than a social geographer or civil engineer. Models are not value-free: moreover, they should not be.



155 We may think of reality as a complex of (sub-) systems (a). This simplification (reduction) of reality is made unconsciously and is not only based on the reach of naturally determined human powers of perception and thought, but also on cultural pre-suppositions, where individual differences apply. From this reality we may separate consciously sub-systems (b), based on intentional considerations and (scientific) cultural pre-suppositions that might differ per individual. The sub-systems may be presented next in models of the reality (c).

22.2 KINDS OF MODELS

In a presentation reality may be put into words, numbers, ‘imagined’ on a scale, or simulated. This leads to a classification of types of models:

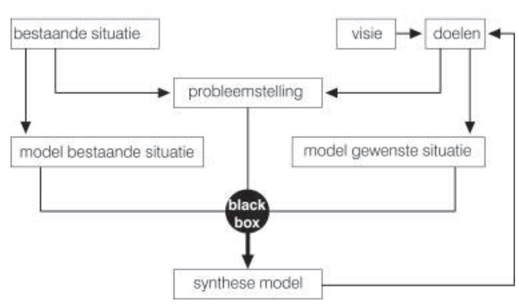
- verbal models
- mathematical models
- spatial models
- mechanical models

A verbal model is a discourse in words (figure 156). In this vein the structure of medical education at Limburg University functioned as a model for re-structuring the architectural education at Delft – with all the pitfalls inherent in the use of an analogue model.[b] A well-known verbal model is *‘Our Common Future’*, the so-called Brundtland Report of 1987.[c]

Who does not understand the Dutch language will find the models in figure 156 incomprehensible; to them the models do not have a communicative function. When words are used with different meanings in scientific (sub)cultures of a (scientific) linguistic community, faulty communication may occur.

156 Example of a verbal model, respectively visualised verbal model: procedure graduation project.[d] In this case visualisation does not make the model a spatial one.

In de door ons gehanteerde werkwijze wordt een onderscheid gemaakt in modelmatige weergaven van de bestaande situatie en mogelijke wensmodellen.
Konfrontatie van beide modellen - these en antithese - leidt vervolgens tot een synthese- en ontwikkelingsmodel. Een ontwikkelingsmodel is te omschrijven als een ruimtelijk kader dat randvoorwaarden stelt en richting geeft aan verdere ontwikkelingen.

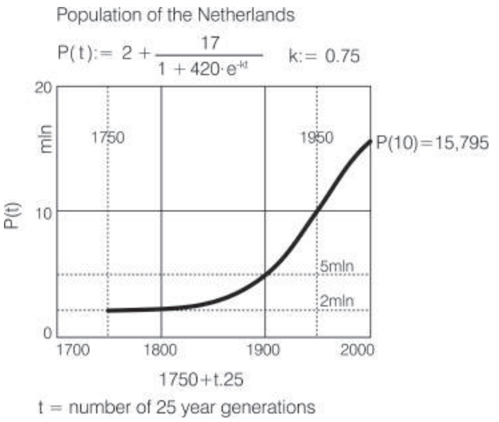


a In the present and following paragraphs use has been made of Bertels, K. and D. Nauta (1969) *Inleiding tot het modelbegrip;* Chorley, R.J. and P. Hagget (1969) *Models in geography, parts I, II and V;* Soest, J.P. van, J. van Kasteren et al. (1988) *De werkelijkheid van het model*.

b Klaasen, I.T. (2000) *Valkuilen bij stedebouwkundig ontwerpen: verwarring tussen model en werkelijkheid*.

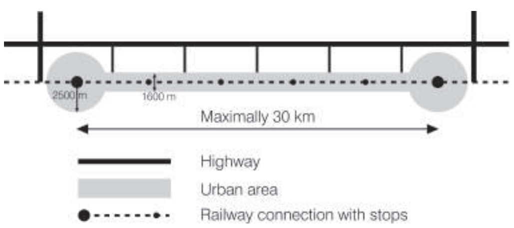
c In contrast to previous comparable ‘world models’, the ‘Brundtland Report’ does not include numerical models, because the pre-supposition that a (global) system may be manipulated is too dependent on thinking along causal lines (Soest, J.P. van, J. van Kasteren et al. (1988) *De werkelijkheid van het model*, p. 86).

d Frijlink, F. and L. Leferink (1991) *Waar het leven goed is - een stedenbouwkundig ontwikkelingsplan voor West Brabant.*



157 Graphic representation of an example of a mathematical model with mathematical contents

158 Example of a spatial model on scale: a (possible, future) articulation of a site.[b] Without explanation of the symbols used (legends) this model is incomprehensible to the 'uninitiated'.



159 Example of a spatial model: a principle model of a city resembling a ribbon

160 Depiction of a mechanical model of a rail system[g]

161 Spatial model of a rail system[h]

a Draaisma, D. (1995) *De metaforenmachine, een geschiedenis van het geheugen*.
b Thüsh, M. (1993) *Almere, uitgeslapen stad*, p.140.
c Draaisma, D. (1995) p.26.
d Bertels, K. and D. Nauta (1969) *Inleiding tot het modelbegrip*, p. 38.
e Citation from an interview with Douwe Draaisma on the use of metaphors. Delft, D. van *Zien en niet geloven; het beeld in de wetenschap beslecht zelden een controverse*. See also Draaisma, D. (1995).
f Frey, G. (1961) *Symbolische und Ikonische Modelle*.
g Source: *Miniatuurbanen* (1963) nr. 6, p.170.
h Source: Jacobs, M.; A. Geerse and I.T. Klaasen (1994) *Snelle trein ontspoort in de Randstad*.

A mathematical model is made up of numbers or symbols (Figure 157). All computer models are mathematical models, even if they are presented as spatial models, like GIS products (Geographical Information System) and 3D models.

The *'Limits to Growth'* report of the 'Club of Rome' of 1972 is an example of a report constructed out of mathematical models. In contrast to verbal models, mathematical models employ a universal 'language', with the proviso that this does not apply for its verbal parts.

A spatial model is a spatial rendering of three-dimensional reality (practically always) on scale. Every three-dimensional model is a spatial model, but so are all architectural designs and maps (figure 158 and 159).

(Sub)cultural agents, often hardly consciously experienced, determine how we visualise and depict something. The use of symbols in a 'picture' language may be compared to the one of words in ordinary language. Unlike the case of numbers the meaning is not universal. Symbols must be explained (legends). *"The image is the result of a long sequence of agreements, conventions, codes, tunings, etc."*[a]

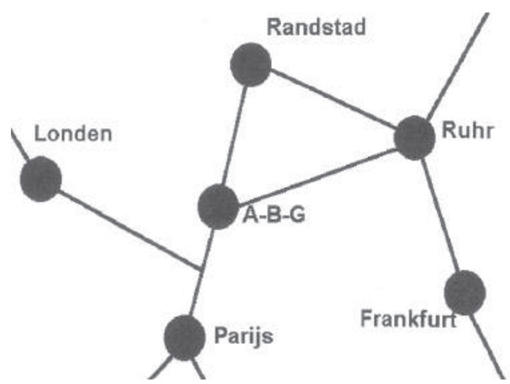
A shape metaphor may be the basis of a spatial model. Examples of shape metaphors are the 'Green Heart' (figure 172) and 'Ribbon City' (figure 159). A well chosen metaphor activates two associative processes at the same time: verbal and visual. This increases the chance that the relevant information is brought across: its communicative value is high.[c] Bertels and Nauta claim that the vague suggestiveness of a metaphor can have great artistic value and cause aesthetic delight.[d]

"Images are cognitively enormously compact, they can accommodate mountains of information; information not readily expressed in words. Consider the effort that is needed to enable computers to recognise faces. Images favour the strong points of the human brain. We made drawings before we wrote, we have a fabulous memory for images, better than for text. Add a summarising metaphor to a text and it will be remembered a lot better."[e]

A mechanical model (figure 160) is a model functioning in analogy with its original. It is a spatial model with real time for fourth dimension. An example is a planetarium, a dynamic model of the solar system. The reality of a mechanical model can only be depicted as a spatial model. Aided by a computer a mechanical model may be simulated; compare this to film, a rapid succession of static images.

Given a system in reality, say a rail system, we may opt for simplifications of different kinds, and consequently for models of different kinds. The choice of a type of model is based on the function the model needs to perform and on the personal preference of the makers. The following example is inspired by G. Frey.[f] We may reduce the reality of a rail system to:

- a model railway net (mechanical model; figure 160)
- the positioning of lines of rail and stations (spatial model; figure 161)
- a schedule of the job allocation of railway employees (mathematical model)
- a description in words of the structure of the system, like types of connections and mutual relations, speeds, frequencies of departure (verbal model)



22.3 TYPES OF MODELS IN THEIR RELATIONSHIPS TO REALITY

Models may also be characterised according to their relation to reality. Models can be:

- concrete (spatial and mechanical models);
- model (conceptual(verbal, mathematical, spatial, mechanical));
- formal (mathematical models).

A model composed of empirical identities is a concrete model. Concrete systems and models correspond with ‘matter’. Concrete models feature spatial dimensions. Concrete models allow realistic experimenting. Examples: model railways (figure 160), urban / architectural models. Sunlighting studies might be undertaken with the help of such a three-dimensional model (figure 162). Another example is a basin filled with clay, sand and streaming water to study patterns of flow. Architect Geoffrey Broadbent mentions an example of a concrete spatial model – an analogue model – of a non-spatial concrete system; a system without length, width and height.

“The national economy has been represented at the London School of Economics by means of a bath into which water flows at controlled rates (representing income) and out through holes of specific sizes in specific positions (representing expenditure).”[b]

A comparable experiment in urbanism with the help of an analogue model is commented on in figure 163.

A conceptual model is a mental construction (theory, sketch) referring to the (past, present, future) reality. A conceptual model is composed of conceptual identities. Bertels and Nauta used in the pre-computer era the evocative term ‘paper and pencil construction’.[d] Conceptual models and systems correspond to ‘comprehension’. Conceptual models only lend themselves to thought experiments, also called ‘thought models’. Examples are construction drawings and design sketches (figure 158 & 159). A classic example:

‘Galilei considered an imaginary experiment involving perfectly spherical balls in motion on a perfectly smooth plane. It would be impossible to achieve these ideal conditions in any actual experiment because of the intervention of friction and imperfections on the spheres. However, this ability to abstract from the conditions of the real world played an essential part in Galileí’s formulation of a new science of motion.’ [e]

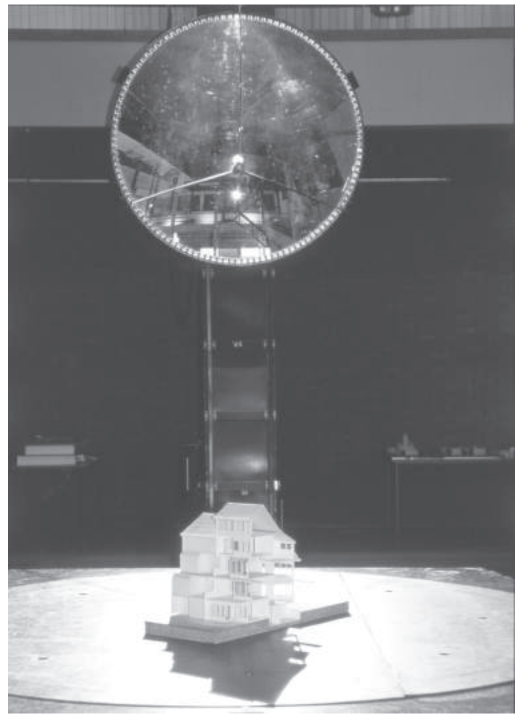
All two-dimensional spatial models, urban / architectural blue-prints included, are thought models. This also applies to three-dimensional models depicted on flat surfaces, and to virtual designs. Conceptual spatial models are, literally, ‘thought images’. (Urban)architects will seldom be in a position to experiment with their products; this in contrast to industrial designers, for instance, who can use conceptual models as well as concrete models. A concrete model of a teapot, for instance, can be tested on efficacy.

Finally, a formal model is an un-interpreted syntactic system of symbols (calculus, algorithm). Geographer David Harvey calls an element of a formal (mathematical) model a ‘primitive term’, and compares it to the – dimension-less – concept ‘point’.[f] Only structure imports, not content. Formal systems and models correspond to abstract names.

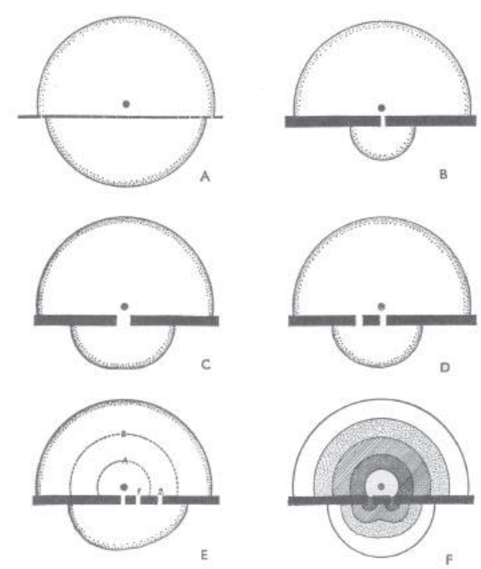
Examples of formal models are un-interpreted mathematical models (consistent conglomerates of mathematical equations) of a concrete hydrological system, for instance (reality is summarised in a series equations) and the formalised conglomerate of Euclid of the conceptual system ‘Euclidian space’.

22.4 FUNCTIONS OF MODELS

Models carry information and are consequently instrumental in communication, study and research. The exchange of information may be directed at the transfer, respectively widening of knowledge, but also at action or eliciting action. By the same token models may also be



162 Sunlighting-experiment aided by a model [a]



Blotting paper analogues simulating gross morphological features of river towns. A suggests that where a river can be crossed easily at many points the town will develop almost equally on both banks. B, C and D are examples of the shapes resulting from various restrictions to easy access to the opposite bank. E is a rudimentary growth model, in that when the solvent front reached the line A bridge 1 was added. Bridge 2 was added when the solvent front reached the line B. This process leads to a marked asymmetry in the shape of the part of the town on the far side of the river. F is a plot of the various stages of growth.

163 Depiction of a concrete blotting-paper (analogue) model of an urban system.[c]

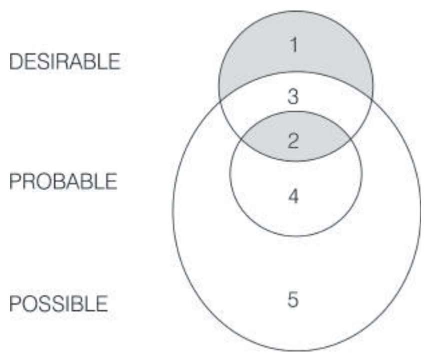
a Source: photographic service, Fac. of Arch. DUT.
b Broadbent, G. (1988) *Design in architecture: architecture and the human sciences*, p. 89.
c Chorley, R.J. and P. Hagget (1969) *Models in geography, parts I, II and V*, p. 763.
d Bertels, K. and D. Nauta (1969) *Inleiding tot het modelbegrip*.
e Commentary on Galilei, G. (1632) *Dialogue Concerning the two Principal Systems of the World*. Source: The Open University (1974) *Science and belief: from Copernicus to Darwin*, unit 3, p. 117.
f Harvey, D. (1973) *Explanation in geography*, p. 452.

interpreted in terms like discussion model, participation model, seduction model, study model, more specifically heuristic model, action model, execution model, etc.

From a scholarly angle models can be classified as follows:

164 Types of models according to their function

function:	*aimed at:*
descriptive model	what (probably) is the case
explicative model	because of what or why that (probably) is the case
predictive or probable-projective model (= trend scenario)	what probably will be the case (probable future)
intentional-projective[a] model (= planning model)	what we have decided that should be the case
(explorative)-potential-projective model (design or explorative scenario)	what possibly can be the case (possible future)



165 In this (spatial, descriptive, conceptual) model the ‘probable futures’ are a subset of the ‘possible futures’ and contains the set ‘desirable futures’[b], parts of the set ‘possible’ and of the set ‘probable futures’.[c]

This survey is related to the modality schema of Taeke de Jong (figure 165). Establishing a relationship between the model-functions and this schema of modalities generates the following classification:

166 Types of models according to their modality

1 desirable, but impossible	model without an application field (fiction)
2 desirable and probable	trend scenario deemed a desirable development
3 desirable, possible, but improbable	intentional-projective model (planning model, design)
4 undesirable, but probable	trend scenario deemed an undesirable development
5 (still) undesirable[d] and improbable but possible	explorative-projective model (explorative scenario) or potential projective model (design)

A descriptive model is a model of an existing situation or process:

- maps of the existing situation (figure 167);
- descriptions in words or numbers of an urban system;
- programmes of requirements;
- description of the procedure followed (figure 156).

An explicative model does not restrict itself to the ‘what’ or ‘how’ question, but addresses the ‘why’ or ‘because of what’; based on insight into the working of processes, traditionally seen from the angle of causal, but also of conditional thought.

An explorative model is used to get insight into:

- what is the case (or what has been the case) or,
- what might be the case (or might have been the case); for instance in the study of the mechanisms of the ‘global warming effect’;
- future (developmental) possibilities.

167 Descriptive model, conceptual[e]

a Instead of ‘projective’ the concept ‘prospective’ may be used. The use of these terms is in the spatial sciences not unambiguous (see, for instance Vught, van and van Doorn (1976) *Toekomstonderzoek en forecasting;* Kleefmann, F. (1984) *Planning als Zoekinstrument*). Here the conceptual descriptions of the leading national dictionary is followed, giving as the first meaning of ‘*projecteren*’ ‘*ontwerpen*’.

b ‘desirable’: desired by a given organisation, such as a lobby, political party, a government – or the designer himself.

c Jong, T.M. de (1992) *Kleine methodologie voor ontwerpend onderzoek*, p. 9.

d (Still) undesirable, since ‘undesirable’ is restricted by ethical boundaries.

e Duijvestein, I (1997) *Mitla*, p.1

Examples of the latter case include (spatial) design studies (study by design) (figure 168) and studies trying to answer the question whether a specific programme might fit, in principle, in an existing plan.

A predictive model (probable projective model) (trend scenario) indicates what probably will happen, given a specific situation, based on insight into the working of processes (figure 169). On the basis of a model of a solar system one can predict that the sun will rise tomorrow, at what time (most probably). Well-known examples of predictive models are those forecasting election results, tomorrow's weather, those of the 'Club of Rome' – among them *'Limits to Growth'* – and the economic models of the Netherlands' Bureau for Economic Policy Analysis.

Descriptive, explicative and predictive models are linked to one another and belong to the domain of empirical sciences.

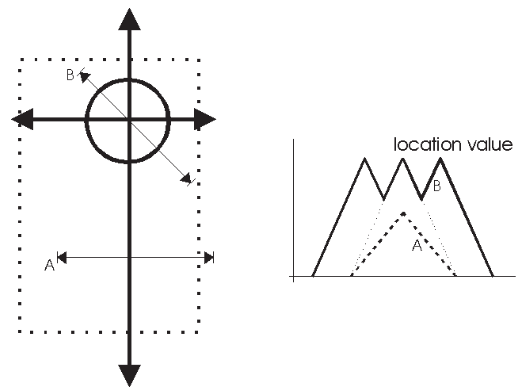
A planning model (intentional-projective model) is a model for a situation, deemed desirable, that does not yet exist, requiring one or more specific actions in order to come in existence ('goal-orientated' design, desirable scenario). Planning models are action models. Examples: the design for a bridge, an educational programme, a holiday trip (figure 170).

In potential-projective, explorative models (designs) possible future situations are rendered, the desirable ones along with those deemed undesirable. Alternative designs are also called 'scenarios'. The original meaning of the word 'scenario' denotes the sequences of actions in the theatre. The use of the term 'scenario' as an alternative projective model was introduced by the American 'Rand' Corporation. 'Rand' theorists, particularly Herman Kahn, developed in the sixties systematically possible futures, to give policy makers insight into (un)desirable effects of policy measures; effect reports, really.[c]

Scientifically understood, a scenario can be viewed as a model with a progressive temporal aspect. From a specific, well described, initial situation, possible future situation are presented in a consistent (logically connected) and plausible way. At the same time usually a trend scenario is made, to lay foundations for recommended changes in policy. Generally, several scenarios are developed to enable comparison. The differences in the scenarios are based on difference in pre-suppositions with regard to factors influencing developments. Figure 171 gives an example of these 'explorative scenarios'.

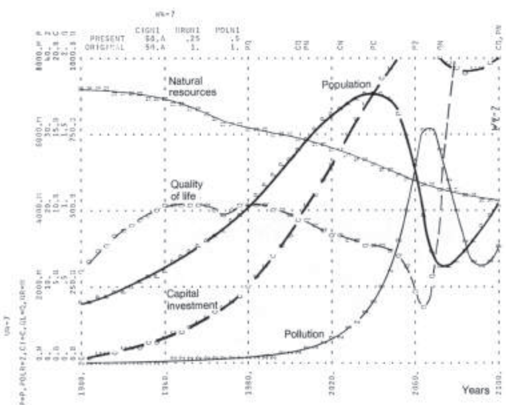
A model for a future reality, or an element of it, is also denoted as a 'concept'. More correct would be 'conception' (both from the Latin *'concipere'*: to take together).

In urban design / architecture the notion 'spatial concept' is used in the sense of a rough design. In the initial stage of a design process it is used to facilitate the designer himself, during the later stages of clarifying the design, for instance when communicating with

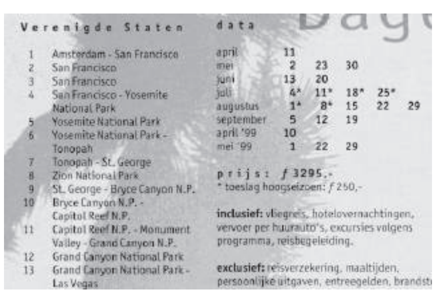


168 Spatial explorative conceptual model, (left) and conceptual, graphically rendered, mathematical model (right).

At the crossroads in a ring road, the location-value based on accessibility is equal to that of the central intersection. It may even become equal to it or greater, depending on the quality of the ring road compared with that of the radial roads inside its perimeter.[a]



169 Graphically rendered mathematical, predictive model, conceptual[b]



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		augustus	1*	8*	15	22	29
5	Yosemite National Park	september	5	12	19		
6	Yosemite National Park - Tonopah	april '99	10				
		mei '99	1	22	29		
7	Tonopah - St. George						
8	Zion National Park						
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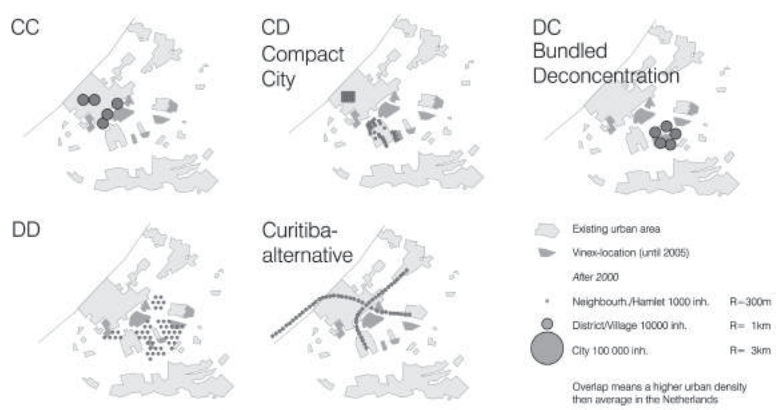
170 Example of a planning model, from a travel brochure (Travel Agency Djoser, 1998)



171 Five scenarios (explorative-projective models) for the way in which the region of The Hague could accommodate a population increase of 50.000 inhabitants in 2005.[d]

a Klaasen, I.T. and M. Jacobs (1999) *Relative location value based on accesibility: application of a useful concept in designing urban regions*.

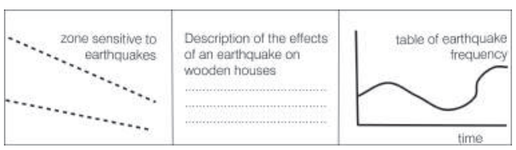
b Lomme, J. , L. Bakker et al. (1988) *Wereldmodellen - een literatuurstudie*, p. 30; source: Meadows, D.L. and D.H. Meadows (1973) *Toward global equilibrium: collected papers*.

c Bell, Daniel (1964) *Twelve modes of prediction: a preliminary sorting of approaches in the social sciences*.

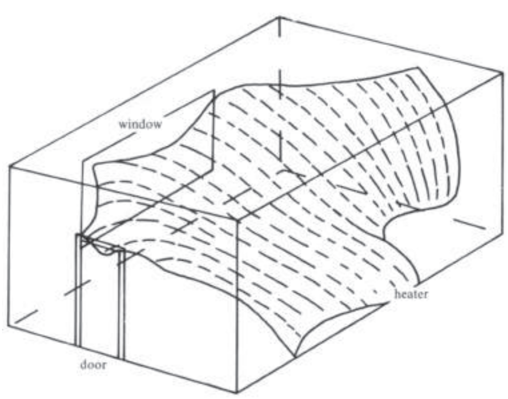
d Source: T. M. de Jong, Delft University of Technology, unpublished.

172 The planning concept 'Groene Hart' (a metaphor).[a]

173 Visualised plan-objective: the proposition "the centre of the city should distinguish itself visually' has been translated into a spatial model.



174 urban / architectural relevant models, respectively:
a spatial differentiation → spatial model;
b spatially undifferentiated qualitative architectural main-lines → verbal model;
c spatially undifferentiated quantitative description earth-quake frequency → mathematical model.



175 Combination of a spatial and mathematical (descriptive) model[c]

parties concerned. In planning, particularly in the policy field the notion 'spatial planning concept' or 'planning concept' is in use. Figure 172 gives an example. It signifies a policy-strategic action concept, and may be purely a means to communicate on the subject with the community and other (government) agencies without (potential) empirical foundation.

Usage of the term 'concept' may cause *mis-understanding* between planners and designers. *"The Randstad/ Groene Hart concept is a strong concept"*, a planner states; and he means to say that the concept has survived decades social-political turmoil and proven to be strategically strong. The urban designer reacts: *"On the contrary, it is a weak concept."*; hinting at the fact that it takes a lot of (political) effort to keep the central area free of urbanisation. The shortest connection between points on the edge of the circle afterall is through the circle.

22.5 USE OF MODELS IN URBAN DESIGN AND ARCHITECTURE

The choice which type of model to use depends on the intention with which reality is approached, the function the model must perform and personal preference of the person making the model. Generalising, we discern in an urban / architectural designing process the following steps (in iterative sequence):

- formulating objectives, programme of requirements, task formulation;
- analysing existing situations, together with its probable developments;
- design study;
- evaluating ('ex ante' and 'ex post').

Models are also used in study by design.

The design approach can be presented in a model: a verbal model (see figure 156). Before the start of the design process this verbal model is a planning or explorative model; when it is finished, a descriptive one (not necessarily the same model). It hinges on the condition that the working procedure demonstrates structure. A listing like 'and then…, and then,…and then…, is just a set of actions, not a model. This verbal model of inter-connections between the actions can be visualised: boxes, arrows, etc.; this is even to be preferred. However, it remains a verbal model, we could call a schema.

A programme of requirements (package of objectives) usually has a plan-like, verbal character. As far as is possible, it is recommended to translate verbal models into spatial ones: the 'medium switch' (figure 173).

While analysing an existing situation we use descriptive, conceptual or concrete models. Sometimes they are, in their turn, based on other descriptive models. A map showing the potentials of an area for playing field development, for instance, will probably be (partly) based on a soil map.[b] Depending on the kind of properties of the existing situation (including socio-cultural, economic and political-organisational conditions) an analysis model can be verbal, mathematical, spatial as well as mechanical. When analysing probable developments within the existing situation (trend prognoses) we use predictive models.

With a model of spatial reality a differentiation applies within the space modelled (this depends on the 'grain' chosen). Then it makes sense to use a spatial model. This encompasses the differentiating qualitative and quantitative attributes: figure 174a (see also figures 167, 175, 178). If spatial differentiation is not relevant, we can use a verbal model to indicate qualitative properties, and a mathematical model for quantitative ones. A verbal model describes, for instance, the existing building condition of a house or neighbourhood; or predicts them for the probable future (compare figure 174b). A mathematical model gives information, for instance, on the development of average occupancy of homes in a community (compare figure 174c).

Combinations of types of models are possible: see figure 175.

a VROM (1996) *Randstad en Groene Hart*.
b A potential chart can also be made on the basis of a designed situation that has not yet been realised: evaluation ex ante – see later in this paragraph.
a source: Hanwell, J.D. and M.D. Newson (1973) *Techniques in physical geography*, p. 78

In the case of design studies descriptive as well as prescriptive models are used (analyses of the existing situation and its developments); but also explorative-projective models. The resulting designs are always spatial models; they are either intentional-projective models (planning models) or potential-projective models (possible spatial futures).

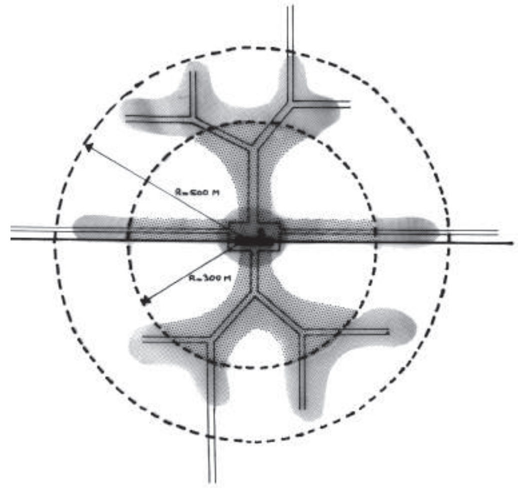
In urban design and architecture evaluation 'ex post' (empirical study) is rather unusual and less feasible with an a (urban) design transcending a certain scale.

An evaluation 'ex ante' of a design may focus on the degree to which the programme of requirements has been met, or on the effects in spatial, social and other terms the design will probably or possibly have after execution: effect analyses. With this type of evaluation one should always watch out for circular reasoning: "The high density of homes around the stations will have a positive effect on the quality of public transportation"; that may well be so, since that was precisely why that high density was chosen! Evaluation of effects not intended makes sense, as well as specifying intended effects.

In this type of evaluation we use predictive models that could in principle be verbal, mathematical, spatial or mechanical. However, verbal models seem to be most appropriate, because of the pseudo certainty associated with numbers, and often with spatial models as well.

In study by design, spatial models are used with an explorative potential-projective function. Effect analysis (evaluation 'ex ante') provides continuously feed-back during the study.

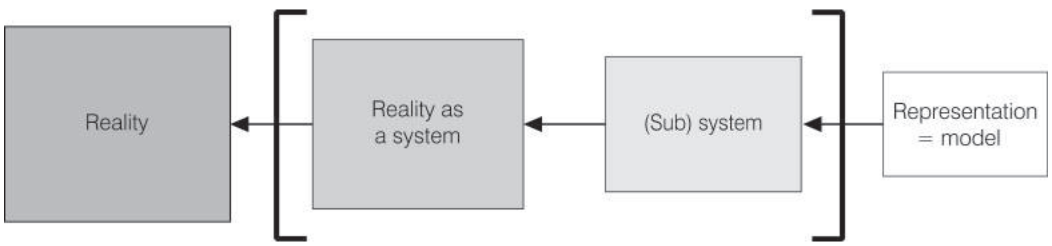
Image forming is a pre-requisite in visual-spatial processes of thought when intending to attain synthesis, states Muller.[a]



176 Principle model for the central part of a central town in a region: high concentration of facilities combined with intensive employment and residential levels round the train-/ regional bus station and along the (radial) main thorough-fares; declining density in the peripheral central areas. Mixing collective functions with the residential function originates in the wish to create conditions for social safety.

22.6 CONFUSING MODEL AND REALITY[b]

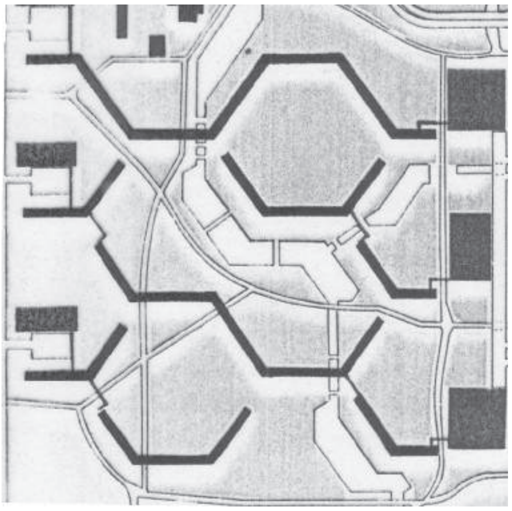
Whenever designers are not sufficiently conscious of the fact that they are working with models only of reality, and/or when they have insufficient insight in the relation between model and reality, mis-conceptions may occur about the possibilities as well as on the limitations of their designs and their analyses: model over-extension. 'The way back', from the model to (future) reality is not taken, or is taken in a wrong way. (figure 177; see also figure 155)



177 Translating a model to (future) reality: 'The way back': 'reductional' restrictions. Compare figure 155.

Before starting to work with a model, reflection is necessary on its application; pre-suppositions in that context should be checked. Outside the field of application, conclusions are not valid. From testing the model of an aeroplane in a wind-tunnel the conclusion may not be drawn, for instance, that the aircraft will fly in reality as well, since the conduct of the pilot has not been considered in the model.[c] In addition, the ratio between the size of the particles in the air with regard to the size of the model plane differs from the one applying in reality. If the model aeroplane is made of a different material that the real machine (think of architectural models) the field of application is even smaller.

a Muller, W. (1990) *Vormgeven, ordening en betekenisgeving*, p.142
b The present paragraph is based on Klaasen, I.T. (2000) *Valkuilen bij stedebouwkundig ontwerpen: verwarring tussen model en werkelijkheid*.
c Soest, J.P. van, J. van Kasteren et al. (1988) *De werkelijkheid van het model*.



178 The (has-been) surface articulation of the Bijlmermeer (Amsterdam South-East) (Source unknown). The design of this area was a good example of model over-extension. The re-structuring is in full swing

179 Circular residential building by Bofill in Marne-la-Vallée, near Paris. Noises in the heart of the circle are amplified many times by the circular construction (Photograph: Author)

For urban designers some relevant types of model over-extension are:

- field of application is lacking;
- no distinction between model and reality;
- insight into the way in which reality has been reduced in the model is insufficient;
- spatial and temporal confusion of scale;
- confusion of stand-points from which observations are made.

An example of confusion between model and reality is depicted in figure 178. In this design the surface articulation is insufficiently adapted to the specific. The result is very monotonous. This pitfall is known as 'stamping' (*stempelen*).

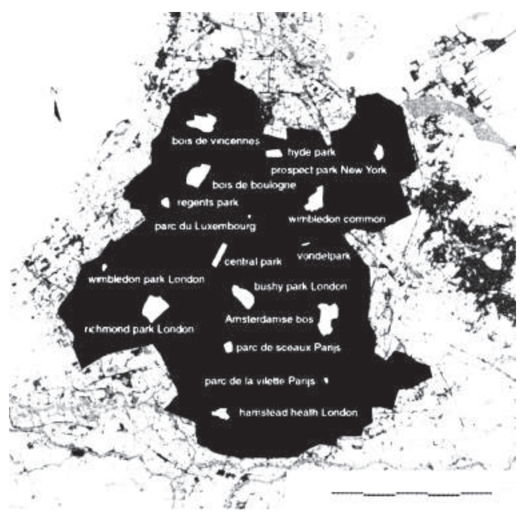
In this example it could also be true that confusion of observation standpoints raises its ugly head. A not uncommon mis-conception among designers – alas – is that the reality designed will be experienced, before too long, from 'above'; just like the designer experiences drawing board or computer screen: 'the drawing board perspective'. The observer-in-reality stands or hangs above an area only occasionally. He stands right in the middle of that reality, or at some distance, moves along it, or through it.

Another example is that designers, traditionally strongly focused on visual perception, neglect noise, smell and physical sensations (wind!) or disregard them, because they are hard to depict. The value of a design – in the sense of its implementation potential, and after implementation, its usefulness – depends on the degree in which specific conditions of a site have been taken into account, even if properties are invisible, not to be depicted in a spatial model (spatial analyses), or reduced out of existence (figure 179).

"The disadvantage of designing by drawing is that problems which are not visually apparent tend not to come to the designers attention. Architects could not 'see' the social problems associated with new forms of housing by looking at their drawings".[a]

When one is conscious of the fact that in a model of a/the (possible, future) reality, a lot of reality has been omitted, it will be clear that two situations can not be compared bluntly. The historical island Marken in the former 'Zuiderzee' (IJsselmeer) may be just about as large as a yet to be constructed, artificial island in the outskirts of Amsterdam ('Haveneiland', part of the development 'IJburg'), but its distance from the centre of Amsterdam, for instance, defies comparison. The same applies of course for its demographic structure.

The failure is complete when referring to another situation spatial scales are confused (figure 180).



180 De Minister of Physical Planning of the Netherlands compared the 'Green Heart' of the 'Randstad Holland' to Central Park, Manhattan, New York City. The 'Amsterdamse Bos' is already two and a half times as large as that New York park! If the scale ratio of Amsterdam and Manhattan are taken into account, and the kind of utilisation qualities that apply, the Amsterdam Vondelpark and Central Park do have many things in common. (Source: Berg, R. van der (2001) *NL Superbia*)

a Lawson, B.R. (1990) *How designers think, the design process demystified, p. 18/19.*

23 VERBAL MODELS

TAEKE DE JONG

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This Chapter discusses verbal models in empirical science. In that context they are logically consistent by definition.[a] However, they will be treated within the context of two other language games just as relevant to Architecture: design and management, (see page 446) where integration and urgency are more important than logical consistency. That is the reason why differences in emphasis on consistency will be discussed as well. Consistency seems to necessitate incompleteness. For analysis this is less dangerous than for synthesis.

Consistency is denoted here by the formal logical model. With this, the relationship of verbal models with reality (reliability) is coming to the fore in the sense of truth or non-truth. However, on a different level incompleteness is remaining a special form of non-truth (half-truth). This demonstrates the restricted contribution of formal logic to the designing of models during the originating stages, when their consistency does not exist as yet, but must be made. That is a different language game. However, this does not detract at all from the importance of formal logic in the discussion of the still varying (increasing and decrasing) consistency aimed at whilst designing. It does not detract at all from the importance of formal logic during the evaluation of the design as soon as it is available in all its completeness. Also in that case the question is raised whether inconsistency is so ‘dangerous’. That is a language game as well. One should never forget that a crystal can not grow without a dislocation in its grid.

Next, the subject of causal consistency comes to the fore obviously. However, this form of consistency will be discussed later in the Chapter *‘Forecasting and Problem Spotting’* (see page 253). The question of incompleteness will get on the agenda again there, then explicitly in the sense of *‘ceteris paribus’*.

23.1 LANGUAGE GAMES

In architecture three distinct language games[b] occur: those of designers, scholars and decision makers (respectively orientated on ‘being able’, ‘being knowledgeable’ and ‘being decisive’). The utterances of these agents in the building process (respectively ‘possible’, ‘true’ or ‘binding’ or not) can not be expressed completely in one another’s language, even when they are using the same words. By this they are causing linguistic confusions hard to disentangle, addressing respectively possible, probable and desirable futures. With it, temporal-spatial completeness, logical consistency and public urgency are becoming topics, respectively.

Grammatically ‘verbs of modality’[c] are reflecting the opinion of the speaker on the relationship to reality of what he is saying: possible (‘can’, ‘may’), probable (‘will’, ‘must’, ‘let’) and desirable (‘will’, ‘must’, and ‘may’; the latter two in a different sense). The language games introduced by this type of verbs are based on different reduction of imaginative realities.

The primary language of design is pictorial. When the designer records the key to symbols (legend) of his drawing, for instance red for urban areas, yellow for agriculture, and blue for water, he reduces the variation within the urban area, agriculture and the water. If he makes his drawing with pre-supposed legend unities, he first selects their site and form (state of dispersion) roughly and subsequently more precisely. So, during the design process he reduces further the tolerances of the design for the benefit of its feasibility.

The empirical researcher reduces reality in more abstract variables (set of related differences), but does not accept that a variable may assume any arbitrary value. He looks for functions between variables to restrict them in their freedom of change in order to make more precise predictions.

The policy maker reduces the problems to a few items on the agenda and tries to reach consensus by arrangements and appointments.

Language games:	being able	knowing	selecting
Modes:	possible	probable	desirable
Sectors:	technique	science	administration
Activities:	design	research	policy
Reductions as to:			
Character:	legend	variable	*agenda*
Space or time:	*tolerance*	*relations*	*appointments*

181 Modal language games

a Nauta, D. (1970) *Logica en model*.

b Wittgenstein, L. (1953) *Philosophische Untersuchungen*. Recent edition: Wittgenstein, L. and G.E.M. Anscombe (1997) *Philosophical investigations*.

c Toorn, M.C. van den (1977) *Nederlandse Grammatica*, p. 91

23.2 MODALITIES

The empirical researcher is using, speaking strictly, exclusively logically consistent models, constructed from well defined concepts and variables.

Already in preparing the legends of his design the designer is disregarding components that do not belong to the legend-unit chosen strictly speaking (like parks in a city, green in red); or he is doing the opposite: over-emphasising details opening up possibilities. Furthermore, the designer is using them during his discourse in a variable significance in order to create intellectual space for the designing. To an empirical scholar the language of the designer is then ambiguous, or poly-interpretable and suggestive. The terms 'red', 'green' and 'blue' are already as variables not well defined, but they are more complete, also given the future possibilities. Utterances on the possible worlds of the design belong partially to 'modal logic', not discussed here.

The agenda of the policy maker is extremely incomplete by necessity. That is the reason why it is an art to get a topic 'on the agenda' of a meeting.

23.3 CHANGE OF ABSTRACTION

If the subject of a sentence is a concrete, touchable, visual, audible reality, the utterance is belonging to the concrete object language; in other cases to a more abstract meta-language (a distinction taken from the logic of classes).[a] Speaking about utterances, as happens in logic and in almost all sentences of the present Chapter, is belonging to a meta-language. This distinction is pre-empting paradoxes occurring if object language and meta-language are mixed within one full-sentence (change of abstraction, see page 37) as in the full-sentence 'What I am saying is a lie'.[b] In written language this is indicated by quotation marks. The change of abstraction may then be indicated by: 'What I am saying' now is a lie. In its turn the meta-language is layered, for if one is talking about logic, one is talking about a meta-language. One is finding oneself then in a high class of abstraction. The object language is layered as well, since one can talk on objects of a different scale, particularly in urban architecture. Changing of scale in a line of reasoning may lead to paradoxes; just think of 'inside' and 'outside', respectively on the levels of room, house, city). This is pre-empted by articulation of scale, as explained on page 37.

23.4 VERBS

Next to verbs of modality (can, shall, may must, will, let be to, dare to, serve to, need to, promise to, threaten to) there are countless independent verbs that may be 'steered' by them ('This building can collapse | has sagged | is being repaired'). By preceding such verbs of modality they can be harnessed for a specific language game (possible, desirable, probable).

Independent verbs are always pointing to a working (a function) or to a 'property' as a result thereof (for instance: 'This building sags'). When the full-sentence is employed in the language game of empirical study a subject from the 'existing' reality is described. It is also possible then to speak of 'models'.

23.5 CONSTRUCTING STATEMENTS

A full-sentence like 'This building is a cube' is providing a (always incomplete) description (predicate) of a subject[c] (in this case in object language a building that may be pointed at). This full-sentence establishes through the verb 'is' a relationship between this special building and more general, compressed earlier experiences ('cube' as an empirical concept of a lower class than the corresponding abstract geometrical concept).[d] 'Buildings are rectangular' is a description of all buildings with, in addition, a more general predicate than 'cube'. What is pronounced in it is not the case, as we know. The world is everything that is the case.[e] Language is also comprising negation of what is the case, and is pre-supposing the capability to imagine; it is possible to speak about what is not the case.

a Whitehead, A.N. and B. Russell (1910) *Principia mathematica*.
b A variant of Emimedes' paradox: 'All Cretans lie, said the Cretan'. If he lies, he speaks the truth; and vice-versa.
c Note, that the building as a real object outside of the sentence is acting as a subject within the sentence. This inversion is characteristic for each form of abstraction.
d Con-cept is Latin for 'taking together'.
e The first proposition of Wittgenstein, L. (1922) *Tractatus logico-philosophicus*. Recent edition: Wittgenstein, L., Pears D.F. et al. (2001) *Tractatus logico-philosophicus*.

A full-sentence always consists of subject and predicate. An utterance is a full-sentence that is the case or not. A design, an order or a vague, ambiguous full-sentence is, for instance, no utterance. Predicate logic is studying the internal construction of utterances and proposition logic their conjoining into assertions (propositions).

23.6 CONJOINING INTO ASSERTIONS

If more predicates are referring to the subject, one must conjoin with words such as 'and', 'not and', 'or', 'neither…nor', 'if…then' single utterances into an assertion:

> 'This building is a cube *and* (this building) is rectangular'
> 'This building is a cube *or* rectangular'
> 'This building is *neither* a cube *nor* rectangular'
> '*If* this building is a cube, *then* it is rectangular'

'If a building is a cube, then it is rectangular' is always true, even if the building we are pointing at is no cube, and even if it is not rectangular.[a] Words that are composing utterances into an assertion, such as 'if…then', 'and' 'or', 'nor' can make the assertion they are composing become true, even if not all parts of the assertion are the case.[b] Proposition logic is studying this truth-determining operation.

23.7 COMPOSING LINES OF REASONING

In their turn, assertions may be composed into a line of reasoning by drawing a conclusion from premises. In contrast with utterances in an assertion, all premises in a line of reasoning must be true in order to draw a correct conclusion. The other way around, correctness of a conclusion is not assured even if all the premises are true.

In the following example the premises (above the dotted line) may be true, but the conclusion (below the line) is not valid.

> 'If this building is a cube, then it is rectangular'
> 'This building is rectangular'
> ———————————————————————— so
> 'This building is a cube'

This line of reasoning can not be endorsed: purely on the ground of its structure, independent from our experience with cubes and straight angles. However, it is useful in the modality of what is possible. Then the conclusion must be: 'This building may be a cube': then it is valid again. The line of reasoning is also valid when the last premise is inter-changed with the conclusion:

> 'If this building is a cube, then it is rectangular'
> 'This building is a cube'
> ———————————————————————— so
> 'This building is rectangular'

A line of reasoning with two premises and one conclusion, is known as a syllogism. A line of reasoning from general to particular is deductive, from particular to general inductive.

An inductive line of reasoning is not valid if the set of premises does not comprise all cases. One can only draw the conclusion that all buildings are rectangular, if one has checked all buildings, while observing for each building: 'This building is rectangular'. There are then as many premises as there are buildings. It is only then that one can draw by complete induction the general conclusion that all buildings are rectangular. Yet, this completeness is virtual. What to do when one is finding buildings in a linguistic environment where the concept 'rectangular' does not exist, or is starting to apply at a certain length of both legs of the straight angle?

Study of the structure and validity of lines of reasoning, independent of their meaning (semantics) is the classical aim of logic (argumentation theory).[c] This entails, in a sequence of a decreasing complexity:

a Note, that 'being the case' relates to parts of the statement and 'being true' to its totality.

b We only talk about (un)truth when talking about statements. (Un)truth is, therefore, always a term from a meta-language.

c See: Eemeren, F.H. van (1996) *Fundamentals of argumentation theory, a handbook of historical backgrounds and contemporary developments*. Dutch translation: Eemeren, F.H. van, R. Grootendorst et al. (1997) *Handboek argumentatietheorie, historische achtergronden en hedendaagse ontwikkelingen*.

- set theory;
- modal logic (language games, modalities);
- class logic (level of abstraction);
- argumentation theory (lines of reasoning);
- proposition logic (assertions), and:
- predicate logic (utterances).

We are starting unconventionally with the smallest unit, the singular utterance, and within it the predicate and within that the full-sentence function.

23.8 FULL-SENTENCE FUNCTIONS AND FULL-SENTENCES

In predicate logic the structure of some assertions discussed here is usually rendered as follows (read for x ‘this building’):

182 Operations with full-sentence functions

Formula:	*Read:*
$K(x)$	being a cube as a working (function) of x.
$R(x)$	being rectangular as a working (function) of x.
$\exists x{:}K(x)$	there exists a x, ($\exists$ x), for which it is valid that (:) it is cubic (K(x)).
$\forall x{:}R(x)$	for each x ($\forall$ x) is valid that (:) x is rectangular.
$\forall x{:}(K(x) \Rightarrow R(x))$	for each x ($\forall$ x) is valid that (:) if x is cubic, x is rectangular as well.

K(x) and R(x) are full-sentence functions, names for a working of their argument (x), but not yet full-sentences themselves. The full-sentence functions are lacking a verb that the working of the argument, possibly on an object, is operationalising. Full-sentence functions are predicates without a verb. In addition a full-sentence is in need of a subject, an instancing of the argument (for example x: = this building).

In the language game of the designer full-sentence functions like villa(landscape) – ‘villa as a working of the landscape’ -, or landscape(villa)’- ‘landscape as working of the villa’ are operationalised only by the design. The working itself is not made explicit with a verb, unless it may be termed a design act. Often verbs like that do not exist; their existence is just suggested by the full-sentence function. In addition only the object of the predicate has been named, so that of the working just the object of operating has been named. These full-sentence functions are so useful particularly in this language game since just the direction of the working between the subject and the object is recorded.

In the language game of policy one is waiting for the verdict of a judge or decision of the board. The relation victim(suspect) must be made by juridical investigation in order to come to a ruling. A policy agenda must be become operational in agreements.

In empirical sciences it is precisely the trick to find for such a full-sentence function a formula or (weaker) a formulation, that is making it operational. Exact mathematical operationalising of a full-sentence function is called modelling.

However, a full-sentence function is in empirical study very useful for a function that has as yet not been made explicit in the problem formulation and the forming of a hypothesis; since assertions are in them not yet expected. For instance, one may surmise that the number of buildings or their volume G is a dependent variable of the population variables p and their prosperity w, considered to be independent for the time being. This working is readily noted as a full-sentence function: G(p,w). In that case the problem formulation is ‘To which degree and how is G dependent on p en w?’ A hypothesis that has become operational may read: G(p,w) = p*w. The operator (*) makes the working explicit; the full-sentence function has become a function.

2.9 FUNCTIONS

A full-sentence function becomes a function, when that function K( ) or R( ) has been made explicitly ‘operational’ (e.g.: ‘K( ) := being a cube.’ Or R( ) := being rectangular). The more explicit operationalisation of the ‘being a cube of x’ is more complicated than of the ‘being the square of x’.

K( ) may be defined, for instance, also as ‘being squared’. This is becoming operational in a mathematical formula by K(x) := x*x. The multiplication sign (*) is a mathematical verb (operator) for ‘multiplied by’ that was not yet explicit in K( ).

The symbol := in this formula means ‘is defined as’ or ‘is per definition equal to’. So it is an operator as well, but it belongs to a meta-language *vis-à-vis* the terms at both sides of this operator. It has an essentially different meaning than the = sign (‘is equal to’ for calculations). By the same token the verb ‘is’ is ambiguous. That is the reason why well defined symbols originate making a distinction between := and =. In the same vein there is a logical :⇔ sign (‘is equivalent to’) that can be used to denote a logical equivalence, for example ‘is defined as’ or ‘is per definition equal to’.

In their turn functions are becoming only an assertion (the case or not the case) if the subject x has been substituted (for instance x := ‘this building’). If a full-sentence function can be translated by substitution in an assertion that is the case, then it is ‘completable’. $K(x):=x^2$ is completable for $x \in \mathbb{R}$ (x as an element of the set of real numbers), but not for x ∈ of the set of buildings.

The full-sentence function may supply more than one subject with a predicate (here p and w). It is then at home on several places. However, if one wants to include more predicates or within them more objects (not just buildings are dependent on population and prosperity, cars as well), there must also be more full-sentence functions. These can be conjoined with the linkage words from proposition logic, like ⇒ to an assertion like K(x) ⇒ R(x).

23.10 A QUANTOR AS SUBJECT

In order to yield a meaningful assertion, the subject of a full-sentence does not need to be one concrete subject. Instead of defining x precisely one to one with reality (name giving) it can also be bound. This is particularly important to mathematics. Also ∃ x (‘At least one x’, existence-quantor) or ∀ x (‘Each x’, all-quantor) may yield a logically acceptable subject. In conjunction with the verb ‘:’ (‘satisfies’) and a full-sentence they may form an assertion. The assertion reads then, for instance, as ∃ x : K(x), ’At least one building satisfies the description ‘cube” or ∀ x : K(x), ‘Each building satisfies the description ‘cube”. The second, more general, assertion pre-supposes excellent scholarly breeding. However a generalising scholarly discipline is always looking for assertions with an all-quantor, since such a general assertion enables in a line of reasoning as a premise a wealth of deductive conclusions:

	Formula	*Read:*
1	$\forall x:(x \in X)$	For each x (∀x) it is valid that (:) x is an element from the set X.
2	$\forall x:K(x)$	For each x (∀x) it is valid that (:) x is a cube.
3	$\forall :(K(x) \Rightarrow R(x))$	For each x (∀x) it is valid that (:) if x is a cube, x is also rectangular.
4	$\exists a:(a \in X)$	There is at least one a (∃a) for which it is valid that (:) a is an element of the set X.
5	$\exists a:R(a)$	There is an (∃a) for which is valid that (:) a is rectangular (R(a)).

183 Quantor functions

Now we know that the second premise is not the case, if we substitute for x ‘building’ as an element of the set *all* buildings X. In order to get nevertheless a relatively general assertion, one must restrict the set, for instance to the set ‘buildings in this neighbourhood’ B, if we know by complete induction that in this neighbourhood all buildings are cubes. Then it is sufficient to change the first premise into $\forall x:((x \in X) \wedge (x \in B))$. The symbol ‘∧‘ in this formula means ‘and’ in a sense well-defined in proposition logic.

23.11 THE CASE OR NOT THE CASE

Formal – mathematical feasible - logic, developed during previous centuries, is a more narrow notion than the concept ‘logic’ used to be in olden days. The word ‘logic’ is derived from the word *'logos'*, a Greek word encompassing two illuminating clusters of meaning: speech itself, and giving account, testimonial. Logic as discussed here corresponds especially to the second cluster, as the lore of the right deductions.[a] The smallest possible 'im-mediate' deduction consists out of two propositions, separated by the two-letterword 'so': Holland is in The Netherlands, so The Netherlands are larger than Holland. More common is the 'mediate' deduction of a third proposition, a conclusion C, from two preceding premises A and B ('syllogism'), usually denoted as: A,B | C or

A If it is winter, I am cold.
B It is winter
______________________________ so
C I am cold.

Logic pre-supposes here, that propositions exist that may be the case, or not, but not both (yielding a contradiction) or both a little. This last restriction is removed in 'fuzzy logic', a branch of modern logic, disregarded in the following.

23.12 THE HUMAN POSSIBILITY TO DENY

A description of observations along these lines is only feasible, if we can imagine facts that are not the case. According to the Swiss psychologist Piaget, this capacity emerges in children when they are some eighteen months old. The capacity is hard to determine when it comes to animals, because they cannot express themselves to us in a way we can understand. Our brain must offer space to the not-here-and-now.

A filing cabinet should be ready there, as if it were, with the image 'it is winter', 'it is not winter', 'I am cold' and 'I am warm. As soon as something is the case, the box is full, as soon something is not the case, the box is empty. This has created space in our imagination for the true *and* false and thus for lies and deceit, but also for abstract thought and for the designing of things that are not (yet) there. Only with such an imaginative capacity (a 'logical space') at our disposal, can we arrive at rather general assertions like: 'If the sun starts shining, then I get warm'.

23.13 IF...

The following paragraphs provide an introduction into proposition logic on the basis of one of the most frequently used, and at the same time most confusing, logical operators, the word 'If…'. The 'if … then …' relation is of great interest to designers, since every design is an image of things that do not exist with an implicit promise: 'If you execute this, then you can dwell!'.

Compare the following assertions:

1 If it is winter, I am cold.
2 If it is winter, I could be cold.
3 If I am cold, then it is winter.
4 If I am cold, then it could be winter.
5 I only get cold if it is winter.
6 I am sometimes cold if it is winter.
7 I am always cold if it is winter
8 If it is winter, then I will probably be cold.
9 If it is winter, then you should turn on the heater, or else I will be cold.
10 I would like it to be winter because I am so warm.

The last expression is a wish, with on its background a lot of logical and causal pre-suppositions. The wish itself and its motivation do not belong to the linguistic game of logic, neither does the command (9) preceding it. In *both* expressions the hidden supposition "I will probably get warm" is a prediction or expectation that only becomes a fact, so true or false, if I really got warm. The grain of time is too small in the case to claim unambiguously a true or a false statement. Logic is necessary to arrive at such an expectation, but expectation itself

a More thorough introductions: Jong, W.R. de (1988) *Formele logika, een inleiding;* Eijck, J. van and E. Thijsse (1989) *Logica voor alfa's en informatici;* Sanford, D.H. (1989) *If P then Q, conditionals and foundations of reasoning;* Benthem, J.F.A.K. van, H.P. van Ditmarsch et al. (1994) *Logica voor informatici*.

surpasses the laws of logic. Assertions 1- 8 may be translated without complications into statements of the proposition-logical type.

23.14 STRESSING THE LOGICAL FORM

In order to study the type and its associated validity of the 'if..then..' relation as such, it is necessary that for the assertions used, any other assertion could be substituted without impairing the validity of the logical form itself.

If somebody makes an assertion, the logical investigation consists, therefore, especially in the search for counter-examples for which that type of the deduction becomes false. The assertions in the deduction are made variable then and the deduction gets the more abstract form 'if p then q'.[a] To avoid possible confusion, we choose an example of the first assertion where the temporal aspect is absent:

'If I am in Holland, then I am in The Netherlands'.

It is remarkable that this assertion can be 'true' if the partial statements are not the case, for instance, if I am in Hamburg. I am not in Holland then, not in The Netherlands, but *if* I am in Holland, I am also in The Netherlands, that stays 'true', even in Hamburg. It is also true when I am in Breda or of course, in Holland. The only case when I cannot uphold my assertion is when I am in Holland and it comes out that I am not in The Netherlands.

The truth-value of the assertion as a whole, depends this way on a specific combination of truth- values of the sub-statements 'I am in Holland' (P) and 'I am in The Netherlands' (Q). This may be summarised in a 'truth-table'. There are four possibilities:

	I am in Holland	I am in The Netherlands	Example	'If P then Q'
	P	Q		$P \Rightarrow Q$
1	Not the case	Not the case	Hamburg	True
2	Not the case	The case	Breda	True
3	The case	The case	Delft	True
4	The case	Not the case	?	False

184 If truth table

23.15 DIFFERENT KINDS OF IF-STATEMENTS.

That was a clear example. But, if one returns to the old example 'If it is winter, then I am cold' and substitutes it, according to this table, I would be allowed to say 'If it is not winter, then I am not cold'.

If someone has difficulty with that, it may be that he still values implicit causal presupposition.[b] It might also be that he envisages another 'If…then..' relation than the one above, to wit 'If and only if' (iff):

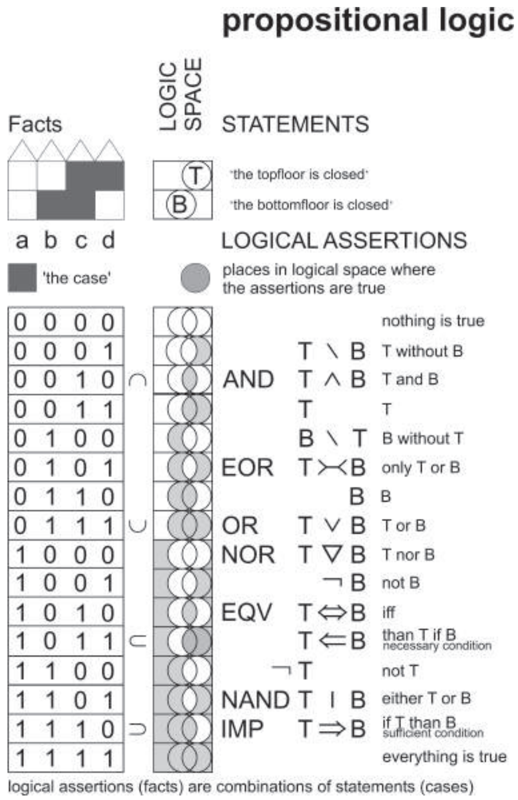
	It is winter	I am cold	Example	Iff
	P	Q		$P \Leftrightarrow Q$
1	Not the case	Not the case	no winter, not cold	True
2	Not the case	The case	no winter, cold	False
3	The case	The case	winter, cold	True
4	The case	Not the case	winter, not cold	False

185 Iff truth table

If we substitute this example again in that of paragraph 23.14 case 1 proves to be not to our liking. Suppose I am in Breda and say to a southerner 'If I am in Holland, then I am in The Netherlands'. He is of the opinion, that I intend this reversibly and answers that that is not true, because I am not in Holland and yet in the Netherlands. Guilelessly, I mean the implication; he thinks that I mean the offending equivalence. I hope he knows the truth tables, for else this mis-understanding will never be sorted out.

a That is, by the way, also the title of a fine book on the history of logic: Sanford, D.H. (1989) *If P then Q, conditionals and foundations of reasoning*.

b Hawkins, D.J.B. (1937) *Causality and implication*.



186 Complete truth table

23.16 DISTINCTION BY TRUTH-TABLES

This shows how useful it is, that formal logic has developed different symbols (*implication* ⇒ and *equivalence* ⇔) and different logical operators for this confusing 'If.. then..' proposition. This distinction was possible by controlling the truth of the 'If P then Q' statement for each of the four states of affairs where P en Q can be combined. For '⇒' it turned out to be the sequence true, true, true and false (simplified by 1110), but for '⇔' it turned out to be true, false, true and false (simplified by 1010).

Do the other combinations like 0000, 0001, 0010, etc. also mean something? It is easy to ascertain, that there are 16 such combinations that we can summarise in a table. A complete table like this appeared for the first time almost simultaneously shortly after the end of WW I in Wittgenstein's '*Tractatus*'[a] and with two other authors.

23.17 SUFFICIENT CONDITION

Just suppose, that four situations a, b, c and d are discerned for a residence under construction expressed in the combination of two assertions: 'The top floor has been provided with a façade ' T, the bottom floor has been provided with a façade B, and the situation in which this is not the case. For these four cases a, b, c and d we verify now the assertion 'If the top floor has been provided with a façade, then the bottom floor has also been provided with a façade', crisply expressed as: 'T ⇒ B':

187 If truth table

	Topfloor closed	Bottomfloor closed	Example	'If T then B'
	T	B		T ⇒ B
1	Not the case	Not the case	a	True
2	Not the case	The case	b	True
3	The case	The case	c	True
4	The case	Not the case	d	False

Only if the top floor has been provided with a façade, and the bottom floor not, we can not validate the assertion 'If the top floor has been provided by a façade, then the bottom floor has been provided with a façade as well'. So we have verified that 'T ⇒ B' is true for the first three cases, but not for the last case: (1110, 'sufficient condition').

23.18 EQUIVALENCE

Now, if a contractor is saying: 'If the top floor has been provided by a façade, *only* then also the bottom floor has been provided with a façade', case b is also invalid (1010, then and only then if, 'taoti', 'equivalence' iff).

188 Iff truth table

	Topfloor closed	Bottomfloor closed	Example	'If T then B'
	T	B		T ⇔ B
1	Not the case	Not the case	a	True
2	Not the case	The case	b	False
3	The case	The case	c	True
4	The case	Not the case	d	False

23.19 NECESSARY CONDITION

However, if the contractor is saying: 'Only if the top floor has been provided with a façade, then the bottom floor has been provided with a façade', then case d is suddenly valid again, but only case b not (1011, 'necessary condition').

189 Then … if truth table

	Topfloor closed	Bottomfloor closed	Example	'If T then B'
	T	B		T ⇐ B
1	Not the case	Not the case	a	True
2	Not the case	The case	b	False
3	The case	The case	c	True
4	The case	Not the case	d	True

a Wittgenstein, L. (1922) *Tractatus logico-philosophicus*.

Each known logical operator like $\Rightarrow$, $\Leftrightarrow$ and $\Leftarrow$, for example 'and', 'or', 'neither..nor', 'either..or', proves to have a place on a truthtable (see diagram). Logical operators are more readily understood as equivalents of the set theoretical concepts $\cap$, $\cup$, $\subset$, $\supset$ or from drawings in which the sets are overlapping.

Symbolical rendering and its definition with the truth-table is now making an unambiguous distinction between the inclusive 'or' ($\vee$, OR) and the exclusive 'either …or' (>-<, EOR, XOR). The confusion of 'and' ($\wedge$), and the inclusive 'and' in the sense of 'and/ or', the logical 'or' ($\vee$) in daily parlance can not occur anymore. These logical operators should not be confused with sequential computer commands in an algorithm, such as the 'IF…THEN…' statement. That belongs to a different language game: the one of commands used for the execution of certain activities.

23.20 MODUS PONENS, TOLLENS AND ABDUCTION

In the examples below we assume that the implication ($\Rightarrow$) is intended throughout.
We accept the following deduction:

(1) If I am in Delft, then I am in The Netherlands.
Well: I am in Delft
———————————————— So:
I am in The Netherlands

We do not accept:

(2) If I am in Delft, then I am in The Netherlands.
Well: I am in The Netherlands.
———————————————— So:
I am in Delft.

Yet we accept:

(3) If I am in Delft, then I am in The Netherlands.
Well: I am not in The Netherlands.
———————————————— So:
I am not in Delft.

This seems obvious with examples directly connectible to enclosing sets (Delft is *in* The Netherlands), but why should we not accept (2) for example:

(2*) If it is winter, then I am cold.
Well, I am cold.
———————————————— So:
It is winter.

if we accept:

(3*) If it is winter, then I am cold.
Well, I am not cold.
———————————————— So:
It is not winter.

In these examples causal explanations are playing a confusing rôle. We know that the examples 2 and 2* are logically not valid, but this line of reasoning is often used in medical practice, historiography, forming empirical hypotheses and in legal matters.

Suppose, a murder has been committed:

(2**) If X commits a murder, one finds his DNA
Well, his DNA has been found
———————————————— So:
X has committed the murder

(3**) If X commits a murder, one finds his DNA
Well, his DNA has not been found
———————————————— So:
X has not committed the murder

Examples 1 and 3 are known, respectively, as *'modus ponens'* and *'modus tollens'*. Peirce has called the logically not-valid line of reasoning of form 2 'abduction'.[a] Abduction is used for finding a cause, even when one can never be sure of it.

a Peirce, C.S. (1992) *Deduction, induction, and hypothesis*

23.21 VERIFYING LINES OF REASONING

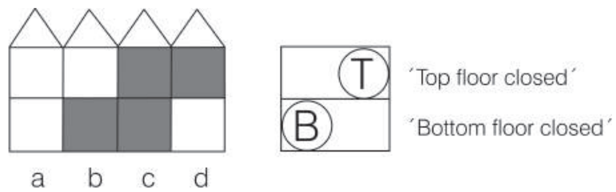
We focus here on an example used previously in which no set theoretical or obvious causal connections are clashing with the logical connections.

(1) If top floor closed, bottom floor closed
Top floor closed (T)
———————————————— So:
Bottom floor closed (B)

(2) If top floor closed, bottom floor closed
Bottom floor closed (B)
———————————————— So:
Bottom floor closed (T)

(3) If top floor closed. Bottom floor closed
Bottom floor not closed (not B)
———————————————— So:
Top floor closed (not T)

Again, we are distinguishing the following state-of-things (situations):



190 Three situations

and render the case and not the case with utterances, true and untrue with assertions both with respectively 1 and 0 in order to verify tree lines of reasoning:

	1 modus ponens			2 abduction			3 modus tollens		
	T ⇒ B	T	B	T ⇒ B	B	T	T ⇒ B	not B	not T
a	1	0	0	1	0	0	1	1	1
b	1	0	1	1	1	0	1	0	1
c	1	1	1	1	1	1	1	0	0
d	0	1	0	0	0	1	0	1	0

191 Modus ponens, tollens, abduction

For T ⇒ B the well-known substitution has been given, the assertions T, B, not T and not B have been derived from the drawing. Lines of reasoning are valid, when the premises and the conclusion are all true; or are ‘the case’ (1). With abduction there is a chance in situation b that conclusion T is not the case, even if both premises are true or the case.

So, one is not permitted to inter-change without damage premise and conclusion. A deduction is ‘valid’ when it is impossible to construct a counter example where the propositions are ‘true’, while the conclusion is ‘false’ (b). If one would accept that, any conclusion would be allowed.

23.22 INDUCTION

Lines of reasoning do not need to make use of an 'if…then..' operator. In the form of examples, we will now make use of the quantors and the 'and' operator in order to add a new form of reasoning. The first three of the following examples are known by now as deduction (1 and 3) and abduction (2). The fourth (4) was noted previously in page 191 as induction; although it is probably an incomplete induction here.

(1) All houses in this neighbourhood are a cube
This house is in this neighbourhood
——————————————————— So:
This house is a cube

(2) All houses in this neighbourhood are a cube
This house is a cube
——————————————————— So:
this house is in this neighbourhood

(3) All houses in this neighbourhood are a cube
This house is not a cube
——————————————————— So:
This house is not in this neighbourhood

(4) This house is in this neighbourhood and is a cube
Also this house is in this neighbourhood and is a cube
Also this house is in this neighbourhood and is a cube
——————————————————— So:
All houses in this neighbourhood are a cube

For the first three forms of reasoning a general rule prevailed, but how to lay hands on such a rule? Example (4) enables this to happen by empirical induction. Since this is seldom complete, empirical science largely consists out of collecting samples. They must be statistically representative for the whole set studied in order to be able to draw a more general probable (not necessary) conclusion (generalisation). The tacit reasoning underlying this pre-supposition looks like an abduction. The more general rule may be used next in its turn in logically valid deductive forms of reasoning as a premise in order to make forecasts.

23.22 INNODUCTION

The example following does not belong to the logical language game, not even anymore to the language game of the empirical. The 'But' is marking an inductive part, the 'So' a deductive part. Without 'But' the reasoning is resembling abduction, but a negation has been inserted that yielded between (2) and (3) already a valid reasoning.

(5) I am not warm.
——————————————————— But:
If I build a house, then I am warm.
——————————————————— So:
I build a house.

This line of reasoning is important to designing. It is a variant of *innoduction*.[a] A line and a new fact (in the original sense of *'factum'*, Latin for 'made') is added to the assertive premise 'I am not warm'. The line between 'But' and 'So' is no premise and no conclusion in the classical sense of the word. It is a new idea and a pre-supposition, construed on occasion of and following from (and so not on an even ranking with) the asserting premise. There also could have stood 'If a build a moderated microwave in my coat, then I've got it warm'. Although no premise, each change in this assertion is affecting the conclusion immediately. The 'But' signifies a shift in the passively asserting language game to an active pragmatic language game. It is introducing a negation of what is the case.

23.24 THE EMPIRICAL CYCLE

Now compare the following description (1), proposition (2), deduction (3) causal explanation. Do they have a 'logical form' in common with the world?

1. It is winter and I am cold.
2. If it is winter, I am cold.
3. It is winter, so I am cold.
4. It is winter, hence I am cold.
5. It is winter, but I am not cold.

In all these cases two propositions are connected: 'it is winter' and 'I am cold'. They have been connected with the word 'and', 'if…', 'so', 'hence' and 'but not', depending on the stage of our intellectual processing of our impressions (the 'empirical cycle', see page 249).

If I have experienced (1) repeatedly, I can conclude (2) for the time being. This kind of conclusion leads from specific statements to a more general one (induction). From this more general proposition, another specific statement (3) may be deducted (deduction). The third statement is an incomplete syllogism, since (2) is not mentioned. In the practice of language there is quite a lot not mentioned.

23.25 TACIT PRE-SUPPOSITIONS

Any reasoning lacks lots of premises, for example 'suppose we are human, suppose we have thoughts and a language to communicate, suppose you want to listen to me, suppose you do not kill me for what I say, suppose this building does not collapse, then I could tell you something'.

Culture contains a huge reservoir of unmentioned pre-suppositions. In the practice of language that is efficient, but it makes different cultures hard to understand. Making cultural pre-suppositions explicit is as hard as to get a description of water from a fish. The fish cannot compare its element with something else: for a description of the water, the possibility of its negation is necessary. Without difference, nothing can be perceived, chosen, described or thought.[b]

Also the general statement 'If it is winter, then I am cold', is only under certain pre-suppositions a fact, as long as we do not turn the heater on, put on warm cloths, take a warm shower, etc. Logic is oblivious of these conditions that are often so interesting to a designer by pre-supposing implicitly that the other circumstances stay equal (*ceteris paribus*).

23.26 PERCEPTION

The *expression* of a perception is closest to the 'world'; *facts* are perceived and expressed in a sentence. Consider the next example:

If the sun starts shining, I get warm
The sun starts shining
———————————————————— So:
I get warm

According to Wittgenstein[a] the world is the totality of the *connections* (facts), not of the *things.* Basically I do not perceive the sun as a thing, but as a 'shining connection', for my first perception is 'something shines' (compare, 'something moves'), next I ask myself: 'What is that?' 'Something shines' can be rendered in formal logic as "there is an x' ($\exists$ x) 'for which holds that' (:) 'x shines' (S(x)). Predicate logic codes that, like $\exists$ x:S(x). By the same token, shining is a function of x. It is still a variable, x: it may be a lamp or a sun, but it does shine. For convenience sake 'on me' is forgotten. That is not without importance, for it establishes a connection, a link. Next I can emancipate 'x shines' as 'the shining of x' that I can envelop by the function B 'beginning': $\exists$ x:B(S(x)). Something starts shining, what is that? The sun: $\exists$ z:B(S(s))! I have now substituted an independent name (s) of something that begins B shining S. What do I have gained here by substituting a noun? Is it not just the name, that other

a This term is suggested by Roozenburg, N.F.M. (1993) *On the pattern of reasoning in innovative design*. as an alternative for 'innovative abduction'. This term was suggested by Habermas for a form of abduction that was not explicit in Peirce. However the form of innoduction presented here does not co-incide with the form Roozenburg uses in his paper.

b Jong, T.M. de (1992) *Kleine methodologie voor ontwerpend onderzoek*.

people have given to the thing, as much as a naming function s=N(x)? My formula extends: $\exists$ x:B(S(N(x))); where is the end? What is named?

By percieving this *connection* I can, for instance, distinguish the shining and the shone unto as active and passive things. Subsequently I can name these things with nouns, make them independent and use them as a subject 'Sun' and object 'that tree' or 'myself' as expressed in a sentence. Barring lies, the fact takes here from the world the barriers of the *impression* and the *expression* to land from that world into the sentence 'the sun is shining'. The fact that someone utters this full sentence is in its turn a new fact that has to take these hurdles again with other people.

A story to match can be told about the second statement 'I get warm' in spite of a number of new philosophical problems, like the meaning of the word 'I', the subjective experience of 'being cold', eventually as a 'property' of the 'I', the possible independence of the concept 'cold', etc. We leave those problems for what they are.

23.27 GRAIN

We assume that both perceptions have landed 'well' into the sentence, and that both are 'the case'; we consider both to be 'true'. They are two facts, combined by the word 'if … then …'. This little word establishes no causal connection like 'hence'; it just denotes that two facts on the same place and within a certain period ('here' and 'now') both are simultaneously 'the case'. That special condition is of importance, because of the local fact that the sun will shine somewhere else, the period imports, while the sun will set before too long. Each perception or observation implies place and time and a size of them both, the 'grain' of it.

In this case the grain was definitely smaller than half the surface of the earth and smaller than half of the 24 hours the earth takes for one spin around here axis, but larger than a point and a moment, since both do not have to occur simultaneously in an absolute sense, but for instance within the period of reliability of the assertion, a short time after one another. The under limit may be determined by asking across which area the observation was extended (in the second statement restricted to 'I'), and for how long the situation (the state of affairs) lasted.

The expression of the observation can now also be made more precise by indicating within the grain of time a sequence:

'First the sun starts shining and then I get warm'. Now suppose that it becomes cloudy next and that I'm getting cold. This expression of the facts is admittedly true, but I leave so many facts out of consideration (a 'half-truth'), that on the basis of this body of facts I can never arrive at a simple hypothesis, education or causal explanation that is known to us now.

a Wittgenstein, L. (1922) *Tractatus logico-philosophicus*.

24 MATHEMATICAL MODELS

TAEKE DE JONG
REIN DE GRAAF

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A curriculum in mathematics for the Architecture Faculty at Delft University, taught for a few years during the nineties by the Mathematics Faculty, was ill-suited to the architecture staff, including its examples and references. So, the students who could not understand what use it was, experienced more nuisance than stimuli while designing, and were avoiding mathematics in the curriculum as a whole, where other disciplines could compensate for low grades in mathematics. The practice of design was doing well with high-school maths with some extensions, so why bother?

The realistic production of form is, as always, superior to the abstract mathematical detour via form description by Cartesian co-ordinates, even if fractal forms are generated. The mathematician and designer Alexander has been more successful with his *'Pattern Language'* [a] than with his *'Notes on the Synthesis of Form'*.[b] The remainder of mechanics and construction physics is being taken care of by specialised consultant agencies and computers following delivery of a sketched design. No senior designer has any recollection of the content of mathematical education (s)he was exposed to during the sixties and seventies, when it was compulsory, while the practice featured nothing that might benefit from remembrance.

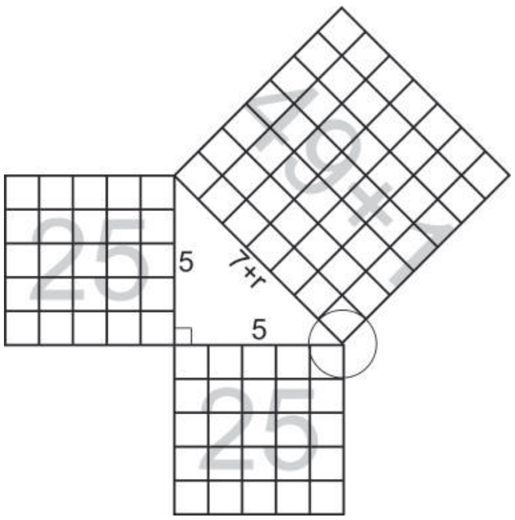
Of the lecture notes on architecture, 'geometry', 'graph theory', 'transformations and symmetries', 'matrix calculation' and 'linear optimising', 'statistics', 'differential and integral calculation' composed and, in the nineties, introduced in a simple form with the problem-orientated education only the latter may be found in the Faculty's bookshop anno 2002. This last relict is due to the tenacity of the sector Physics of Construction. During the more mature years of building management matrix calculation for optimising exercises is being brushed-up from high-school maths. Then one is lacking the lost foundation in the first year. With the slow filtering-through of end-user friendly computer applications, as there are spreadsheets and CAD (pixel and vector presentations of form) during designing a new interest is dawning. Computer programs like Excel, MathCad, Maple or MatLab do ease experimenting with mathematical formulae as never before.

From these mathematical ingredients, also adopted by Broadbent[c] as relevant to architectural design, maybe a new mathematics for architecture might be composed. Architecture itself and civil engineering, it should be remembered, were standing at the cradle of mathematics. This Chapter does not pretend to stand at the birth of a new building mathesis. As urban architects, its authors fall short of the proper attainments. However, it gives a global survey of mathematical forms that may be employed in architectural design – with a reading list – providing linkage with a new element as a point of departure: combinatorics. Whoever wants to brush up on high-school maths[d] or to get a bird's eye view of mathematics as a whole[e] is referred to publications pertaining thereto. Experimenting with the Excel computer programme is especially recommended. In spite of that, Euclid's answer to the question whether there would not be a simpler way to study geometry than his 'Elements'[f] is still applicable: "There is no regal road to geometry'.

24.1 ORIGINS

Mathematics is a language developed in order to describe locations, sizes (geometry), numbers (arithmetic)[g] and developments from observations (measurements and counts), to process these descriptions and to predict new observations on that basis. In this vein, until 500 BC, for the founding of cities in Greek colonies a square of 50 x 50 plethra (a 'plethron' is some

a Alexander, C. (1977) *A pattern language*.
b Alexander, C. (1964) *Notes on the synthesis of form*.
c Broadbent, G. (1988) *Design in architecture: architecture and the human sciences*.
d Kervel, E (1990) *Prisma van de wiskunde 2000, wiskundige begrippen van A tot Z verklaard*.
e Reinhardt, F., H. Soeder et al. (1977) *dtv-Atlas zur Mathematik*. Dutch translation: Reinhardt, F. and H. Soeder (1977) *Atlas van de wiskunde*.
f A complete interactive version of the 'Element', the 'Bible' of geometry, may be found on the internet: http://aleph0.clarku.edu/~djoyce/java/elements/usingApplet.html.
It is argued on this site and its links that omitting the Euclidean method in education and exchanging it for a derivation from set theory is detrimental to the logical deduction of conclusions from axioms via propositions to new propositions. The site is interesting by the possibility to change geometrical schemes. This demonstrates the operational character of mathematical propositions and formulae.
g A distinction that was made by the Greek mathematician Proclus.



192 Pythagoras

30 metres) was paced off thanks to a diagonal of 70 plethra, in order to realise straight corners.

Pythagoras (580 – 500 BC) – or one of his pupils – next provided the well-known proof that the ratio should be slightly larger than 7 to 5; while the square of 7 is just one unit less than 25 + 25. From the womb of geometry, thus, the arithmetical insight was born that real numbers $\mathbb{R}$ exist – such as the square root of 2, or the real length, derived therefrom, of the diagonal – not to be attained by simple partitioning (rational, $\mathbb{Q}$) of natural numbers ($\mathbb{N}$ = 1, 2, 3 …). Geometry – literally: 'measuring of the land' – owns its first described development to the annual flooding of the river Nile. In ancient Egypt, floods wiped out the borders between estates and property, so that each year it had to be determined geometrically who owns what; as seen from standpoints which were not flooded. Arithmetic has roots in Phoenician trade. The Greek Euclid (around 300 BC) collected knowledge in both senses of his day and age in his book *'Elements'*, (via the Arab world) the cornerstone of all education in geometry well into the twentieth century. Euclid used earlier texts, but derived as the first the propositions of geometry by logical reasoning from 5 axioms.[a] With this, a process was completed emancipating mathematics from empirical practice of measuring and counting examples.

24.2 THE MATHEMATICAL MODEL IS NO REALITY

Since then, mathematics can, like logic, without observing the sizes of input (non-empirically, a priori) come to new forms of insight (synthetic judgements). Logical proof dominating, they are accepted as new insight also without observing sizes of output. The philosopher Kant (1724 – 1804) struggled with the question how that is possible at all: synthetic judgement *a priori*.[b] For an empirical-scholarly theory always states that in the case of an independent observation from a set X, a corresponding independent observation from a set Y follows. If the elements of X and Y may be put in a corresponding, following, order so that they form the variable x and y, one may interpolate observations that have not been performed, while at constant conditions (*ceteris paribus*) – not included in the model – extrapolating to the future. A regularity of their correspondence is regarded as a working between them: y(x). Probable (causal) as well as possible (conditional) workings exist. If, for instance, the size of the population (x) is increasing, the number of buildings (y) is increasing as well following a hypothetical working y(x).

The larger the number of observations (n), the more convincing the theory. If one can demonstrate, by way of one hundred time sequences[c] (n = 100) that between the two of them a working (function) may be defined that convinces more than one single correspondence in the past year (n = 1). At larger numbers of independent input observations, the scholar can look for a mathematical working y = f(x) (e.g. y = ½ x) producing the same results as dependent observing (modelling). A just input and output must show a perceptible relation to reality, not the mathematical operations employed by way of a model. Different (mathematical or real) workings (functions) may yield the same result. As soon as other facts are observed than those predicted, the empirical theory is rejected; not necessarily the mathematical discourse playing a rôle therein; although the two occasionally get confused. If the predictions are confirmed, in its turn, the mathematical working model is often regarded as 'discovered' reality ('God always calculates').[d] However, this is not necessary in order to accept a theory (until the opposite has been proven).

24.3 MATHEMATICS IS THE LANGUAGE OF REPETITION

Just as in daily parlance, in logic and mathematics as well, concepts (statements, expressions) are used and composed into a model (declarations, sentences, full-sentence functions[e], workings, functions) with operators like verbs and conjunctions. Logic is using these operators particularly in the case of conjunctions (for instance: if P, then Q) mathematics the verbs

a Non-Euclidean geometry rejects one or more of these axioms.

b Kant, I. (1787) *Critik der reinen Vernunft*. The programme of this study is summarised particularly clearly in the Preface to this edition. A short introduction to Kant: Schultz, U. (1992) *Immanuel Kant*.

c A well-known publication of CBS is '*X years in time series*', e.g. Centraal Bureau voor de Statistiek (1989) *1899-1989 negentig jaren statistiek in tijdreeksen*.

d 'Ηο θεοσ αει γεομετρει', according to a famous Greek statement.

e See Reinhardt, F., H. Soeder et al. (1977) *dtv-Atlas zur Mathematik*.

(functions such as adding and summing). Logical deductions in mathematics usually have the logical linguistic form: 'if working P, then working Q'. However daily parlance has the capacity to name *unique* performances. This primal declarative function of everyday language has the character of a contract. Only when the performance has been witnessed anew is there a rational ground to start counting. *What is repetitive is food for mathematics*. For all mathematical operating, name-giving, as in everyday language, is – usually implicitly – pre-supposed. However, mathematics is of no use in unique performances. If one stone weighs one kilogram, then two stones weigh only two kilograms if they are 'equal'. This equality (here in terms of size and material) can only be agreed on by normal words. Sub-dividing dissimilar stones in equal fragments (transforming sizes in numbers, analysis) can make unique specimens elective for counting, and then for mathematical operations based on that counting. The question whether that can be done will always remain; as in Solomon's judgement: two times half a child means no child anymore. In analysis a connectivity, incorporating the essence of the architectural object, may get lost and will remain lost during synthesis to a different magnitude (counting). The scale paradox may be a nuisance while sub-dividing an object again and again in increasingly smaller parts, in order to compose next from this a different order of magnitude or to predict it (infinitesimal calculation, differential and integral calculation). The other, lost properties – in a specific context – may be taken into account next in the formulae (increasing validation, see page 258), but this is just shifting the problem.

24.4 SERIAL NUMBERING

Serial numbering (sequential numbering) just pre-supposes difference in place, not similarity in nature. I can enumerate the total of different objects in my room (or letter them) in order to be able to see later whether I am missing something, but this serial numbering does not allow mathematical operations. In spite of the fact that they are mathematically greatly important, since ordered difference of place (sequencing) is crucial in number theory.[a] The serial number serves as a label, name, identification (identification number, ID number, or index, in the case of variables) that may prevent exchanging, missing and double counting. By the same token, it is impossible to calculate with these numbers, although 'serial numbering' is pre-supposed silently in the case of 'counting'. Sequencing of numbers has in principle no other purpose than that it is staying the same, even if the numbered objects are changing later in place. The number stabilises differences in place ever witnessed as if on a photograph.

Nevertheless, the sequence, in which one is numbering, often gets, in practice, a meaning (for instance: in the order of arrival), allowing conclusions. Although one is inclined to introduce with an eye on that some logic in a numbering (categorising), it is halted sooner or later, while numbering is incurring lapses or lack of space. At current capabilities of information processing, this is why it is advisable, for instance, to open in a spreadsheet a database next to the column with sequential identification numbers (always to be produced independent of the shifting row and column numbers!) new columns in order to distinguish categories on which one wants to sort. For the mathematics it is important that it is possible to number input and output numbers, to index, identify and retrieve from a database in a fixed sequence and combination. The serial number is the carrier of the difference of place in a database. A reliable database is carrier of differences in nature and place in the reality (not that nature and place itself). To ensure that an identification code will always be pointing at the same object it should be invariant during the existence of that object. Additionally it is not allowed to have meaning in terms of content, such as a postal code and house number combination, for this changes in the case of home-moving the corresponding 'object'.

24.5 COUNTING

Observations can only be expressed mathematically if they are occurring more than once (in a comparable context) and may be harboured as 'equal' in any sense in a set. Only then can

a Russell, B. (1919) *Introduction to mathematical philosophy*. Frege, G. (1879) *Die Grundlagen der Arithmetik, Ein logisch mathematische Untersuchung über den Begriff der Zahl*. English translation: Frege, G. (1968) *The foundations of arithmetic: a logico-mathematical enquiry into the concept of number*. Dutch translation: Frege, G. (1981) *De grondslagen der aritmetica, een logisch-mathematisch onderzoek van het getalbegrip*.

one count them. The equality pre-supposition of set theory and mathematics is sometimes forgotten, or all too readily dissolved, by analysis (changing scale by concerning smaller parts). In this way an area of 1000 m^2 can be measured by counting, but each square metre has in many respects a different value that makes the area found in itself (without weighing) meaningless. The equality pre-supposition can lead to mathematical applications without sense, when the set described is too heterogeneous qua context or object for weighing. In this vein I can count the number of objects in my room, but each and every mathematical operation on this number alone does not lead to useable conclusions. Some objects are large, others small, some valuable, others not; or not elsewhere. From the number I might perhaps derive the number of operations in case of moving home, but these actions will be differing in their turn with the nature of the objects. Still I can say: "If I throw away something, I have less to move." If I throw away a moving box with that argument, I have less to move, but this has no relation anymore with the effort (larger without the box) of moving that may have fostered the argument. Mathematical modelling would be misleading here and requires a comparable context.

So: some equality in nature is already pre-supposed when it comes to counting. Curiously enough, a difference is also pre-supposed: when counting, I am not allowed to point at the 'same' object twice (double counting). The objects pointed at should differ! What this difference in identity exactly is, is left here undecided;[a] for reasons of convenience we call it 'difference of place', although this does not cover, for instance, the problems involved in counting moveable objects like butterflies on a shrub, mutually exchanging places. A number, or variable, therefore pre-supposes equality of nature and difference of place, even if that place is not always the same.

The equality in *nature* pre-supposed when it comes to counting does require the definition of a set (determining which objects we count or not). This definition is pre-supposing within the set defined equality, but at the same time to the outside difference with other sets. This paradox is explained elsewhere in the book as a 'paradox of scale' or 'change in abstraction, see page 37. In addition the 'nature' is not possible to change during counting. It is not allowed to use one century for counting one basket of apples, since it is likely that after an age like that the apples will not exist anymore.

The difference of place pre-supposed at the moment of counting does require a unique indication of place (which objects were already dealt with or not yet). By the same token such an indication of place is pre-supposing distances between the places (intervals) or between their centres (core to core distance); if that would not apply they would not differ and be unique. The size of the places indicated (scale) should be equal to the one of the objects placed (extension); if that would not apply one place could contain several similar objects, pre-empting identification of the objects themselves. Rather paradoxically, difference of place (uniqueness) is also pre-supposing an equality of scale (unit or order of magnitude) in order to guarantee that uniqueness. An object without scale (a point) may still be identified by mutual intervals. If these are small enough, points may produce a line, a surface or a volume.

24.6 VALUES AND VARIABLES

Therefore, we are counting by pointing at similar objects differing in their various places (in order to avoid double counting). Since that place may change, we make a snapshot, stating for the differences of place a randomly chosen, but fixed sequence, numbering(serial). To each indication a different name is given, number(serial). The final number is the number(quantity) or figure. Sometimes the sequence is of no importance, so that it is possible to restrict oneself to a uniquely identifying naming (nominal values). When the sequence imports, the values are termed 'ordinal'. Next, when the intervals between the objects numbered are equal, we call the number an 'integer'. This enables operations with interval-values such as adding, subtracting, multiplicating and dividing, without the need to indicate, count

a Frege, G. (1879) *Die Grundlagen der Arithmetik, Ein logisch mathematische Untersuchung über den Begriff der Zahl*. This problem is described in paragraphs 34 to 54 and proves to be not as easy as it seems.

or re-count the objects and their places. With this different counts may be predicted from certain counts, but the result may also be a 'non-object'. By naming this outcome 'zero' according to a price-less discovery of the world of Islam, and even by extending it after boundless subtracting with 'negative numbers' calculating is not restricted to objects accidentally present.

This is opening the road to calculation without reference to existing objects. By taking zero for a point of departure with at both sides the same interval as between the other numbers, the distance of this zero point to two numbers can provide a relational number (rational value). This is the foundation for measuring. Sometimes this results in fractions, that may be expressed in 'rational numbers'. If they are represented as points on a line of numbers, it becomes apparent that in between the numbers resulting from division of integral numbers still other values exist (real numbers). Ratios can also yield a relation between numbers of different kinds of objects (for instance inhabitants per residence: residential occupation). The set of values of one kind is called 'variable'. Function theory tries to work out arithmic rules for predicting, from the development of one variable x, another variable y. It is often difficult to determine, whether x and y entertain also a causal relationship, or are just demonstrating some connection (correlation); for instance on the ground of a common cause, a third variable, or if a great many causes and conditions are at work.

Particularly in probability calculus chances and probability distinguishing between these various kinds of values is important.[a] Then large numbers of results of a process are taken into consideration and the chance of a few of them is calculated (event). Extreme values are occurring less often as an outcome of many natural processes than values in-between. As a measure for the distribution between the extreme outcomes the average and its mean, the median and the modus are used. The average of a large number of values can only relate to interval values or rational values. In the case of ordinal values, no average exists, but a median (as many outcomes with a higher as with a lower value). In the case of nominal values it is also impossible to calculate a median; then a modus can be used (the number of values occurring most frequently).

When a set of values X is now compared to a different set Y in order to find between both a correlation, various statistical arithmetic methods (tests) exist, depending on the kind of values representing X and Y. In figure 193 the nominal values have been distinguished in dichotomous (yes – no) or non-dichotomous (multi-valueous).

	Y		
X	Nominal	Ordinal	Interval & ratio
Nominal (dichotomous)	Cross-table	Mann-Whitney	- Big sample: t-test/ANOVA - Small sample, Y normally distributed: t-test/ANOVA - Small sample, Y not normally distributed: Mann-Whitney
Nominal (not dichotomous)	Cross-table	Kruskal-Wallis	- Big sample: ANOVA Small sample, Y normally distributed: ANOVA - Small sample, Y not normally distributed: Kruskal-Wallis
Ordinal	Not applicable	Rank-order correlation	- Rank-order correlation
Interval & ratio	Not applicable	Not applicable.	- Big sample: Pearson-correlation - Small sample, X and Y normally distributed: Pearson correlation - Small sample, X and Y not normally distributed: rank-order correlation

193 Summary of tests on paired chance variables X en Y[b]

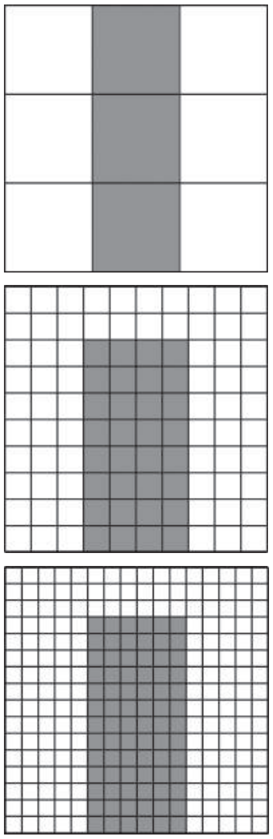
24.7 COMBINATORICS

As soon as counting has been mastered, one may name on a higher level of abstraction the internal categories (k), pre-supposed to be homogeneous, and unique places (n) themselves; number and count them. Allocating over the available places the kinds and within it the number of kindred cases (p) is the subject of combinatorics. It is pre-supposed in numerical systems and, therefore, a fundamental root of mathematics. This way the number of possible arrangements of 10 names over 2 places equals 100, over 3 it is 1000. Due to the Islamic discovery the notation of large numbers by combination of cipher names has become simple and more accessible to calculations.

Combinatorics may also be regarded as a basic science for architectural designing. More generally, one may calculate the number of possible arrangements, without any restriction, of k categories over n niches as kn. When it is supposed that 100 different kinds of building materials (among them air, space) may be used on a site of $100m^2$, with 1 mln inter,connecting allocation possibilities of 10 x 10 cm, this is yielding already in a flat surface many more design possibilities ($100^{1000000}$) than there are atoms in the universe (10^{110}). The designer is travelling, so to speak, in a multiple universe of possibilities, where the chance of

a Stevens, S.S. (1946) *On the theory of scales of measurement*.

b Leede, E. de and J. van Dalen (1996) *In en Uit. Statistisch onderzoek met SPSS for Windows*.



n = 9 k=3, scheme

n = 100 k=32, rough draft

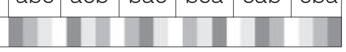
n = 256 k=78, Windows-icon

194 A programme, approx. 1/3 of the site, spread over the ground in 3 resolutions.

aaa	aba	aca	baa	bba	bca	caa	cba	cca
aab	abb	acb	bab	bbb	bcb	cab	cbb	ccb
aac	abc	acc	bac	bbc	bcc	cac	cbc	ccc

195 V(3,3) = 3^3 = 27 variations

niches n	just as much *different* colours as niches					
1	a					
2	ab		ba			
3	abc	acb	bac	bca	cab	cba



196 P(n) = n! : 1, 2 en 6 permutations

4	abcd	acbd	bacd	bcad	cabd	cbad
	abdc	acdb	badc	bcda	cadb	cbda
	adbc	adcb	bdac	bdca	cdab	cdba
	dabc	dacb	dbac	dbca	dcab	dcba

197 P(4) = 4! = 24 permutations

198 Combinatorial explosions

meeting a known design is practically nil. With this, to all practical purposes, infinite number of possibilities no rational choice is possible by taking them all into account. Also, when restricted by a programme of requirements, in which the units 'space' must be positioned in certain amounts in inter-connection, the number of possibilities is still practically infinite. The various mathematical disciplines passing muster in the following paragraphs as possibly relevant for architecture, are described there as rational restrictions of this number of possibilities.

The designer, faced by a white sheet of paper or a blank screen, is asked to indicate on it difference in place (state of dispersion; form) and in kind (colour). The differences in nature are contained in a range to be generated (the 'legend', in its original connotation) spread in k units of colour (e.g. the programme) over n niches (appropriate fields, for instance on the grounds of the site); the differences in place.

How many different states of dispersion can be generated in total? In mathematics the branch of combinatorics deals with that kind of 'arrangements' in finite sets and with counting the possibilities of arrangement under suitable conditions.[a]

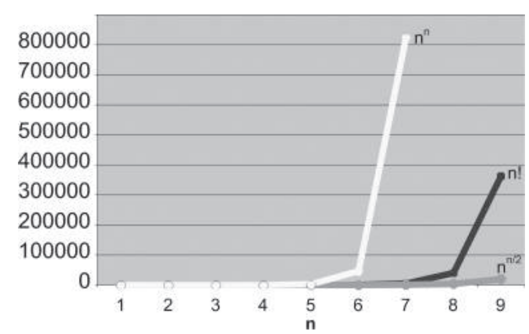
If a range of k = 3 colours is available for colouring n = 3 fields, $3^3 = 27$ variations apply. In this way the colours red, white and blue combined with three fields yield 27 distinct flags. More generally: $V(n,k) = k^n$ (variations with repetition). With as many colours as niches the expression reads n^n.

Among them, however, there are many cases in which colours have been repeated or omitted. True 'tricolores' are but few. The first condition to be added amounts to the presence of all three colours. Colour one has 3 positions available; for the second 2 remain; and for the third 1. This limits the number of cases to 3 x 2 x 1 = 6, abbreviated to 3!, so-called permutations. More generally, one may write: P(n) = n! (permutations without repetition).

24.8 TAMING COMBINATORIAL EXPLOSIONS

Permutations restrict the ultimate number of variations: 3!, is less than 3^3; n! rises less fast than n^n, but faster than, for instance $n^{n/2}$, when only half the amount of niches is used for the number of colours 'Factorials limit the largest powers'. Yet, permutations do increase fast enough to lead to a 'combinatorial explosion' on higher values for n.

How to select from all these possibilities? To many people this is a crucial question in personal life and in designing. With so many possibilities a conscious choice does not apply actually, so that during the reduction of the remaining possibilities, still not yet imaginable, one is guided by contingency, emotion of the moment, sensitivity for fashion, or routine. Deelder says: *'Within the patches the number of possibilities equals those outside them."*; as long as the resolution is lowered.[b]



a Formulation derived from Reinhardt, F., H. Soeder et al. (1977) *dtv-Atlas zur Mathematik*.
b Deelder, J.A. (1991) *Euforismen*.

Although mathematics is used generally in order to arrive at *singular solutions*, where different possibilities are not taken into account to the dismay of the designer, this discipline may also be used to *reduce the remaining possibilities* within exactly formulated, but in other respects freely varying conditions (for instance a scale system of 30 x 30 x 30 cm)[a] and to survey them. The branches of mathematics relevant to architecture are therefore seen here as the formulating of these *restrictions on the total amount of possible combinations*.

- *Systems of measure* are mathematical sequences reducing the sizes of components to convenient sizes.
- *Geometry* restricts itself to connected (contiguous) states of dispersion (shapes) such as lines, planes, contents, which can be described with a few points and with a minimum of information as to their edges.
- *Graph theory* restricts itself to the connections between these points (lines with or without a direction) regardless of their real position.
- *Topology* formulates transformations of forms and surfaces, so that one form can be translated in another one by a formula. This involves the direct vicinity of each point in the set of points determining this surface. For minimising the curved surfaces between the closed curves (soap skins) the forgotten mathematical discipline of the calculation of variances is necessary.[b] This mathematics is also applied to the tent roof constructions of Frei Otto. However, this discipline is too complicated to be introduced in this context.
- *Probability theory* and its application in *statistics* restricts itself to the occurrence of well-defined events, for instance the happening of one or more cases among the possible cases in a given space or time.
- *Optimising* by *linear programming* aided by *matrix calculations* formulates the remaining exactly restricted possibilities as the ‘solution space’.

Next to these mathematical restrictions there are any number of intuitive restrictions causing the disregard of possibilities. If one recognises, for instance, in the illustrations of figure 194 the image type of a door in a wall, the basis of the door (programme k) should lie in the lower part of the wall (the space present n), unless a staircase is drawn as well.

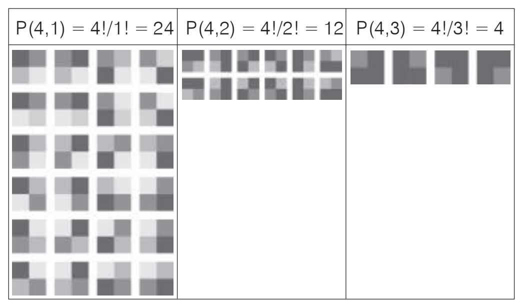
24.9 THE PROGRAMME OF A SITE

With variations (k^n) and factorials or permutations (n!) alone it is impossible to determine the number of cases when a particular colour has to be present in a predictable amount (a quantitative ‘programme’ per colour). With a programme for an available space n, where one colour (for instance black) has always to be present at least p times P(p,k) = n!/p! possibilities exist (permutations with repetition). The p! in the denominator of the ratio restricts the explosive effect of n! in the numerator at higher values of n, except of course, if it equals 1. That applies in the first column of the figure below; if p = 1 (so p! = 1) it has no effect on the ratio; the number of permutations P remains, as in the previous example with letters, 4! = 24. At p = 2 (p! = 2) and p = 3 (p! = 6) the number is restricted.

The larger the programme p of one colour (in this case black) with regard to the total space available, the less possibilities remain for other colours to generate cases. Evidently, p, the number of colours surfaces to be distributed may not exceed the space n available. The formula pre-supposes that the niches remaining will be filled with as many different colours. They generate the additional cases, not requiring attention from a programmatic point of view.

If not only the programme p, but also the complementary remainder n - p is combined into one colour, no other colours remain to generate additional cases. A permutation with two colours, p_1 and p_2 equals the formula $n! / p_1!p_2!$ The possible arrangements of programme p1 and the remainder p2 develops into n! / p!(n-p)! (Newtons binomium)[c].

Within both surfaces, the sequence in which the niches are filled in with one colour is now irrelevant. The only consideration of importance left is the number of different instances in which one quantity p is allotted to n possibilities without a rôle for its own sequencing.



199 Permutations in 4 niches, with at least k = {1,2,3} black elements combined with other hues.

a An example of a measure-system with equal measures also applies, by the way, to the choice of a resolution.

b Hildebrandt, S. and A. Tromba (1985) *Mathematics and optimal form*. Dutch translation: Hildebrandt, S. and A. Tromba (1989) *Architectuur in de natuur: de weg naar optimale vorm*.

c In case 0!, 0! = 1 by definition.

C(4,1)	C(4,2)	C(4,3)
1x2x3x4/ ((1x)(1x2x3)) = 4	1x2x3x4/ ((1x2)(1x2)) = 6	1x2x3x4/ ((1x2x3)(1)) = 4
abbb,babb, bbab,bbba	aabb,abab,abba, baab,baba,bbaa	aaab, aaba, abaa, baaa
		

200 Combinations in 4 niches of 2 colours

This entity plays a leading part in statistics. Factor p then is the number of possibilities of an event, n – p is its complement of possibly not happening that event. So, not only possibilities in space apply: n may also reflect time. A little counter-intuitively the common expression is ‘combinations’; and one simplifies the formula P (n, p_1, p_2) = n! / p_1! p_2! to C (n,p) = n! / p! (n - p)! ; or shorter still: $\binom{n}{p}$, ‘n over p’ (combinations without repetition).

Without fail, one will experience the highest level of freedom of design if a one-colour programme occupies half the site available (p1 = p2); a sound reason to plead with the principal for as much open space as space built. At higher resolutions as shown in the diagrams on page 208 even the number of black-white combinations rises again explosively C(9,3) = 84, C(100,32) = 140 000 000 000 000 000 000 000 000 (1,4E+26 for short in Excel), the Windows-icon shown (16pixels x 16pixels = 256pixels), amounts at 78 black pixels C(256,78) already to 1,2149E+67 images. Excel calculates this with the command = COMBIN(256,78).

Again, the greatest freedom to design is found in an equal distribution between black and white pixels: C(256,128) = 3,50456E+75. If 256 colours can be used the variance formula is 256^{256} (10^{616} or 2^{2048}). This exceeds the number of atoms in the universe by far (10^{110} or 2^{365}). If one studies, for instance, 5625 locations of 1 km^2 in the colours ‘built’ (815) and ‘open’ (4818), approximately 10^{1008} or 2^{3349} alternatives exist for the present ‘Randstad’ area in the Netherlands. The average PC cannot deal with this. Excel digests in this case at great pains p=156.

In this sense the designer is travelling in a multiple universe of possible forms (states of dispersion). The chance one runs into a known form is, in this combinatorical explosion, negligible

The number of elements of the programme (legends units, colours, letters) may be raised above 2; within the total sum of n, of course. This enables the calculation of the numbers of cases belonging to a specified programme on a location. If one wants to allocate for instance 4 programmatic elements to n fields, there are $P(n,p_1,p_2,p_3,p_4) = n!/p_1!p_2!p_3!p_4!$ design possibilities. The total of the colour surfaces $p_1+p_2+ \dots$ desired is again not allowed to exceed, of course, the surface available in total n. With a computer programme such as Excel one can calculate, for instance, that the possibilities of allocating 4 types of usage of $25m^2$ each to an area of 100 m^2 is 2,5E+82.[a]

24.12 THE RESOLUTION OF A MEDIUM

A pen, pencil or ball-point draws points and lines occupying a minimal surface, corresponding with the thickness of the material. Surfaces may then be depicted by filling in the surface with such lines. A computer screen is building an image from tiny surfaces (picture elements, pixels) that suggest contiguous lines or surfaces. This resembles the difference in the history of art between the Florentine (line-orientated) and the Venetian (point-orientated) ‘disegna’. A usual screen features 1024 x 768 pixels.

A pixel-orientated program (paint- or photoprogram) just supports the colouring of these points like in painting. If a ‘line’ or ‘surface’ is over-written, the only way to restore it is to fill in the previous colour of the over-written pixels. More often than not foreground and background are not distinguished in layers allowing display layer by layer, or in combination.

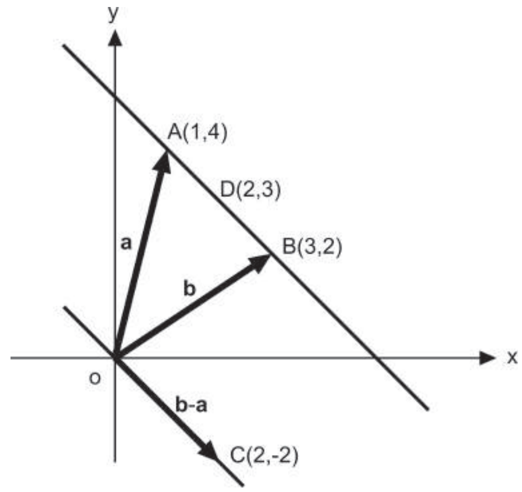
A computer drawing program (vector program, drawing program or CAD) just takes together the essential points of a drawing into a matrix of co-ordinates and translates the drawing into pixels between these points as soon as it is activated. The pixels in between are not being remembered during storage, so that mass-storage space requires less space than a pixel image. In the case of an enlargement of the drawing or of a detail, the resolution adapts itself automatically, so that the lines are not blurred.

a This can be done with the formula: =FACT(100)/(FACT(25)*FACT(25)*FACT(25)*FACT(25))

A vector is a matrix with one column (or row) that in the flat plane just needs two co-ordinates to be defined from an origin. In the figure alongside three vectors a, b, and b-a have been drawn. They illustrate the calculation rules that are of great service in drawing programs and applied mechanics. The line segment AB is represented by the co-ordinates of the vectors a and b; the points in between are calculated by giving a co-efficient I between 0 and 1 a sequence of values provided by the formula l (b-a)+ a.

vector substraction			l =*0,5*	
A	**B**	**b**-**a**	l(**b**-**a**)	l(**b**-**a**)+**a**
1	*3*	2	1	2
4	*2*	-2	-1	3

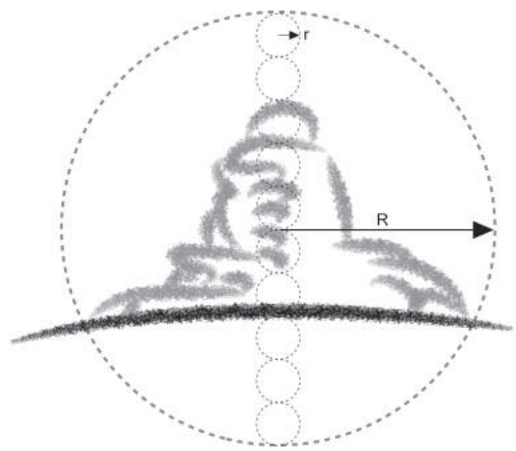
This way I=0,5 yields the point in between D(2,3). The larger the number of values calculated, the greater the resolution of the line segment AB.



201 Defining line segments by vectors

The combinatoric explosion of possibilities is already drastically reduced during the initial stage of the design by the coarseness or, in reverse, the resolution of the drawing. That is something different than the scale of a drawing.

The position of a deliberately coarsely sketched line must be judged according to commonly understood conventions within certain margins. The size of such a margin surrounding a drawn point we call ‘grain’. We call the radius R of the circle inscribed in the drawing as a whole ‘frame’, and the ratio of the radius r of the grain to R ‘resolving capacity’, or ‘resolution’. The ‘tolerance convention’ could be interpreted as ‘any sketched point may be interpreted within a radius r, by that interpretation transforming the rest of the drawing accordingly’. The tolerance of the drawing is expressed by r.

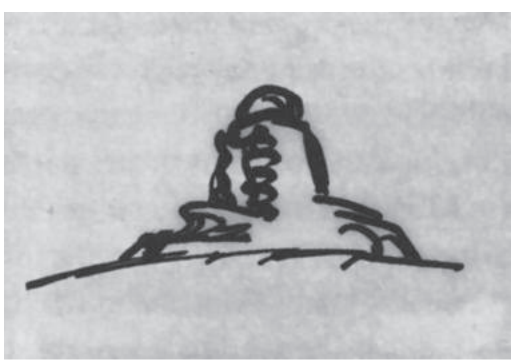


The radius r of a grain here is approximately 10% of the radius R of the frame.

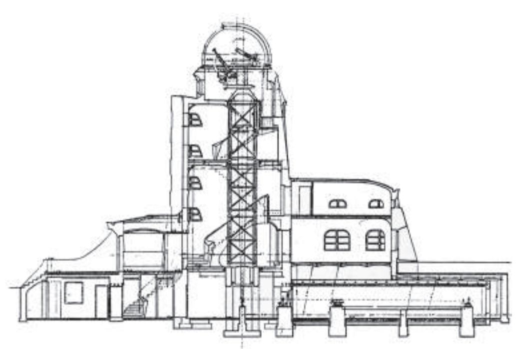
A sketch with a grain of roughly 10% of the frame is known as a loose sketch. It is used in an early concept, a type or a schema. It is often produced with a felt-tipped pen; with the same order of magnitude as the grain of the drawing, by that means stressing the tolerance convention. A ‘design’ hardly has a smaller resolution than 1%. A blue-print or computer screen does not exceed 0,1%. Only at this level things like details in the woodwork of a door and its frame in a wall are displayed. The total concept of a work of architecture is highly influenced by details, observed by the zooming eye of an approaching user.

a Source: Media-centre, Fac. of Arch. DUT. After: Zevi, B. (1970) Erich Mendelsohn, opera completa, p.65.

202 Mendelsohn *Einsteinturm* (Potsdam, 1920) [a]



Sketch approx. 10% resolution



Drawing approx. 1% resolution

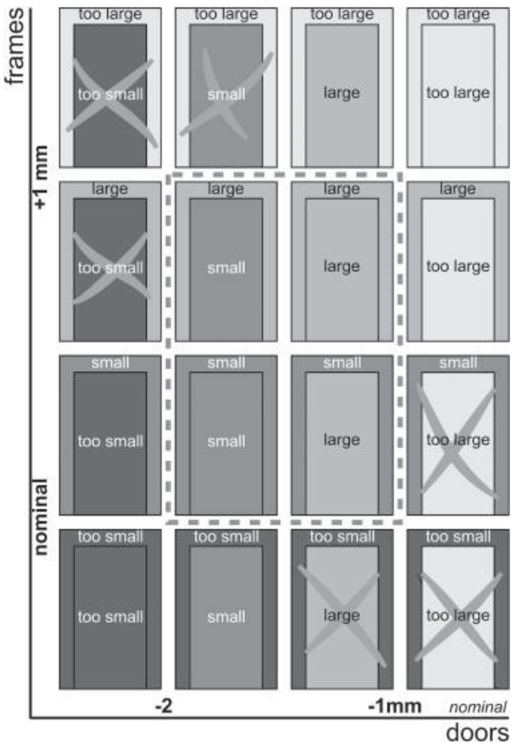
Screen approx. 0,1% resolution

24.11 THE TOLERANCE OF PRODUCTION

A door should be slightly smaller than its frame in order to acquire a functional fit. In addition a carpenter or machine makes frames and doors respectively slightly larger, or smaller, than the nominal size written on the blue print.

This is also taming the mathematical use of positions behind the comma for architecture and technique in general. One is not allowed to think anymore in terms of numbers representing a point on the line of numbers: they are representing a margin, a band-with, a distance or class on that line. The nominal size is indicated on the blue-print, but it is certain that this precise size will never be delivered.

The frame should be equal or larger than the nominal size, the door should be smaller, but how much? The limits put to this product tolerance must be calculated by weighing the price of the precision and the performance of the door as a closure. If the tolerance of the frame opening is 0 to + 1mm, and the tolerance of the door –1 to –2 mm, the crack-width will be 1 to 3 mm, divided over two sides.



203 Acceptible and not acceptible sizes

In order to decide whether to accept a batch of doors and frames or to send them back, no absolute measures are taken into account, but rather margins, classes such as ‘too small’, ‘small’, ‘large’ or ‘too large’. They may vary within limits of tolerance. In the case of the frames of the example alongside these limits are *vis-à-vis* the nominal size M:

‘too small’ < M + 0 mm < ‘small’ < M + 0.5 mm < ‘large’ < M + 1 mm < ‘too large’,

while for the doors the nominal measure:

‘too small’ < M – 2 mm < ‘small’ < M – 1,5 mm < ‘large’ < M – 1 mm < ‘too large’.

In first instance one assumes that they are not too small, nor too large (zero-hypothesis, dotted contour in the drawing) while a few are measured in order to see whether they are belonging to these classes yes or no. However on the basis of a random selection one can refuse the batch frames and doors on false grounds (fault of the first kind) or accept them on false grounds (fault of the second kind). The first fault is a producer’s risk, the second fault consumer’s risk. Since two different risks apply, both faults can not be minimised by taking the minimum of their sum. For instance the consumer’s risk may be systematically smaller than what one may derive from the second fault. If the producer, for instance, is delivering systematically too large or too small frames and doors, one may settle for a smaller refusal. If both are within reasonable margins (too) large, they will still have an acceptable fit and width of the crack (above right in figure 203). This also applies for a systematic deviation to the smaller side (below left). Only when the frame is (too) small and the door at the same time (too) large (below right), or vice-versa (above left), does the sizing cease to be acceptable.

Normal dwelling			Flat building Van Tijen (1932)	
n	v	h	v	h
9			2.85	27.15
8			2.85	24.30
7			2.85	21.45
6			2.85	18.60
5			2.85	15.75
4			2.85	12.90
3			2.85	10.05
2	2.6	8.0	2.85	7.20
1	2.6	5.4	2.85	4.35
0	2.8	2.8	2.70	1.50
				-1.20

204 Arithmical series in building

24.12 NOMINAL SIZE SYSTEMS

A restriction to multiples of 30 cm (little less than a foot) is a well-known bridle to sizes in construction of buildings. It reduces the combinatoric explosion of design possibilities. A grid like that is used to localise foundations, columns and walls without design efforts for smaller sizes. A grid may have a different size in any one of the three dimensions. A preceding analysis of usage may yield an appropriate size of the grid for a specific function. The distinct multiples of the size of the grid yield distinct functional possibilities. In the preceding paragraph a grid is implicitly used.

An arithmatical series has an initial term a, a reason v and a length n. The terms increase along a straight line by steps of v, starting with a. An example is the height of the first floor a of a building and its successors with a height of v, resulting in the series of heights h (normal dwelling and flat building[a], $a=v_0$).

a Van Tijen, *Bergpolderflat* (Rotterdam, 1932) N.V. Woningbouw; Barbieri, S.U. and L. van Duin (1999) *Honderd jaar architectuur in Nederlands, 1901-2000*, p. 194

Another example concerns a building with one oblique wall, like the KPN building by Renzo Piano in Rotterdam. The oblique wall commences on first floor level. The initial term is representing here the fixed surface per floor (supposed to be 100 m^2). The n-th term indicates the surface at the level of the n-th floor. The sum of the terms is the total surface of floors at the oblique wall.

In these examples the reason remains equal, in the next example the reason changes each next n. A sequence well-known from architecture and other arts is Fibonacci's sequence: a new term equals the sum of two previous terms (figure 206).

The reason v is variable now, but the ratio r between two adjacent terms converges at last to the 'Golden Rule', 'Golden Section' or 'Divine Proportion': the smaller number (minor m) than has a fixed ratio to the larger number (Magior M). The Magior M, again has the same ratio to the sum of both:

$$m : M = M : (m + M).$$

From $\frac{m}{M} = \frac{M}{m + M}$ follows: $M := \frac{1}{2} \cdot m + \frac{1}{2} \cdot \sqrt{5} \cdot m$

For m = 1 follows for M/m and m/M:

$$\frac{1}{2} + \frac{1}{2} \cdot \sqrt{5} = 1.6180340 \quad \text{and:} \quad \frac{1}{\frac{1}{2} + \frac{1}{2} \cdot \sqrt{5}} = 0.6180340$$

In the case of a geometrical series the ratio r is a factor of multiplication, so that the ratio of two adjoining terms is a constant. A geometrical series can be continued for negative values of n while the array remains positive for real architectural purposes.

Applications of geometrical series are found in the financial world, like in compounded interest and in annuities. An investment of € 1000,- at an interest of 5% yields after 1 year € 1000 * 1.05 = € 1050. After two years this is € 1050 * 1.05 or € 1000 * 1.05 * 1.05 = € 1102.50.

Experiencing sound is also an example of the geometrical series. An increase of sound to the amount of 10 dB is experienced as twice as loud; an increase of 20 dB as four times.[a]

length	starting term	reason	array	sum
n	a	v	a+v*n	
9			190	1450
8			180	1260
7			170	1080
6			160	910
5			150	750
4			140	600
3			130	460
2			120	330
1			110	210
0	100	10	100	100

205 Arithmical sequence with sum

length	starting term	reason	array	ratio
n	a	v	$v_{n-1} + v_n$	r
9		5,5	8,9	1,62
8		3,4	5,5	1,62
7		2,1	3,4	1,62
6		1,3	2,1	1,62
5		0,8	1,3	1,63
4		0,5	0,8	1,60
3		0,3	0,5	1,67
2		0,2	0,3	1,50
1		0,1	0,2	2,00
0	0,1	0,1	0,1	1,00
		0	0,1	

206 Fibonacci's sequence

length	starting term	ratio	array
n	a	r	$a*r^n$
9		2,000	51,20
8		2,000	25,60
7		2,000	12,80
6		2,000	6,40
5		2,000	3,20
4		2,000	1,60
3		2,000	0,80
2		2,000	0,40
1		2,000	0,20
0	0,1	2,000	0,10
-1		2,000	0,05
-2		2,000	0,03
-3		2,000	0,01

207 Geometrical sequence

a See also Lootsma, F.A. (1999) *Multi-criteria decision analysis via ratio and difference judgement.*

length	starting term	ratio	array	
n	a	r	a+r*n	
9		1,618	7,60	
8		1,618	4,70	
7		1,618	2,90	
6		1,618	1,79	
5		1,618	1,11	
4		1,618	0,69	
3		1,618	0,42	▲
2		1,618	0,26	m+M
1		1,618	0,16	M
0	0,1	1,618	0,10	m
-1		1,618	0,06	M-m
-2		1,618	0,04	▼
-3		1,618	0,02	
-4		1,618	0,01	
-5		1,618	0,01	

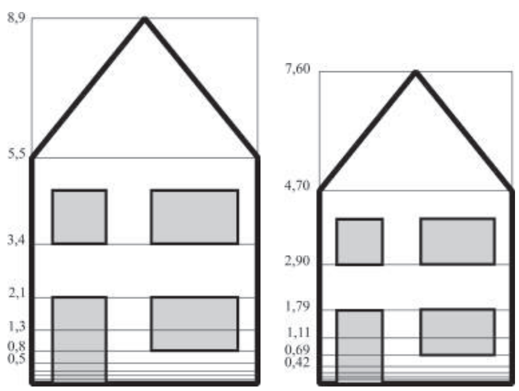
208 Golden Section

209 Measure systems of Le Corbusier

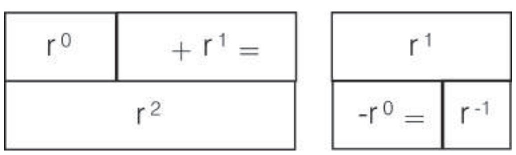
< 1947	Red array			Blue array
0,2	0,2			0,3
0,3	0,4			0,4
0,5	0,6			0,7
0,9	0,9			1,1
1,4	1,5			1,8
2,3	2,4			3,0
3,7	3,9			4,8
6,0	6,3			7,8
9,8	10,2			12,6
15,8	16,5	Modulor		20,4
25,5	26,7	27		33,0
41,3	43,2	43		53,4
66,8	69,9	70	86	86,3
108,2	113,0	113	140	139,7
175,0	182,9	183	226	226,0
283,2	295,9			365,7
458,2	478,8			591,7
741,3	774,7			957,4
1199,5	1253,5			1549,0
1940,8	2028,2			2506,4
3140,2	3281,6			4055,4
5081,0	5309,8			6561,8
8221,3	8591,5			10617,2
13302,3	13901,3			17179,0

length	starting term	ratio	array	
n	a	r	a+r^n	
9		1,325	1,26	
8		1,325	0,95	
7		1,325	0,72	
6		1,325	0,54	
5		1,325	0,41	
4		1,325	0,31	▲
3		1,325	0,23	$r^n+r^{n+1}=r^{n+3}$
2		1,325	0,18	
1		1,325	0,13	r^{n+1}
0	0,1	1,325	0,10	r^n
-1		1,325	0,08	
-2		1,325	0,06	
-3		1,325	0,04	
-4		1,325	0,03	$r^{n+1}-r^n=r^{n-4}$
-5		1,325	0,02	▼

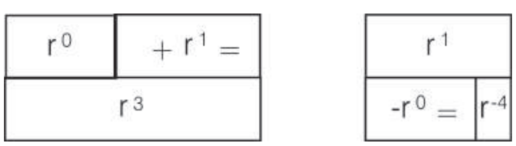
213 The plastic number



210 Fibonacci house & Golden Section house



211 Golden Section



212 Plastic Number

For architectural applications most geometrical series are not very useful, because adding architectural elements next to eachother (juxtaposition) produces new sizes, not recognisable anywhere else in the series. However, when we choose the Golden Section as a ratio we get figure 208. Now, every adjacent pair of sizes is flanked by their sum and difference, just as in the non geometrical Fibonacci sequence. Adding and subtracting of adjacent terms do not produce new sizes. The differences in use of both proportion rules are only visible in the smallest stages.

Many attempts have been made to recognise the Golden Section in nature and the human body. Until 1947 Le Corbusier took the human length of 1.75 m as a point of departure. Later, he started at 182.9 (red array) and 2.26 m (blue array): the reach of a man with a hand raised as high as possible.

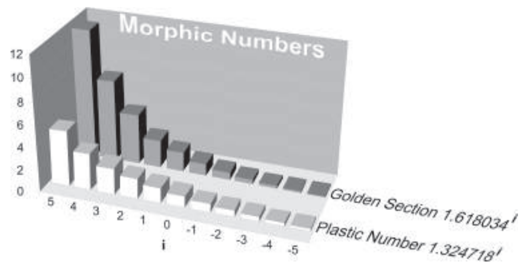
The red and the blue columns are featuring steps with a stride too wide between the terms for architectonic application. For this reason, Le Corbusier combined them and rounded them off in the Modulor. There are many other attempts in making the possibilities of the design with scale sequences easy to survey and well to maintain in the design decisions with a pre-supposed, built-in beauty.[a]

The problem of too large steps with the Golden Section was later solved by Dom van der Laan. He selected the ‘plastic number’ r = 1,3247180 for a ratio. This is a solution of the equation $r^1 + r^0 = r^3$ as well as of the equation $r^1 – r^0 = r^{r-4}$

Since any number may be substituted for the initial term a, for instance another term from the series, the more general formulae $r^{n+1}+r^n=r^{n+3}$ and $r^{n+1}-r^n=r^{n-4}$ may be employed. In the row below this means that the formulae can be shifted, while keeping the mutual distance constant.

a Kruijtzer, G. (1998) *Ruimte en getal*.

The sum and the difference between two consecutive terms are returning in the series as at the Golden Section, albeit 2 places upward or 4 places downward, rather than by 1 each time. Therefore they are forming with addition and subtraction no new measures while preserving the ratio r. Aarts *et al.* have demonstrated that this ratio is, next to the Golden Section, the only one with this property.[a] Together they are called ‘morphic numbers’.



214 Morphic Numbers

24.13 GEOMETRY

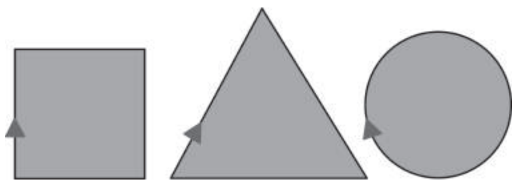
It is impossible to imagine a door distributed in tiny slits across a wall. Geometry restricts itself to contingent states of dispersion (lines, surfaces, contents) that can be enveloped by a few points, distances and directions. This pre-supposition of continuousness lowers the amount of combinatorial possibilities dramatically. The extent to which this geometrical point of view restricts the combinatorial explosion, is the subject of combinatorial geometry: it studies distributions of surfaces and the way these are packed.

However, the often implicit requirement that points in one plane should lie contingent within one, two or three dimensions is more obvious and self-evident in the case of a door, than in the one of a city. That is the reason why urban design is interested in non-contingent states of dispersion. The possibilities are restricted geometrically, following the often implicit pre-supposition of rectangularity. Particularly efficient production suggests this pre-supposition.

When one limits oneself to enclosed surfaces or enclosed spaces and masses, three simple shapes may be imagined in a flat plane: square, triangle and circle. Why are they so simple? They survive as geometrical archetypes in geometry and construction everywhere.

Their simplicity may be explained by the minima added in words to the diagram. This gives at the same time a technical motivation for application. A minimal number of directions is for production - e.g. sawing and size management - an effective restriction. Any deviation influences directly the price of the product. A minimal number of directional changes (nodes) is constructively effective (also from a viewpoint of stiffness of form). A minimal variation in directional changes(one without interruption, smooth) is effective with motions in usage; when one keeps the steering wheel of a car in the same position, a circle is described at last. They have been drawn in the diagram, with an equally sized area (programme). Their circumferences then are roughly proportioned like 8 : 9 : 7.

Our intuition meets its demasqué, by keeping the area equal: the triangle wins out. This visual illusion may be used for spaces needing particularly spatial power.

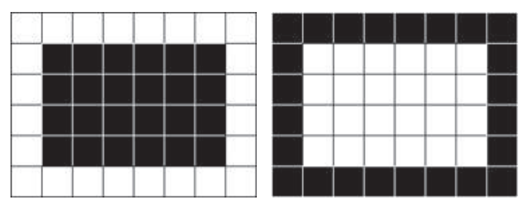


Minimal number of *directions* in one loop | Minimal number of *changes of direction* in one loop | Minimal *variation* of changes of direction in one loop

215 Simple shapes

A second example of the capacity of geometry to lead to counter-intuitive conclusions about areas is the seeming difference in surface between the centre and the periphery as Tummers emphasises.[b] In the diagram alongside they equal one another.

Functions characterised by the importance of inter-connectedness at all sides - like greenery, parks - tend to be better localised centrally, while differently structured functions (e.g. buildings) are better placed peripherally.



216 Apparent difference in surface between centre and periphery

The triangle is playing an important rôle in geometry because all kinds of shapes on flat or curved surfaces and stereometrical objects may be thought of as being composed out of triangles. Measuring the surface, also on bent surfaces (geo-metry), is dependent on insight in the properties of triangles (triangulation) since it establishes a one-to-one relationship between lines and angles (trigonometry). In the measuring of angles (goniometry) next to the triangle the circle plays a crucial rôle. Descriptive geometry[c] makes images of three-dimensional objects on a two-dimensional surface (projections), so that they may be reconstructed eventually on a different scale. With this, descriptive geometry is a fundamental discipline for architecture and technical design in general; for this description enables in its turn a wealth of mathematical and designing operations. The triangle also plays an important rôle in the technique of projecting.

a Aarts, J.M., R.J. Fokkink et al. (2001) *Morphic Numbers*.
b Tummers, L.J.M. and J.M. Tummers-Zuurmond (1997) *Het land in de stad. De stedebouw van de grote agglomeratie*.
c Berger, M. (1987) *Geometry I and II;* Wells, D.G. and J. Sharp (1991) *The Penguin dictionary of curious and interesting geometry;* Aarts, J.M. (2000) *Meetkunde*. Dutch translation: (1993) *Woordenboek van merkwaardige en interessante meetkunde*.

These subjects have already been described thoroughly and systematically by Euclid in his 'Elements' around 310 BC. Until in the twentieth century this book has been the basis for education in 'Euclidean geometry'. The work is available on the internet with interactive images and it is still providing a sound introduction into elementary geometry, as always.[a] Together with the co-ordinate system of Descartes geometrical elements became better accessible to tools from algebra such as vectors and matrices (analytical geometry).

When objects can be derived with rules of calculation in a different way than by congruency, equal shape or projection, when they can be represented or shaped into another, 'topology' is the word. The properties remaining constant under these transformations – or oppositely change into sets of points – can be described along the lines of set theory or with algebraic means. This is leading to several branches of topology. What is happening during a design process, from the first concept via typing to design, is akin to topological deformation, but it is at the same time so difficult to describe, that topology is not yet capable of handling.

On the other hand, the existing topology is already an exacting discipline, pre-supposing knowledge of various other branches of mathematics, before the designer may harvest its fruits. Nevertheless, it is conceivable that a simple topology can be developed, restricting the combinatorical explosion of design possibilities rationally in a well-argued way, in order to generate surprising shapes within these boundary conditions. It would have to describe constant and changing properties of spaces, masses, surfaces and their openings in such a way, that complex architectural designs could be transformed in one another via rules of calculation. With this, architectural typology and the study by transforming design would be equipped with an interesting tool. The computer will play a crucial rôle in this.

When a design problem can be described in dimension-less nodes and connecting lines between them, graph theory could be a predecessor.

	input		output				
figure	nodes	connections per intersection (valence, degree)	branches = nodes x valence/2	planes = branches - nodes + 2	connections = branches x 2	boundaries = branches x 2	boundaries per plane (valence) = boundaries / planes
tetrahedron	4	3	6	4	12	12	3
cube	8	3	12	6	24	24	4
octahedron	6	4	12	8	24	24	3
K3,3	6	3	9	5	18	18	3,6
K5	5	4	10	7	20	20	2,9

217 Nodes and connections in regular solids

24.14 GRAPHS

When one notes in a figure only the number of intersections (nodes) and the number of mutual connections per intersection (valence, degree of the node) we deal with a graph. A graph G is a set 'connecting points' (nodes, points, vertices) and a set of 'lines', branches (arcs, links, edges), connecting some pairs of connecting points together. A branch between node i and node j is noted as arc (i,j), shortened arc^{ij}. Length, position and shape of the connecting points and branches are without importance in this. (In architecture the relative position of the connecting points *vis-à-vis* one another will probably be of importance.)

Among many figures corresponding types may be discerned wherein neither length nor area play a rôle (e.g. designing structure, not yet form and size). This enables the study of formal, technical and programmatical properties in space and time even before the sizes of the space or the duration in time are known.

A cube may serve as an example. It has 8 nodes. Each of them has 3 connections. This fixes the number of connecting lines (branches) : 8 x 3 / 2 = 12. For each connecting branch occupies 2 of the 8 x 3 connections in total.

Following the formula of Euler, the number of planes = number of branches - number of nodes + 2. As soon as the planes (still without dimensions) come into the picture, we deal with a map. By the same token it suffices to count in a figure the intersections and their valencies in order to be able to calculate the number of branches and planes in the map. The regular solids can be represented in a plane by a graph.

a Euclides (310 BC) The Elements (http://aleph0.clarku.edu /~djoyce/java/elements/usingApplet.html)

Based on this, the terminology of graph theory is readily explained. These are single graphs: there are no cycles arriving at the same node as the one of departure; or multiple connections between two nodes. Furthermore, they are regular: in each node the number of connections per graph is equal.
The tetrahedron has a complete graph (K) unlike the other two, where possible connections - the diagonals - fail. The graph of the cube clearly demonstrates that the outer area should be in calculated in order to count 6 planes. It is as if the cube is ‘cut open’ in one plane, in order to ‘ex-plane’ it on the page. It is immaterial which plane serves as outer plane. Graph theory does not yet distinguish between inside and outside.

The graphs of the tetrahedron and the cube are ‘planar’: the flatland does not feature crossings without intersection. Figure 219 shows to the left an ‘isomorph graph’ for the octahedron where the number of nodes equals the number of valencies. Compare the octahedron graph with the one of figure 218.

The branch between nodes 6 and 1 of the octahedron may be ‘contracted’ in such a fashion, that one node remains, where all other branches end previously ending in 6 and 1, with whom they were ‘inciding’ or ‘incident’ in the parlance.

If a graph yields to contraction to K5 or K 3,3, it can be proven that it is non planar. Architectonically this is especially important: by the same token no blueprint can exist that relates all the relationships as recorded in the graph.

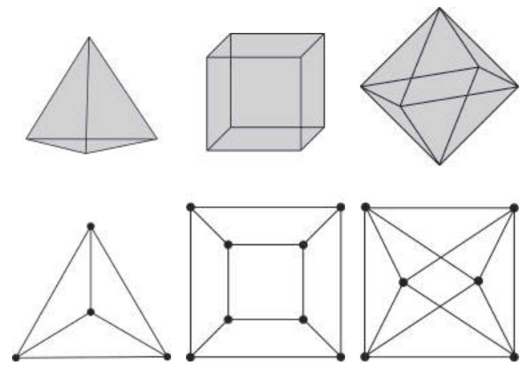
According to K4, each of the four rooms, may be linked by 3 openings mutually among themselves. The left shows the solution with two openings where one may circulate. The corresponding graph (a circuit) has also been drawn. The middle one demonstrates a solution where only two of the four rooms have 3 doors. To solve the complete graph K4, a ‘dual map’ must be drawn. To do that, K4 is made isomorphically planar, and the planes are interpreted as ‘dual nodes’. The outer area of the graph is involved as a large, encircling dual ‘node’.

These dual nodes (white in the drawing below) should be connected in such a way by dual branches (dotted lines) that all planar branches will be cut through just once:

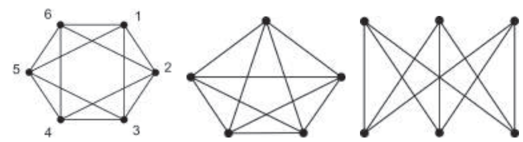
These cuttings through have become ‘doors’ (or windows) in the dual graph. The dual lines are ‘walls’ of a dimension-less blue-print and the dual points are constructive inter-connections. If this prototypical blue-print is regarded as ‘elastic’, it can be transfigured in an isomorphic way into a design, by giving the surfaces at random forms and shapes. Type K4 is usual for bathrooms and museums.

From a fundamental point of view, no solution exists in flatland for 5 rooms, each sharing one door with all other rooms (K5): no planar graph could be drawn of K5.

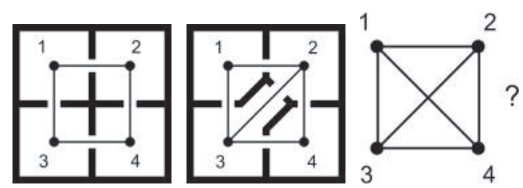
By regarding each space firstly as a node between other spaces, the design possibility of programmatic requirements can be verified along the lines of graph theory. Suppose, that the programme of relations between rooms in a dwelling results in the following scheme:



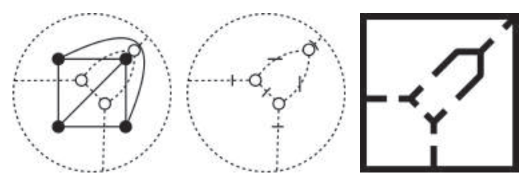
218 Regular solids as a graph



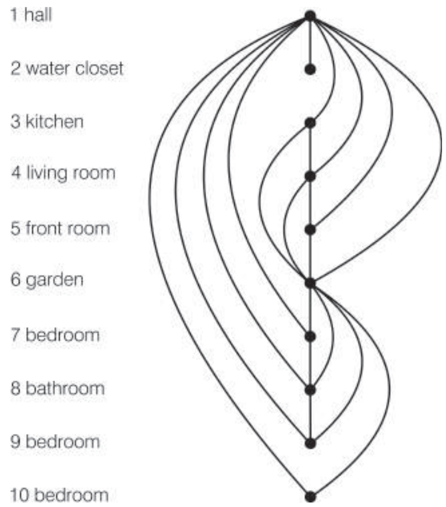
219 Octahedron, K5, K3,3



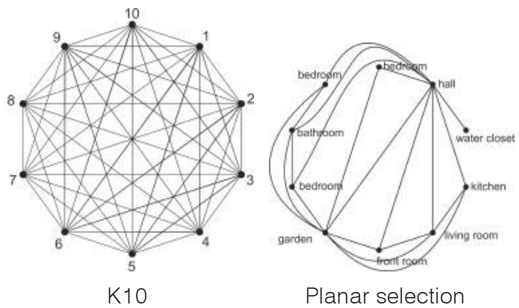
220 Four connected rooms



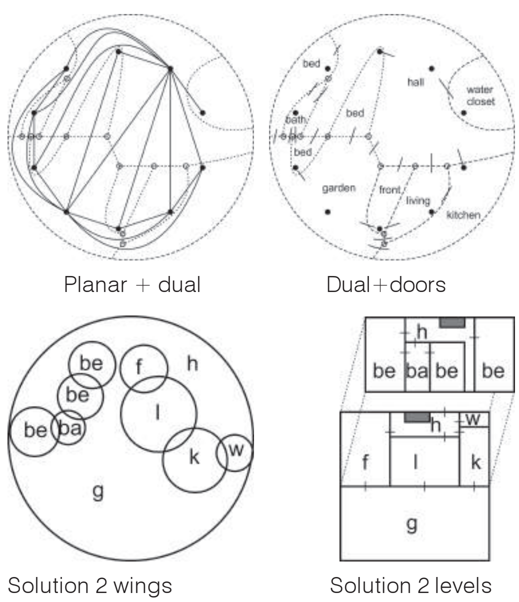
221 Dual graph



222 Possible relations between rooms



223 Planar selection of possible relations



224 Different solutions of the same dual graph

a With a 'network' usually a directed network is implied.
b Amongst others, used in programmes like Route Planner, to calculate the shortest path from A to B.
c For instance: in order to calculate the time required to realise a building or site.
d This form of representation is known as an AON-network: Activity On Node. Initially only AOA-networks were used with the activities on the arrows (Activity On Arrow). In that case the flow on the arrows represent the temporal duration.

This graph demonstrates that a third bedroom (10) can not be connected to the bathroom if it is already connected to the hall (1) and the garden (6). When the requirement that all bedrooms should give to the garden is skipped (the connection 9-6), a solution exists in which all bedrooms give access to the bathroom.

At 10 rooms, combinatorially speaking 10!/2!8! = 45 relations exist (K10). They can never be established directly (made planar).

Yet, the selection of the relational scheme can be made planar, therefore, it has a solution. With their high valence (number of doors), the hall (1) and the garden (6) are crucial. If these nodes are removed, a 'non-conjunctive' graph originates. Following that, it may be decided to give the rooms 6 to 10 a separate floor; or even its own location. If a node alone allows this freedom, it is termed a 'separational node' The minimal number of nodes n that can be taken away with the incident branches in order to make them non-conjunctive makes the graph 'n-conjunctive'. It is an important measure of cohesion in a system. In the figure following possible realisations are given by drawing the dual graph (figure 224).

A map of the national roads of our country is an example of a network. Each branch is denoting a stretch of road, each node a crossing or roundabout. By supplying branches with a length the user can determine quite simply the distance between his point of departure and that of his destination. If the traffic streams across the network and their intensities are known it is easy to determine the traffic load on each branch. A network like this may also represent the traffic deployment within a building, where the branches stand for corridors, stairs and elevators. Then it can be determined, for instance, how many students change places in between classes in the building. This gives a basis for deciding on dimensioning the corridors e.t.q.

The organisational structure of an enterprise may also be depicted in a network. If this structure should be expressed in the building at the design of a new office, this network may form a point of departure; one may derive from it which departments are directly linked to one another with the wish to realise this physically in the new building.

By the same token networks enable the structure of a building or an area, without describing the whole building or the whole area.

A graph with a weight (stream, flow) of any type (time, distance), on its branches, is called a network.[a] If this flow displays a direction as well, the network is termed a directed graph. A path is a sequence connecting branches with a direction and a flow. A network is a cyclical network, if a node connects with itself via a path. The length of the path is determined by the sum of the weight of the paths concerned. The length of the shortest path between two nodes may be determined by following, step by step, a sequence of calculating rules, the shortest-path algorithm.[b]

An algorithm in this vein exists for the longest path, the 'critical-path method'. This method is used in 'network planning' in order to determine, within a project, the earliest and latest moments of starting and finalising each and any activity of the project; and, consequently the minimal duration of the project.[c] In this, the nodes represent the activities, the branches the relation between the activities.[d] The start of an activity (successor) is determined by the time of finalisation of all preceding activities (predecessors). With regard to the network planning, it is feasible to monitor the progress of (complex) projects; as well as to survey the consequences of retardation of activities. However, the problems caused by retardation are not able to be solved. In order to achieve such a solution one has to employ other techniques; like linear programming (LP).

24.15 PROBABILITY

An event is a subset of results A[a] from a much larger set of possible results Ω[b] (a set much larger) that might have yielded different results as well. The chance of an event of A well-defined instances taking on ‘what had been possible’ Ω is - expressed in numbers - A/Ω. Often it is not easy to get an idea of what would have been possible; certainly when Ω has sub-sets dependent on A, for the event may influence the remaining possibilities.

Say, that the number of possibilities for filling in a surface with 100 buildings from 0 to 9 floors is 100^{10}. Two of these possible events have been drawn in figure 225: an example of ‘wild’ and one of ordered housing. The chance that with a maximal height of 9 floors one of the two will be realised exactly is 2 in 100^{10} (summation rule).

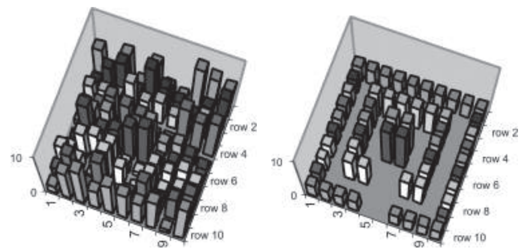
The wild housing leaves the elevation of a building completely to contingency. As a consequence in 100 buildings the average elevation is approximating the middle of 4,5 between 0 and 9. The average of 2.2 floors of ordered housing is deviating more from the mean than the one of wild housing and, therefore, less ‘probable’. In the matrix alongside the elevations from the drawing of the wild housing have been rendered.

It is as if a dice with ten faces has been thrown one hundred times. The rows form every time a sample of 10 ‘throws’ from the 100. Such a partial event in the first row of buildings can already yield 10^{10} (10 000 000 000) averages with only 10! = 3 628 800 *different* averages (less than 0,4%!). If one studies from among them all the 10 averages comprising the natural numbers (0 to 9) one must conclude that for 4 and 5 the maximum of possible combinations exist (9!/4!5! = 126).

In order to get any other average a lower number of combinations is available. For a result of an average of 0 or 9 , for instance, just one thinkable combination exists (the improbable events of 10 zeroes or 10 nines. The averages are condensing themselves between 4 and 5. The combination possibilities of the above are representing this way the chance density. In the column in question a Gaussian curve is, therefore, manifesting itself.

Obviously, this applies to the columns (events) as well. In figure 228 the mean, median and mode of the columns are *cumulatively* rendered by taking each time more columns into account.

The larger the number of throws, the closer the average approximates 4,5. However this does not apply as yet for the median (as many results above as below) and even less for the mode (the highest result). Their deviations of the mean are indicating asymmetrical, skew distributions.



225 Wild and ordered housing

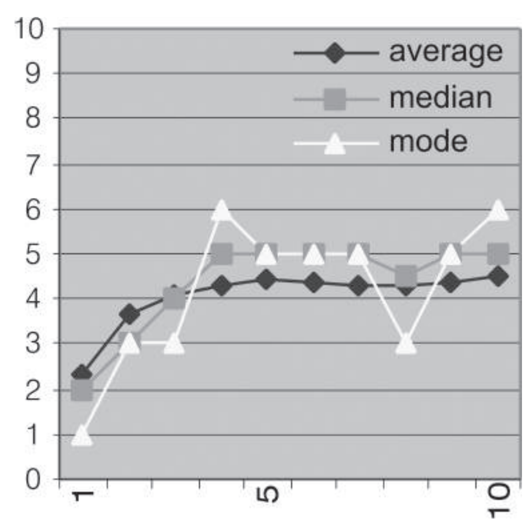
columns	1	2	3	4	5	6	7	8	9	10	mean per row
rows											
1	5	0	6	1	6	2	1	8	7	6	4,2
2	1	5	7	2	8	5	3	1	4	7	4,3
3	3	3	3	7	4	5	3	1	5	1	3,5
4	2	3	6	5	4	0	0	9	7	8	4,4
5	1	6	6	1	5	7	3	4	2	0	3,5
6	0	9	5	6	8	9	2	3	6	4	5,2
7	1	6	7	6	2	1	5	4	6	4	4,2
8	5	3	0	8	5	0	6	3	5	8	4,3
9	3	8	1	9	0	3	8	4	6	9	5,1
10	2	7	9	5	8	7	6	8	1	9	6,2
			Average for the total:								4 , 5

226 An average of means

rows			mean per row
9!/0!9! =	1	0 0 0 0 0 0 0 0 0 0	0
9!/1!8! =	9	combinations for the mean	1
9!/2!7! =	36	combinations for the mean	2
9!/3!6! =	84	combinations for the mean	3
9!/4!5! =	126	combinations for the mean	4
9!/5!4! =	126	combinations for the mean	5
9!/6!3! =	84	combinations for the mean	6
9!/7!2! =	36	combinations for the mean	7
9!/8!1! =	9	combinations for the mean	8
9!/9!0! =	1	9 9 9 9 9 9 9 9 9 9	9
		Average for the total:	4 , 5

227 Possibilities to compose averages from the 10 numbers from 0 to 9 an average

n =	10	20	30	40	50	60	70	80	90	100
mean	2,3	3,6	4,1	4,3	4,4	4,4	4,3	4,3	4,4	4 , 5
median	2,0	3,0	4,0	5,0	5,0	5,0	5,0	4,5	5,0	5,0
modus	1,0	3,0	3,0	5,0	5,0	5,0	5,0	5,0	5,0	5,0



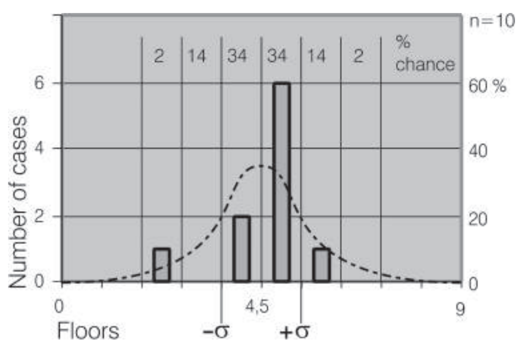
228 More results stabilise the mean

a This Latin capital is properly chosen, in view of its form-equivalence with the stream that ‘comes out’ of an urn.

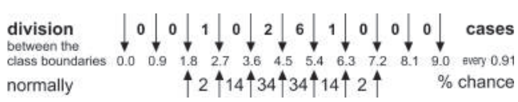
b This Greek capital is properly chosen, in view of its form-equivalence with the urn, proverbial in statistical textbooks, turned upside-down and producing arbitrarily red and white marbles.

The mean reduces the variation of a large set of numbers to one number. The variation itself is then very partially acknowledged with the 'standard deviation' sigma (σ). Some 2/3 of the cases differ usually less from the mean than this standard deviation. Some 95% lies within 2 x σ from the mean (95% probability area). This gauge σ only makes sense in the cases condensing themselves by combination possibilities around a mean. This is not applicable, for instance, for the individual cases of the example above. They have each an equal chance for 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 floors. The mathematically calculated 'standard deviation' on this basis would amount to some 5 floors at each side of the 4,5. Within this wide margin, not carrying meaning, all results are falling; not just the two thirds of them. However, the 10 columnar averages do concentrate themselves at the total average; for from average values like 4 and 5 more combinations of individual elevations could be composed than from extremes such as 0 or 9. If we consider the spreading of these 10 more 'obedient' outcomes, then the standard deviation may be calculated from the sum of the outcomes and the sum of their squares (squared sum):

Column-number	1	2	3	4	5	6	7	8	9	10	sum
outcome (means)	2,3	5,0	5,0	5,0	5,0	3,9	3,7	4,5	4,9	5,6	44,9
square	5	25	25	25	25	15	14	20	24	31	209,8
Standard deviation σ = ROOT((sum of squares – sum of outcomes ^ 2/n) / n)										n=10	0,91



229 Classes of observations



230 Chance within class boundaries

Mean μ as well as standard deviation σ may be used for the comparison of the event in the bar graph with a corresponding normal distribution (μ,σ) of the chance density that is approached in the case of very many events. This distribution represents the total of probable possibilities Omega Ω at a given μ and σ against a back-drop from which the event A is one outcome.

At both sides of the average with a sigma of 0,91 floors five classification boundaries have been distinguished with 0 and 9 for extremes. The number of cases between 4.5 and 5.4 floors is above expectancy.

Employing classification boundaries in this manner it may be seen at once in how far these results could have been expected on the basis of a normal distribution of chances. The case of a columnar average of just 2,3 floors (in the left column of the wild housing area) is rather far removed from the total average. Each average number of floors under 2,7 or above 6,3 lies outside the 95% probability area (14+34+34+14 = 96).

Now the usual mistake of policy makers and statisticians is neglecting these cases; or even using them as a point of departure for establishing norms. Contrariwise the designer is specialised in improbable possibilities from the available combinatorial explosion of possibilities, albeit within a context ruled by probabilities.

24.16 LINEAR PROGRAMMING (LP)

Imagine we want to invest on a location of 14.000 m^2 within 16 months in as much housing (D units) and facilities (F units) as possible:[a] (see also page 301)

asked: number of units	mln Dfl investment	1000 m^2 surface	months building time per unit
D	5	2	1
F	8	1	2
maximize	Z	14	16

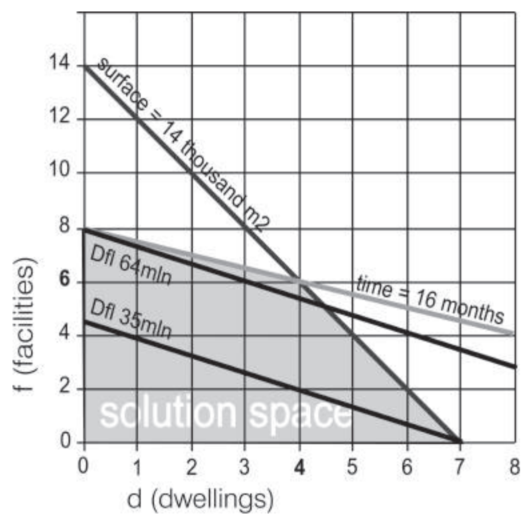
231 LP problem

What to build? In other words: what are F, D and Z? The objective (Z maximised) implies making 5D + 8F as large as possible (maximising it) under the following boundary conditions: (also known as restrictions or constraints)

Max!	$Z = 5D + 8F$			(maximize this investment Z within
	$2D + 1F$	$\leq$	14	x1000 m^2 surface and
	$1D + 2F$	$\leq$	16	months, while
	F	$\geq$	0	F and D are not negative)
	D	$\geq$	0	

232 LP operationalisation

All points (combinations of D and F) within the solution space satisfy all constraints, but not all fulfil the requirement of optimisation of a maximal Z. When no facilities are to be built (F=0) the surface restriction 2D+1F ≤ 14, determining the maximal value of Z, is restrictive, resulting in D = 7 dwelling units to be built in 1D + 2F = 7 months. The investment will be 5D + 8F = 35 mln. If no dwellings are to be built (D = 0), the maximum duration of building becomes the limiting factor, realising 8 units of facilities within 16 months. The maximum investment Z would be 64 mln. This is not yet maximal. Considering only the surface constraint, F = 14 units of facilities could be built on the site resulting in an investment of 112 mln, but that would take too much time: 28 months.



233 Solution Space

Next, we want to know for which point within the solution space the investment represented by the function 5D + 8F, is maximal.

In the origin (D=0, F=0) the investment will be zero, so Z = 0. Moving the line to D > 0 and / or F > 0 will increase Z. We will find the solution after moving this line as far as possible from the origin *without* leaving the solution space. In this case this point (4,6) The investment will be 5 * 4 + 8 * 6 = 68. The lines through this point define the solution. The boundary conditions are then called 'effective'. Removing or changing one of the boundary conditions will amount to changing the solution. Were there, for some reason, 15.5 months available, we could realise 4.2 dwelling-units and 5.7 facility-units. If a feasible solution has been found, the solution will always find itself in a corner point (intersection of two lines); unless the object function (objective) is parallel to the boundary condition determining the solution.

This simple example already shows how sensitive optimisations are *vis-à-vis* their context (two weeks less building time results in a different solution) and how important it is to readjust continuously the boundary conditions, which are quickly considered stable, or to vary them experimentally in sensitivity-analyses with regard to different perspectives, changing contexts. This requires tireless calculation in order to be able to react to changing conditions. If such boundary conditions are within one's own sphere of influence they may also be considered as objectives. This way the choice of what is called an end or a means gets a different perspective.

Often designers manage to annihilate boundary conditions deemed stable: suddenly, different objectives become interesting. This sensiti-vity with regard to context only increases, if more boundary conditions or objectives are taken into account; for instance, the valuation of occupants of facilities (e.g. 3) in contrast to dwellings (5) : then maximize 3F + 5D, while keeping, for example, the budget constant. The solution space now has 5 rather than 4 corners, which might be more optimal in one way or another.

a Convention used: names of unknown variables in capital letters, names of known ones in under-case. In multiplication known precedes unknown, so: a * X

With a problem with n variables and m restrictions, the cornerpoints number m+n / m!n!.

In case of a LP problem with 50 variables and 50 restrictions some 10^{29} equations must be solved. That is a time-consuming task even for the fastest computer. Luckily, it is not necessary to study all corner points. Proceeding from an initial solution obeying all conditions (and admitted or feasible solution) far fewer corner-points have to be investigated in order to find a solution; approximately m+n. The procedure to follow is known as the Simplex method (see page 223), that finds an eventual solution in a finite number of steps. With this the inequalities are transformed into equalities by adding 'slack variables' or 'remainder' variables before the inequal sign:

$$\begin{array}{lcl} 2D + 1F \leq 14 & & 2x_1 + 1x_2 + x_3 = 14 \\ & \text{becomes} & \\ 1D + 2F \leq 16 & & 1x_1 + 2x_2 + x_4 = 16 \end{array}$$

This way unknown variables have been added. With two of them, such as F and D, optimisation can still be visualised on a piece of paper. If more than two dimensions of decision are to be made variable, one is restricted to the outcome of an abstract sequence of arithmetical operations like the Simplex method, a method employing matrix calculation.[a]

24.17 MATRIX CALCULATION

Take a system of equations, for instance:

$$\begin{array}{ccccccccc} 2x_1 & - & 3x_2 & + & 2x_3 & + & 5x_4 & = & 3 \\ 1x_1 & - & 1x_2 & + & 1x_3 & + & 2x_4 & = & 1 \\ 3x_1 & + & 2x_2 & + & 2x_3 & + & 1x_4 & = & 0 \\ 1x_1 & + & 1x_2 & - & 3x_3 & - & 1x_4 & = & 0 \\ \hline a_1x_1 & + & a_2x_2 & + & a_3x_3 & + & a_4x_4 & = & b \end{array}$$

Usually the co-efficients, the unknown variables and the results are symbolically summarised as **Ax** = **b**. In this **A**, **x** and **b** represent numbers concatenated in lists (matrices) or columns (vectors)[b], in this case:

$$\mathbf{A} := \begin{bmatrix} 2 & -3 & 2 & 5 \\ 1 & -1 & 1 & 2 \\ 3 & 2 & 2 & 1 \\ 1 & 1 & -3 & -1 \end{bmatrix} \qquad \mathbf{x} := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \mathbf{b} := \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

This way the sum of the products $2x_1 - 3x_2 + 2x_3 + 5x_4$ equals the first number of **b** ($\mathbf{b}_1 = 3$), the next one $1x - 1x_2 + 1x_3 + 2x_4$ equals $b_2 = 1$, etc. This shows how matrix multiplication of each row of **A** with the column vector **x** works. In the imagination the column must be toppled in order to match each a with its own **x**. In order to calculate such a sum of products at all, the number of columns should of course, be equal to the number of rows of **x**. Considering that the number of rows of **A** conversely is not equal to the number of columns of row vector **x**, vector multiplication **xA** is impossible here: in matrix calculation **Ax** is not the same thing as **xA**. One cannot divide matrices and vectors[c], but often it is possible to calculate an inverse matrix $\mathbf{A}^{-1}$, so that one may write $\mathbf{Ax} = \mathbf{b}$ as $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$.[d] By that, one can not only determine n unknown variables in n equations by substitution, much writing and many mis-calculations, but also by applying the Gauss-Jordan method; or in the case of more solutions **b** by the (inverse matrix):

$$\mathbf{A} := \begin{bmatrix} -14 & 37 & -3 & 1 \\ 17 & -45 & 4 & -1 \\ -5 & 13 & -1 & 0 \\ 18 & 47 & 4 & -1 \end{bmatrix} \qquad \mathbf{x} := \mathbf{A}^{-1} \cdot \mathbf{b} \qquad \mathbf{x} := \begin{bmatrix} -5 \\ 6 \\ -2 \\ 7 \end{bmatrix}$$

The point of contact of the two equations of paragraph 24.16 may be determined easily both graphically and by substitution, but in the matrix guise the solution would look like this:

$$\mathbf{A} := \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \mathbf{b} := \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad \mathbf{x} := \mathbf{A}^{-1} \cdot \mathbf{b} \quad \mathbf{x} := \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

a Further reading: Hillier, F.S. and G.J. Lieberman (2001) *Introduction to operations research*.

b X is a column vector, XT the same sequence as a row vector

c Horssen, W.T. van and A.H.P. van der Burgh (1985) *Inleiding Matrixrekening en Lineaire Optimalisering*. describe in which cases that is possible.

d For instance in Excel A-1 is calculated by the function {=MINVERSE(A2:D5)} and x by {=MMULT(F2:I5;K2:K5)} if you indicate a field of 4 x 4, call the function, select the necessary inputfields A-1 and b and close with ctrl-shift.

Matrix A can also be used for calculation of the area and number of times needed to realise any combination of number of dwellingunits and number of facilityunits. If we want to know how may units can be realised within a given area and time, we have to solve the problem $\mathbf{x} = \mathbf{A}^{-1}\,\mathbf{b}$. In real life the number of variables and constraints are not equal. Neither is it necessary to consume all available space and time. So inequalities will be introduced. The result is that the number of solutions may be infinite. Therefore the linear objective-function has been added. Among all solutions find that one that results in a minimum or maximum value of the objective function as shown in 24.16.

The essential operations, with regard to the solving of a system of equations by means of the Gauss-Jordan method are: multiplying all co-efficients of one row with an identical factor adding a multiple of a row to a (different) row.[a] These operations are also essentially important with regard to the Simplex algorithm, to be discussed next.

24.18 THE SIMPLEX METHOD

The Simplex Method comprises two stages:

Stage 1: Look for a starting solution obeying all restrictions. Such a solution is termed an 'admitted solution'.

Stage 2: As long as the solution is not optimal: improve the solution whilst continuing to obey all restrictions.

The algorithm will be explained by way of a problem with just 2 restrictions. The origin is then an admitted starting solution, so that Stage 1 can be passed.[b] The problem from paragraph 24.16 serves as an example. First of all the inequalities are re-written as equalities with remainder variables:

	max		F		D		remainder variables				
i	$a_{i,s0}$ Z		$a_{i,1}$ x_1		$a_{i,2}$ x_2		$a_{i,3}$ x_3		$a_{i,4}$ x_4		b
0	1 Z	+	-8 x_1	+	-5 x_2	+	0 x_3	+	0 x_4	=	0
1	0 Z	+	1 x_1	+	2 x_2	+	1 x_3	+	0 x_4	=	14
2	0 Z	+	2 x_1	+	1 x_2	+	0 x_3	+	1 x_4	=	16

The crucial data have been re-written as a 'Simplex Tableau' below.

j			0	1	2	3	4	
i	Con	Bas	Z	F	D	Surf.	Time	b
0	Z		1	-8	-5	0	0	0
1	Surf.	3	0	1	2	1	0	14
2	Time	4	0	$\boxed{2}$	1	0	1	16

Simplex Tableau 1

The rows are numbered as i = 0…2, the columns as j = 0…4. Row 0 represents the target function, rows 1 and 2 the restrictions. Column 0 belongs to the Z value, columns 1 and 2 to the 'real' variables, dwellings D and facilities F, columns 3 and 4 to the remainder variables of the restrictions. Column b contains the values of the basic variables x_3 and x_4 named in column Bas (establishing together the basis); in the present case these are the initial remainder variables with a factor $a_{i,3}$ or $a_{i,4} = 0$. The value of the variables with a factor $a_{i,3}$ or $a_{i,4} = 0$ are not mentioned in this column (non-basis) and equals 0.

Next, it will be attempted to improve upon this initial solution by repeatedly exchanging a non-basic variable against a basic variable to be removed.

Step 1: Optimising test

If all elements within the row are larger than zero (Z>O), the basic solution belonging to this tableau is optimal. If row Z contains negative values, select column c with the most negative value, in this case column 1, variable F. This column is termed pivot column (axle). The appropriate non-basic variable is now introduced into the basis.

a A more elaborate introduction in matrix calculation in: Lay, D.C. (2000) *Linear algebra and its applications*.

b The algorithm for stage 1 is with a small addition equal to the one of stage 2.

Step 2: Selection of the basic variable to be removed

To determine the maximum value that this variable can take under the condition that all restrictions continue to be obeyed. Select the row r for which the ratio of the value b_r and $a_{r,k}$, with $a_{r,k}$ larger than zero, is minimal. This row becomes the axle or pivot row, element $a_{r,k}$ the axle or pivot, in this case $a_{2,1}$ with value 2. The basic variable belonging to this row (column Bas) has now been removed from the basis.

Step 3

Transform pivot column into unit vector with in the place of the pivot $a_{2,1} = 1$: multiply the pivot row by 1 / pivot (normalising pivot row):

row							
2	0	2	1	0	1	8	times 0.5
	0	1	0.5	0	0.5	4	

Now subtract an appropriate multiple of the pivot row from the other rows, so that the pivot column becomes 0 (a_{ik}) in each case. Then go back to step 1.

row							
0	0	- 8	-5	0	0	0	
substract	0	- 8	-4	0	-4	-64	times -8
	0	0	-1	0	4	64	
1	0	1	2	1	0	14	
substract	0	1	0.5	0	0.5	8	times 1
	0	0	1.5	1	-0.5	6	

The tableau now has the following appearance:

j			0	1	2	3	4	
i	Con	Var	Z	F	D	Surf.	Time	b
0	Z		1	0	-1	0	4	64
1	Surf.	3	0	0	$\boxed{1.5}$	1	-0.5	6
2	Time	1	0	1	0.5	0	0.5	8

Simplex Tableau 2

Since row 0 still contains negative elements, the solution found is not yet optimal. Steps 1 through 3 are repeated once again. The variable to be introduced is now 2, the variable to be removed the slack (3) with the surface restriction (row1). The pivot row now becomes

row							
1	0	0	1 .5	1	-0.5	6	times 0.67
	0	0	1	0.67	-0.33	4	

The other rows become:

0	0	0	- 1	0	4	64	
substract	0	0	- 1	-0.7	0.33	-4	times 1
	0	0	0	0.67	3.67	68	
2	0	1	0 .5	0	0.5	8	
substract	0	0	0 .5	0.33	-0.17	2	times 0.5
	0	1	0	-0.33	0.67	6	

The tableau now becomes:

j			0	1	2	3	4	
i	Con	Var	Z	F	D	Surf.	Time	b
0	Z		1	0	0	0.67	3.67	68
1	Surf.	2	0	0	1	0.67	-0.33	4
2	Time	1	0	1	0	-0.33	0.67	6

Simplex Tableau 3

This solution is equal to the graphical solution found previously (homes 6, facilities 4, investments 68).

24.19 FUNCTIONS

In colloquial speech sentences are formulated in which an active subject x operates on a passive object y. The verb in the sentence describes this operation. In logic a sentence like that, telescopes in a 'full-sentence function' y(x): 'y as operation of x' . In the same way mathematical formulae can be made which translate an input x (argument, original) according to a well-defined operation (function, representation instruction, e.g. 'square') into an outcome y (output, function value, image, depiction, e.g. the square). The outcome is represented as an operation of x: $y = f(x)$; e.g. $y = x^2$. The value of the independent variable x is chosen from a set of values X (the domain), the definitional range of the function.

Each value from this set corresponds to only one outcome from a set Y (range, co-domain, function value field) with possible values of y. With a given input the outcome is therefore certain.[a] That is why a function value x in a graph never runs back, like in the graph of a circle. There are just decreasing, increasing or unchanging values of y in the direction of x in the graph, allowing in one way unmistakable differentiation (rating growth) and integration (summing from x_1 to x_2).

Because of this function, it is the ideal mathematical means to describe causal relationships. It supposes that each cause has one consequence, but the same consequence may have different causes. If the sun starts shining, the building warms up, but the heat in a building may also result from the function of a heating system.

However, when cases should be studied in which one architectural design can have different effects, workings, functions according to circumstances, each effect should be modelled as a separate function.

Functions like that are alternately activated by steering functions according to circumstances. Since these functions change with the context, this context should be introduced as a set 'exogenous variables' and be included as parameters (for instance co-efficients, factors) in the functions. These exogenous variables to be introduced may be modelled next in a wider context as output of other functions. It results in a dynamic system of functions and magnitudes.

A computer programme for mathematical operations like MathCad, Maple, Mathematica or Matlab should then be extended with a modelling programme, say Simulink or Stella.

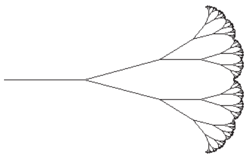
24.20 FRACTALS

Fractals are mathematical figures exhibiting self-similarity. A central motif returns in every detail. They result from rather simple functions; but these are being calculated many times using their own output; say 100.000 times.

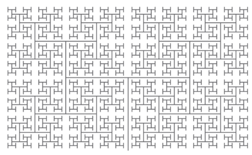
In practice one is forced, while working with a computer, to stop calculating after a few rounds, given the resolution capacity of the screen (for instance a high-quality colour screen). By using an artificial trick it is possible to zoom in on a detail: that is to say, to depict a detail enlarged, simply by introducing a smaller, well-chosen 'window'. Then it is possible to make more rounds (details of a higher order) visible. As a rule, one sees the central motif back in the details. With a computer of modest performance characteristics one is hindered in this enlarging by storage capacity as well as processing speed.

Professional study in the area of Fractals is conducted in institutes with access to very powerful and fast computers and video screens of large size and very large numbers of pixels. (Amsterdam, Netherlands; Bremen, Germany; Atlanta, Georgia, US).

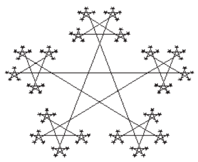
a In the reverse, however, an outcome may be produced by different values of the input.



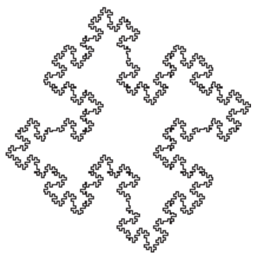
Branching



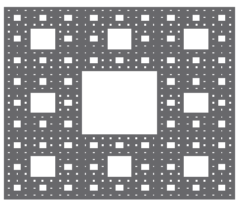
Branching 90 ^{0}



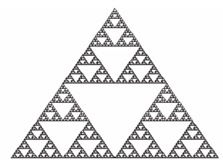
Branching star



Koch's replacements



Sierpinsky's carpet



Sierpinsky's triangle

234 Fractals

One may distinguish, according to the way they are generated, 4 kinds of fractals:

- Repeated branching (tree fractals)
- Replacement ('twists', 'turning curves')
- JULIA- and CANTOR- sets, based on the formula: z1 = z2 + C

As well as an extraordinary type:

- The MANDELBROT set.

In the case of repeated branching beautiful structures are emerging, based on a triangular star, a H, internal triangles (sieve of Sierpinski), forking, pentagram star, respectively a star with 7 points. There is an analogy with biological structures like a tree under an angle caused by the wind, an arterial system, the structure of a lung.

With repeated replacement a straight line segment — is replaced by, for instance -^- , or ^, or |‾| a meander. This way 'twists' are originating, turning curves, curves baptised with the name of their discoverers, like Levy, Minkovsky, Koch, etc.[a] In the latter there is an analogy with a coastal line; it may be helpful to determine the length of a coastal line and of other whimsical contours.

With repeated transformation (displacement, turning, disfiguring, including diminishing) figures originate resembling leaves, or branches with leaves. Applying a large number of such appropriately chosen transformations may generate images of non-existing forests, non-existing landscapes with mountains and fjords and even of a human face.

Even 'fractalising' an image of a realistic object can be done: that means to say to determine for a sufficient number of transformation formulae each time the 6 parameters in such a way, that they will render that image with great precision.

Further development of this technique is of importance for transmission of images; since these are already determined by a small set of numbers.

In the case of repeated, iterated, application of a formula like z = 2 + C, (where z and C are so-called 'complex numbers', that can be represented by a point {x, y} respectively {A, B} in the complex plane, thus allowing calculation of a new point z as the sum of (the old) z squared and the constant C) results in helixes; shrinking or expanding, depending on the initial value of z. For certain initial values of z border-line cases originate. That is causing then 'curves' with a bizarre shape: a JULIA set. Sometimes it is a connected curve, sometimes the curve consists out of loose, non-connected parts, depending on the value of C.

A set of points C (A,B) establishes the MANDELBROT set. If the computer is ordered to make a drawing of all points C that might deliver a connected JULIA set, another interesting figure is emerging displaying surprisingly fractal character as well. Whilst zooming in on parts of that figure one discovers complex and fascinating partial structures where in the details de key motifs materialise again and again. Compared to the other fractals discussed here the MANDELBROT fractal is of a higher order.[b]

24.21 DIFFERENTIATION

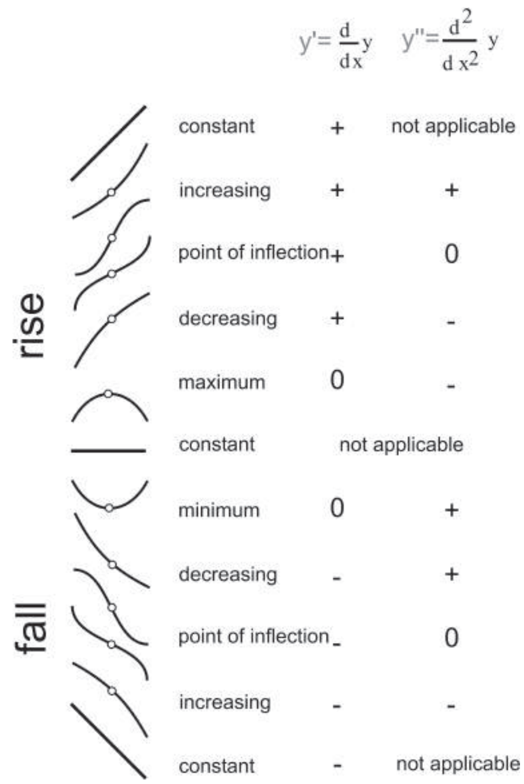
From the difference of two evenly succeeding values in a sequence in space or time one can derive the change at a given moment or position. Also, these derived differences can be put into a sequence. If all 'holes' between two contiguous values in a sequence are filled (Cauchy-sequence; for instance the sequence of values of a continuous function y), one does not speak any longer of a 'difference', Greek capital delta (Δ), (for this has reached its limit to zero), but of a 'differential' (d). If the difference in value dy that has approximated zero is divided by the distance in space or time dx between both values that has also approximated zero a 'differential quotient' dy/ dx results.

a See http://library.thinkquest.org/26242/Full/index.html
b Mandelbrot, B.B. (1983) *The Fractal Geometry of nature*.

In the graph of a function this quotient stands for the inclination at a certain place or moment. How flat or how steep a motor road is, is also indicated by the ratio between height and length (in percentages). However, in the case of the derived function both are approaching zero, so that they relate only to one point of a curve, instead of a trajectory. The quotient y'= dy/ dx is positive in climbing, negative in descending (see corresponding figure).

A lot of rules exist in order to formulate for a given function for all values at the same time a derived function. Since Leibniz and Newton these derivations can be proved by and large; but it is not strictly necessary, to know that proof in order to use the rules. Still the question remains, whether the mathematical insight aimed at in an education benefits from absence of proof in the Euclidean tradition. However this insight does not benefit either from applying without comprehension these rules learned by heart. Computer programs like MathCad, Matlab, Mathematica or Maple do apply these rules automatically and unerringly, so that all attention can be concentrated on the external behaviour of the functions and their derivations.

In the table of figure 235 one may see that minima, maxima, and turning points can be found by putting the derived function y'= 0 or the derived function of the derived function y'' = 0, and then calculate the corresponding y. This is especially important for optimising problems. However if one knows of a certain function for instance just that is rises increasingly, one can search contrariwise for a formula for y for which y' as well as y'' is positive. For this reversed way (integration) a body of calculation rules is also available.



235 Properties of derived functions

24.22 INTEGRATION

Imagine, that one determines for a symmetrical ridged roof for the time being the width to cover, for instance b = 10m, the height of the gutters (gh) to left and right gh1 = gh2 = 4; the height of the ridge rh1 = rh2 = 9 and the ridge position rp = 0.5b. Then the slope of the surfaces of the roof is depending on this. In Mathcad the calculation looks as follows:

Width and ridge position of a roof:

b = 10	rp = 0,5b
roof-plane left	roof-plane right
gh1 = 4, rh1 = 9	rh2 = 9, gh2 = 4

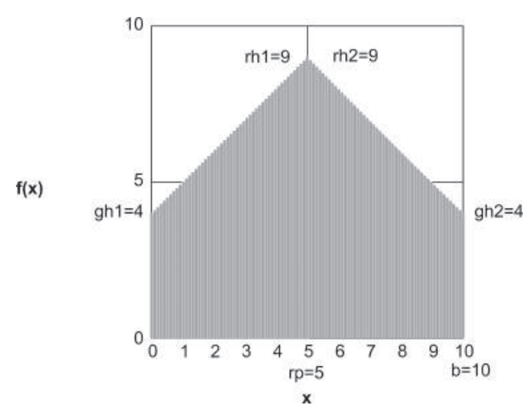
The roof-slopes are now:

$$h1 := \frac{(rh1 - gh1)}{rp} \qquad h2 := \frac{-(rh2 - gh2)}{(b - rp)}$$

The mathematical formula describing the course of the left half between x = 0 (left gutter) and x = rp (ridge) is h1x, to which is added the height of the gutter. However, the part to the right had a descending slope h2 and therefore a different formula between ridge x = rp to the gutter at the right (x = b). Mathcad can assemble both formulae to one discontinuous function f(x) with an 'if-statement': 'if x < rp, then f(x) = gh1 + gh2, else f(x) = rh + h2(x – rp)'. This is described as follows:

Wall function: $f(x) := \text{if}(x \le rp,\ gh1 + h1 \cdot x,\ rh2 + h2 \cdot (x - rp))$

Summed surface area of the wall from 0 to b: $\int_0^b f(x)\,dx = 65$



236 A wall as a function

The integral over the width gives the surface area of the front wall. When one is now also taking into account de depth y, this surface may be integrated one more time over y in order to get the cubic content.

Suppose, that the ridge position with the depth y is varying according to a randomly chosen formula rp(y), then the roof slopes h1(y) and h2(y) also vary in depth y:

$$rp(y) := \frac{10 - y}{3} + rp$$

$$h1(y) := \frac{(nh1 - gh1)}{rp(y)} \qquad h2(y) := \frac{-(rh2 - gh2)}{(b - rp(y))}$$

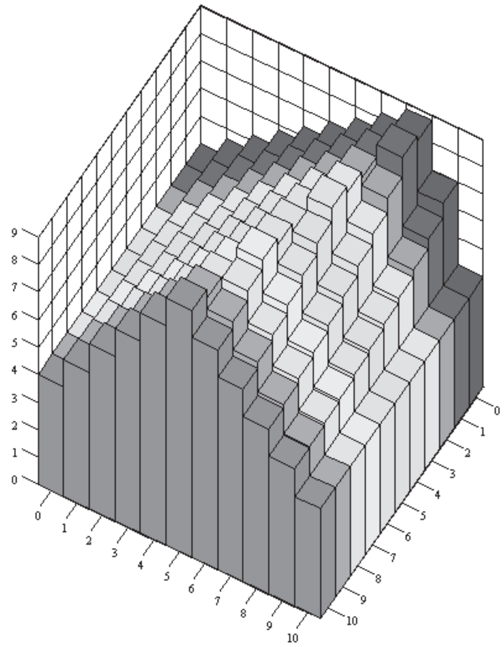
The wall function now becomes a building function with two variables f(x,y). In the wall function one should only substitute for the constant np the variable rp(y) and do the same for h1 and h2:

$$f1\,(x\,,\,y) := \text{if}\,(\,x \le rp\,(y)\,,\,gh1 + h1\,(y)\cdot x\,,\,rh2 + h2\,(y)\cdot(x - rp\,(y))\,)$$

When one substitutes for x and y discrete values from 0 to 10, one can store the values in a 10 x 10 matrix Mx,y = f1(x,y). It can be made visible as in figure 237:

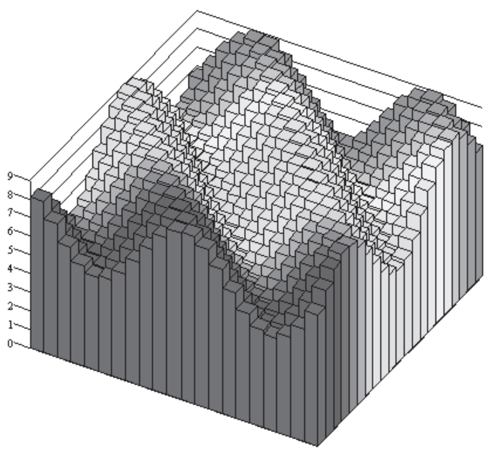
$$\sum_{x=0}^{10}\sum_{y=0}^{10} M_{x,y} = 757.067$$

$$\int_0^{10}\int_0^{10} f1\,(x,y)\,dx\,dy = 649.997$$



237 A house as a function

When the values over rows and columns are summed in the matrix one gets a first impression of the content; the double integration of the function f1(x,y) over x and y obviously yields a more exact calculation. This is particularly convenient, when one is opting for smoothly flowing roof-shapes, for instance the house in figure 238.



238 A house with waved walltops as a function

Differential and integral calculus may be understood along the lines of spatial problems; but it is more often applied in the analysis of processes. In that case the differential dx is often indicated by dt and the growth (or shrinkage) on a particular moment in time by:

$$\frac{d}{dt}f\,(\,t\,)$$

A well-known example is the acceleration with which a body rolls from a variable slope, like a raindrop from the roof. In architecture calculations for climate control are mainly the ones using differential and integral calculus.

24.23 DIFFERENTIAL EQUATIONS

Population growth offers an example of growth that may grow itself, sometimes constant (k) with the size of the already grown population P(t) itself:

$$\frac{d}{dt}P\,(\,t\,) = k\cdot P\,(\,t\,)$$

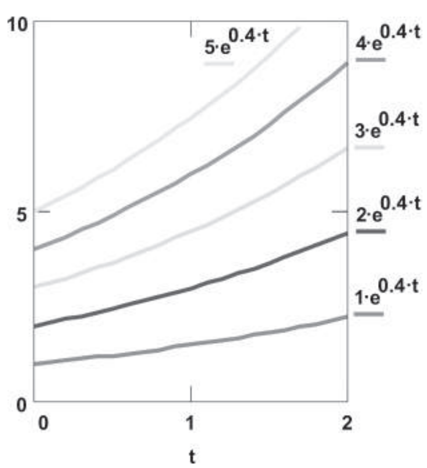
One may verify this even when not knowing a formula for P(t). Such an equation, where an unknown function is occuring together with one or more of its derivations, is called a differential equation. Also in daily parlance primitive differential equations are used: 'If the population is increasing, the population growth is becoming correspondingly larger'.

Solving a differential equation entails finding formulae for the unknown function (here P(t)). Since the derived function of e^{kt} is equal to ke^{kt},

$$\frac{d}{dt}e^{k\cdot t} = k\cdot e^{k\cdot t}$$

the derived function of e^{kt} (ke^{kt}) is easy to calculate. This is the reason for a preference to express functions as a power of the real number e = 2,718.

However, if one solution for the differential equation is $P(t) = e^{kt}$, this does not need to be the only solution. Usually, there is a whole family of solutions. We could have substituted, for instance, Ce^{kt}; for its derived function is Cke^{kt}, so that the differential equation also holds in that case. The solutions in the following are by the same token just a few of an infinite number of possible solutions for k = 0.4 (e^k=1,5), for instance as in figure 239.



239 Powers of e

In order to get to know which formula of this family is the right one, we must know the outcome on any specific moment in time, for instance t=0 (initial value). Supposing that we know that the population was initially 2 people, then we can calculate C from Ce^{kt}=2.

Given that $e^0=1$, C=2 applies for each k. In this family of equations C is always the initial value. From this follows the only formula that is correct: $P(t)=2e^{kt}$. However, this would mean that if every person would have after 25 years (a generation) 1.5 children on the average (three per couple), there would be after 2000 years (80 generations) $1.579 \cdot 10^{14}$ children.

The hypothesis of constant growth with regard to the population used in our differential equation is not correct. There are limits to the growth put to it by the sustaining power of the environment. This is demonstrated by many other species than *Homo sapiens* and the real course of the population since 1750 in millions of inhabitants.

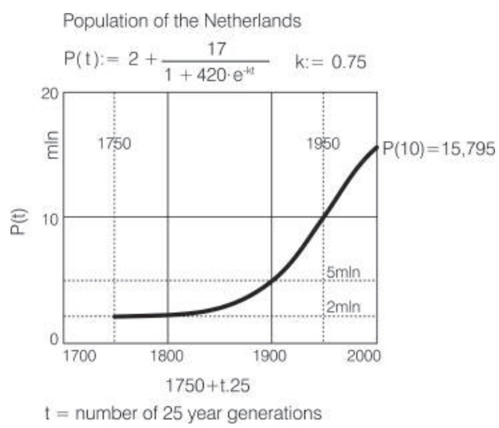
Exponential growth is only valid in the case of relatively small populations. Nearing the boundaries G – established by the size of the habitat – the growth slackens (see also page 257). We can add a term to the differential equation realising this:

$$\frac{d}{dt}P(t) = k \cdot P(t) \cdot \left(1 - \frac{P(t)}{G}\right)$$

If P(t) is small compared to G, the term within the brackets approaches 1, so that our original hypothesis remains valid; in the other case the growth becomes 0, or even negative. Logistical curves like the following comply:

$$P(t) := 2 + \frac{17}{1 + 420\, e^{-k \cdot t}}$$

The function is a variant of the function mentioned as an example on page 257. Differential equations are employed to generate from a hypothesis families of functions, allowing selection based on known initial values.



240 Simulated population of The Netherlands

23.24 SYSTEMS MODELLING

Any system has a border; beyond it exogenous variables serve as input for the first functions, they meet within the system. The output of these functions serves on its turn as an input for subsequent functions, sometimes operating in parallel. The system as a whole delivers in the end an output of tables, images, animations or commands in a process of production.

If a system is modelled on a computer, it is a program that just like a word processing program, awaits input and eventually a command to execute (e.g. the command ‘print’). A programming language like Basic, Pascal, C or Java, contains, except mathematical functions, a host of control functions, like ‘print(programming language)’ , and control statements like ‘do…while’, ‘if …then… else’.

The ‘if…then…’ function, as applied to the ‘wall-function’ on page 227 is a good example for understanding how during the input in a system and during the course of the process – depending on the circumstances – it may be decided which function should operate on the incoming material next and to which following function the result should be passed next as input.

25 VISUALISATION AND ARCHITECTURE

ALEXANDER KOUTAMANIS

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25.1 RÔLES FOR VISUALISATION

Visualisation of real and imaginary space is a traditional strong point of architectural education and practice. Even when architectural design is removed from the influence of visual arts, the architect makes extensive and intensive use of visual methods and techniques in the development of a composition, the specification of a design product, the communication of more abstract concepts, and the analysis of design ideas. As a result, knowledge of the world's architecture stems more from published images than personal experience.[a]

Emphasis on visual matters in architecture is not accidental. Human inter-action with natural and built environments is predominantly visual. A wide spectrum of human activities, from aesthetic appreciation to planning of actions, relies heavily on visual information and makes use of visual means to analyse and formulate states and conclusions. Visualisation was a significant aid to understand and control complex processes. Widespread employment of pictorial instructions for e.g. assembling a piece of furniture, putting on a life jacket or tying a tie in a Windsor knot demonstrates the extendibility of relatively simple visual representations.[b]

Recent technological and cultural changes form a new context for a re-evaluation of the significance of visualisation for architecture. Pictures are re-emerging as vehicles for storage, manipulation and communication of information, especially in relation to the visual environment.[c] Such changes are a useful antidote to the aesthetisation of pictorial representation — of which design disciplines are often found guilty. Moreover, they agree with the primary dual purpose of visual representations in designing:

- registering input and output to cognitive processes: internal mental representations are refreshed and reinforced by creating external versions and subsequently internalising them again through perception.
- communicating design ideas: from visual / geometric specification of forms to be built to analysis of functional patterns.

25.5 THE DEMOCRATISATION OF COMPUTER TECHNOLOGIES

The re-emergence of pictures as information carriers relates to computerisation. Current developments suggest that we are entering an initial phase of the computer era. The most striking feature of this phase is democratisation of information technology. After two decades of relatively slow development, restricted to the initiated, the computer is becoming a ubiquitous appliance linked to a new information infrastructure. Computerisation of the workplace was followed by an increasing presence of computers in entertainment and at home.

While the computer's value in increasing efficiency has been amply proven, as in production and management of building documents, its applications have yet to lead to higher quality and performance in designing the built environment or in the built environment itself. The availability of computational power is not matched by methodical utilisation of computing for improvement of current practices. Most computing applications in architecture and planning are sporadic *ad hoc* transfers of technology that may resolve isolated problems, but do little to relate the solutions they provide to their wider context. The transition from analogue to digital media was restricted initially to two-dimensional representations (line drawings) matching limitations of available technology and priorities of architectural practice. Subsequent addition of the third dimension to two-dimensional drawings and production of photo-realistic renderings on the basis of three-dimensional models was also geared to efficiency and productivity rather to than new forms of expression.[d]

a Evans, R. (1989) *Architectural projection*.
b Gombrich, E. (1990) *Pictorial instructions*.
c Lopes, D. (1996) *Understanding pictures*.
d Mitchell, W.J. (1992) *The reconfigured eye;* Mitchell, W.J. and M. McCullough (1995) *Digital design media.*

241 Drawing by an eight-year-old (1996, KidPix on a Macintosh Powerbook 165c)

Still, the new technologies are already having a profound influence on architectural visualisation in three significant ways. The first is that, by making computational power available, affordable and relevant, they provide more efficient and economical implementations of pre-existing analogue techniques, as well as new, complementary tools. Younger generations are particularly proficient in digital visualisation. Figure 241 is a casual drawing by an eight-year-old with the program KidPix on a Macintosh Powerbook 165c. Despite the added difficulty of having to master the trackball of the particular computer model, the drawing comes very close to the child's drawings on paper. Even the use of standardised elements in the computer programme echoes her application of self-adhesive and stencilled figures.

The early familiarity of today's children with computer visualisation, their natural acceptance of cognitive and manual ergonomics, as well as their high exposure to related media, like video and arcade games, suggest that digital tools will soon cease to be an alien technology in architectural education and practice. Even thorny issues like digital sketching (cf. figure 241) will be resolved simply by the future users' proficiency in both the digital and analogue versions.

A second potential contribution of modern visualisation technologies is provision of sharper, more reliable and hopefully more intuitive geometric tools. The practical and conceptual necessity of describing three-dimensional objects with coherence, accuracy and precision created a strong but strained relationship between architecture and geometry. A frequent complaint is that orthographic projections may fail to register salient features of their subject. Consequent rebellion against the "tyranny of the box" oscillates between giving up geometry altogether and adopting other, more complex geometries —choices with an outcome never fully explored.[a]

The third influence of democratisation of computer technologies on architectural visualisation lies in that it opens a wide and exciting new market for visualisation in information systems. Graphical interfaces are frequently developed for spatial forms, as for example the Internet with VRML. The architects' experience in representing spatial patterns visually has led to the assumption that design of these interfaces and of inter-action in information space adds to the scope of architects who are arguably better suited to such subjects than other design specialists today.

25.3 REPRESENTATION: A DEFINITION

A suitable working definition of what a representation is and what it does can be derived from Marr.[b] According to this, a representation is a formal system for making explicit certain entities in a transparent manner, i.e. together with an explanation of how the explicitness is achieved. The product of a representation as applied to a specific entity is a *description*. Familiar examples of representations include Roman and Arabic numerals (decimal or binary). Figure 242 contains alternative descriptions of the number 17 produced by different representations.

242 Alternative representations

In each of them a number is described on the basis of a finite set of symbols and a rule systems for composing a description from the symbols. Arabic decimal numerals use the following set:

$$S_A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

These symbols are correlated to a number in the following manner of positional notation:

$$n_n * 10^n + n_{n-1} * 10^{n-1} + \ldots + n_1 * 10^1 + n_0 * 10^0 \Rightarrow n_n n_{n-1} \ldots n_1 n_0$$

For example:

$$1 * 10^1 + 7 * 10^0 \Rightarrow 17$$

a Evans, R. (1995) *The projective cast, architecture and its three geometries*.
b Marr, D. (1982) *Computer vision*.

Arabic binary numerals make use of a smaller set of symbols and corresponding decomposition rules: $_B = \{0, 1\}$

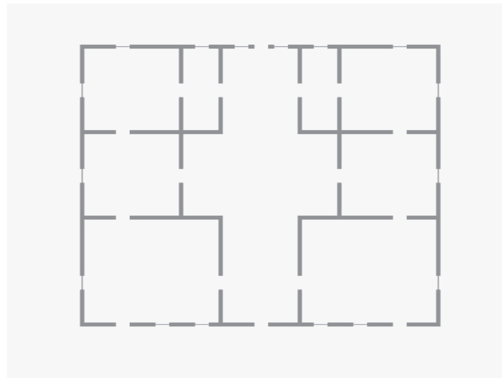
$$n_n * 2^n + n_{n-1} * 2^{n-1} + \ldots + n_1 * 2^1 + n_0 * 2^0 \Rightarrow n_n n_{n-1} \ldots n_1 n_0$$

For example:

$$1 * 2^4 + 0 * 2^3 + 0 * 2^2 + 0 * 2^1 + 1 * 2^0 = 10001$$

Architectural representations are essentially similar in structure. They consist of symbols for spaces and/or building elements, relations between the symbols and correspondence rules for mapping the symbols and their relationships to the subject of representation. Figure 243 depicts the symbols of a basic set of building elements. The set is sufficient for describing orthogonal floor plans, like the one in figure 244, as two-dimensional arrays comprising generic building elements.[a]

243 A basic set of symbols for floor plans.



244 Floor plan created with the symbols of figure 243.

25.4 IMPLEMENTATION MECHANISMS

The significance of spaces and building elements in architecture has not been realised in practical design computing and visualisation. Drafting and modelling programs generally employ lower level geometric primitives, like points, lines and simple surfaces for the outlines of building components. Moreover, these geometric symbols are seldom grouped together into a coherent description of a component and have few, if any, explicit relations to other elements.

A useful distinction, also from Marr[b], is the one between representation and implementation. For every representation there are several alternative implementations, usually depending on the context of the application. For example, binary numbers can be represented with Arabic numerals (1 or 0) or with states of switches (ON or OFF). Both refer to the same representation: the implementation mechanisms change; not the actual symbols used in the representation.

The elevation of implementation mechanisms like lines and surfaces to primitives of architectural design is symptomatic of two general conditions in computerisation of architecture. The first: most digital techniques are direct transfers of analogue practice. This almost always includes unquestioning acceptance of the implementation mechanisms of an analogue representation as the basis of its digital equivalent. The second: an underlying mystification tendency, confuses implementation mechanisms and visualisation techniques with spatial form and perception. The use of spaces and building elements as primitives of architectural design representation is too prosaic to allow far-fetched associations and loose metaphors, which can be easily accommodated in neutral geometric justifications.

25.5 ELEMENTS AND RELATIONSHIPS

Research into the structure of symbolic representation focuses on two issues:

- which primitives should be employed and at what level,[c] and
- the possibility of units (chunks, partitions, clusters more structured than simple nodes and links or predicates and propositions.[d]

The primitives issue can be resolved by the analysis of existing representations of the built environment. These traditionally assume a direct, atomistic form. A conventional representation like a map or floor plan comprises atomic elements like individual buildings or building components. These elements appear at an abstraction level appropriate to the scope of the representation. Depending on the scale and purpose of a map, buildings are depicted individually or concatenated into city blocks. Similarly, a floor plan at the scale of 1:50 depicts building components and elements that are ignored or abstracted at 1:500. Most other aspects of built form remain implicit, with the exception of those indicated as annotations by means of colouring and textual or symbolic labels conveying information like grouping per sub-system, material properties or accurate size. Relations between elements, like the align-

a Koutamanis, A. (1995) *Recognition and retrieval in visual architectural databases*.
b Marr, D. (1982) *Computer vision*.
c Brachman, R.J. and H.J. Levesque (1985) *Introduction*.
d Brachman, R.J. (1985) *On the epistemological status of semantic networks*.

ment of city blocks or the way two walls join in a corner are normally not indicated —unless, of course, they are the subject of the representation, as in detail drawings.

Using formalisms like semantic networks, frames, scripts and objects, elements are brought together in associative symbolic representations that share the following features:

- representation consists of objects and relations between objects;
- objects are described by their type, intrinsic properties and extrinsic relations to other objects;
- properties are described by constraints on parameters;
- relations are described by networks of constraints linking objects to each other.

A comparison of such representations with conventional architectural drawings reveals that architecture is handicapped by omission of explicit relationships between elements. The reasons for this omission have to do with representational complexity and range from the user's unwillingness to input multiplicity of relevant connections, to the computer's inability to handle them efficiently and effectively. Consequently, architectural associative symbolic representations has restricted greatly focused generative systems where structure and intention can be controlled.

More ambitious representations have attempted to integrate all relevant aspects and entities. Their main intention: resolving real design problems as encountered in practice. However, large or holistic representations have a size and complexity that often make representations unmanageable both for computers and humans. Problematic maintenance and lack of predictability in the behaviour of such representations, especially following modification and augmentation, limit severely their applicability.[a]

25.6 ELEMENTS

Architectural composition is often equated to arranging items chosen from a finite set of 'solid' building elements and/or 'void' space forms. Building elements traditionally attract more attention than spaces. Especially within the confines of a single formal system we encounter relatively compact and well-ordered collections of building elements which form the *sine qua non* of the system. The best example of such collections is the orders of classical architecture, where canonisation of the system was achieved by standardisation of a small subset of building elements. The conspicuous presence of these elements in classical buildings led to the view that a building with classical proportions cannot be classical if it does not contain elements from the classical orders.[b]

The attention paid to the arrangement and articulation of a specific subset of building elements has propagated a particular image of architectural design that is more akin to fine arts than to engineering. Even after the classical orders were dismissed by modernist architecture and replaced by abstract systems based on proportion and standardisation, this image remained an implicit yet powerful part of architectural methodology. Probably the best examples are the ideas on industrialisation in building developed and applied after WW II. These were dominated by standardisation of building elements in size and type and modular co-ordination for the arrangement of these elements. The resulting hierarchical system of building elements and positioning constraints bears similarities with the classical orders. The image of architectural design as arrangement and articulation of a finite set of building elements has been influential in computer-aided architectural design. It suggested a graphic system where the designer selects objects from a database and integrates them in a design by means of simple geometric transformations.

A significant issue relating to the elements of architectural representations is the duality of 'solid' building elements and 'void' spaces in the representation of the built environment. Spaces are less frequently chosen as the atomic elements of architectural composition than building elements. This is frequently attributed to the implicitness of spaces in conven-

a Gauchel, J., S. van Wijk et al. (1992) *Building modeling based on concepts of autonomy.*

b Summerson, J. (1980) *The classical language of architecture.*

tional analogue representations like floor plans. A possibly more significant reason is the equation of spatial arrangement to a fixed pattern locally elaborated, annotated and studded with building elements. Such an interpretation appears to underlie traditional design practices, as well as computational design studies including space allocation[a], shape grammars[b] and similar generative systems.[c]

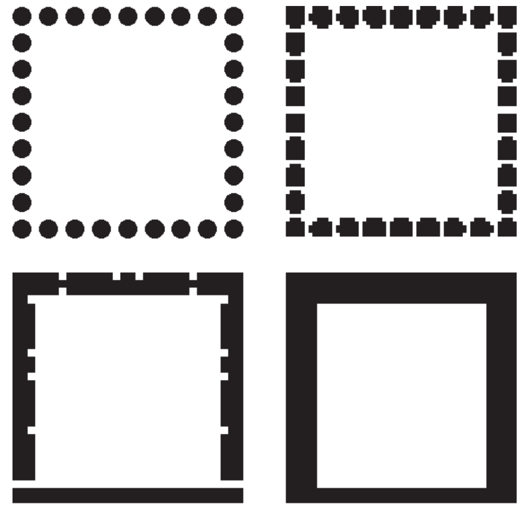
In cognitive studies the representation of objects by their parts and modules has been a common hypothesis for computational and cognitive studies of vision and visual recognition.[d] According to this hypothesis, a visual scene is parsed into components, normally corresponding with the canonical parts of the objects depicted in the scene. The representation derived from a scene has a multi-level structure, each level corresponding to a different abstraction level. At the highest level an object is represented as a single component analysed into smaller components at subsequent (lower) levels. For example, a human form starts as a single component, then sub-divided into components for head, torso, arms and legs. Each component is further sub-divided, e.g. an arm into upper arm, forearm and hand. Again the hand is analysed into components for palm and five fingers.[e]

Elements are straightforward to define and recognise in a multi-level structure; but their applicability is limited to a small range of abstraction levels. In figure 245 the actual elements (top left) are thirty two bullets arranged along the sides of an imaginary square. Nevertheless, the image is normally described simply as a square. Rather than describing the atomic parts we group them in one pattern denoting the overall configuration. The same effect can be achieved by lowering resolution of the image, as in figure 245 (sequence: top right, bottom left, bottom right). By doing so, the individual parts progressively lose individuality and fuse into a solid square.

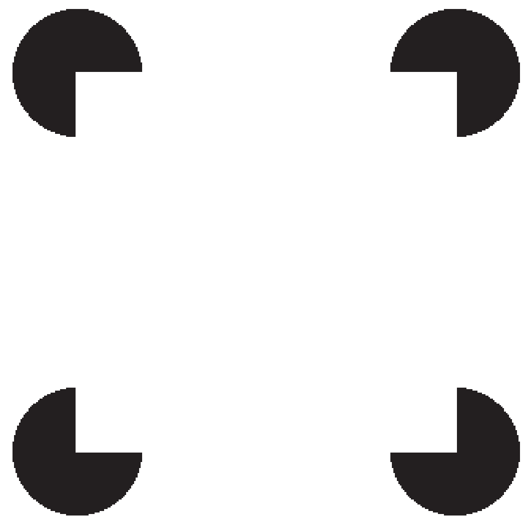
In other situations the actual elements are interpreted as something different than what they actually are. In figure 246 the four incomplete disks are interpreted as four complete black disks partially occluded by an illusory white square.[f] The instability and degradation of elements suggest that beyond certain levels of abstraction atomic elements are replaced by co-ordinating devices. These devices can be derived by purely visual processes (figure 245). This, however, does not preclude the existence of these co-ordinating devices as separate entities existing independently of the elements and which may appear in representations with or without elements.

Despite limitations in applicability of elements, it is assumed that, at a low level (before significant abstraction occurs), the representation of complex visual scenes can be based on a small set of basic components. Biederman proposed that this set can be reduced to 36 simple components, called *geons*.[g] Similar principles have been employed for recognition of line drawings of three dimensional scenes where the repertory of possible edge junctions was reduced to a small number of configurations labelled with respect to convexity/concavity.[h] In an austerely trihedral environment the number of possible junction configurations is just 18.[i] The same applies to representation of spaces in orthogonal floor plans (figure 243 and 244), where 8 types suffice.[j]

The arrangement of elements is normally represented in terms of an associative structure linking discrete components to each other with spatial/geometric relationships.[k] As with the number of elements, it is proposed that the number of basic relationships is quite low. Geons relate to each other by means of five edge properties.[l] In line drawings correlation of edge junctions takes place on basis of the labelling of each edge with respect to convexity / concavity in an iterative constraint propagation procedure.[m] In orthogonal floor plans each space corner is linked to two other corners, one in horizontal, one in vertical direction. For a given space corner type there are two possible types of corners it can be linked to in either direction.[n] This also suggests that certain relationships are implicit in the type of the elements: each element is characterised by specific expectations concerning type and position of elements to which it relates.



245 Elements and abstraction



246 Elements and illusory contours

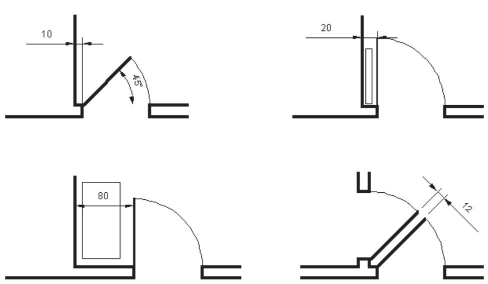
a Eastman, C.M. (1975) *Spatial synthesis in computer aided building design*.
b Stiny, G. and W.J. Mitchell (1978) *The Palladian grammar*.
c Hersey, G. and R. Freedman (1992) *Possible palladian villas*.
d Brooks, R.A. (1981) *Symbolic reasoning among 3-D models and 2-D images;* Marr, D. (1982) *Computer vision;* Tversky, B. and K. Hemenway (1984) *Objects, parts, and categories;* Biederman, I. (1987) *Recognition by components: a theory of human image understanding;* – (1995) *Visual object recognition*.
e Marr, D. (1982).
f Kanizsa, G. (1979) *Organisation in vision. Essays on Gestalt perception preager*.
g Biederman, I. (1987, 1995).
h Guzmán, A. (1966) *Computer resolution of three dimensional objects in a visual scene;* Clowes, M. (1971) *On seeing things;* Huffman, D. (1971) *Impossible objects as nonsense sentences;* Mackworth, A.K. (1973) *Interpreting pictures of polyhedral scenes;* Waltz, D. (1975) *Understanding line drawings of scenes with shadows*.
i Winston, P.H. (1992) *Artificial Intelligence*.
j Koutamanis, A. and V. Mitossi (1992) *Automated recognition of architectural drawings;* Koutamanis, A. (1995) *Recognition and retrieval in visual architectural databases*.
k Marr, D. (1982); Winston, P.H. (1992).
l Biederman, I. (1987, 1995).
m Waltz, D. (1975).
n Koutamanis, A. and V. Mitossi (1993) *Computer vision in architectural design;* Koutamanis, A. (1995).

25.7 LOCAL CO-ORDINATING DEVICES

While representation of elements in both analogue and digital design practice is explicitly supported by symbolic techniques, less importance has been attached to the way in which elements are integrated in a design. This is normally left to the designer who has to position and connect each new element in a building representation with little help from his instruments. For example, many drafting and modelling systems still fail to address the physical impossibility of two objects occupying the same space, let alone attempt to interpret the designer's intentions in overlapping objects. In analogue design media, this is a logical consequence of their implementation structure. An analogue representation is perceived, recognised and interpreted by the human viewer. Computerised representations, on the other hand, are not limited by human interpretation. On the basis of explicit relationships between objects the computer can provide meaningful feedback on the basis of qualitative and quantitative analyses complementing and supporting the designer's creativity.

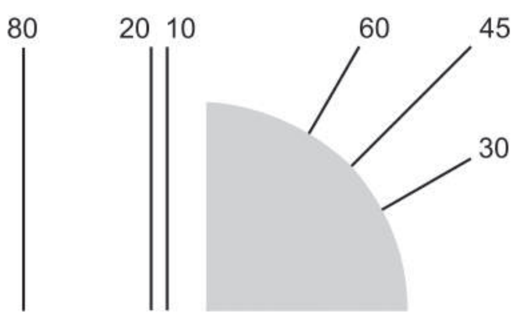
Frequent absence of meaningful explicit relationships between elements in architectural representations does not imply lack of knowledge on the subject. Architectural and building textbooks deal extensively with relationships between building elements and components. The positioning of one element relative to another derives from formal, functional and constructional decisions and has consequences for the articulation and performance of the building. Textbooks provide guidelines ranging from ergonomically sound distances between chairs and tables to correct detailing of joints in roof trusses. Frequent and faithful use of textbook examples has resulted in a corpus of architectural stereotypes. Even though stereotypes may lead the designer to repeating known solutions, they help reach levels of reasonable performance in designing and in the built environment. By obeying underlying rules and reproducing textbook stereotypes, the designer ensures conformity with norms of building regulations, professional codes and general empirical conclusions.

In textbooks, aspects of a recommended configuration are usually presented separately in a proscriptive manner, by means of sub-optimal and unacceptable examples. These are annotated with the relevant relationships and usually ordered from general to specific and from simple to complex. It is assumed that the reader of the textbook makes a selective mental aggregate on based on the aspects that apply to the problem at hand. Despite that, incompatibilities between different aspects and examples are seldom addressed in textbooks. Forming an aggregate representation is generally a straightforward hill-climbing process. For example, in designing a door, one starts with basic decisions relating to the door type on the basis of spatial constraints and performance criteria. Depending upon the precise type, the designer proceeds with constraints derived from adjacent elements and activities. In the case of a single inward opening left hinged door of standard width (figure 247), these constraints determine position and functional properties of the door, i.e. the distance from elements behind the door, and the swing angle, orientation and direction enabling the projected entrance and exit requirements. These can be adjusted by other factors unrelated to the initial decision. For example, the existence of a load-bearing element in the initial place of the door may necessitate translation of the door and hence a re-formulation of the initial design problem.



247 Textbook representation of local co-ordination constraints

Similarly to textbooks, templates offer useful insights into stereotypical interpretation of local co-ordination constraints. In templates, building elements usually appear as holes or slits. Each one is accompanied by annotations in the form of dents, notches and painted text. These facilitate geometrical positioning of a form, as well as geometric interpretation of spatial constraints. The configuration of forms and annotations typically represents a simplified fusion of parameters reduced to typical cases (figure 248). Even though superimposition of different patterns makes the template less legible than the more analytical textbooks, the template comes closer to the mental aggregate of the designer.



248 Template representation of local co-ordination constraints

The manner in which local constraints are centred on elements, the connections between elements and their stereotypical treatment in designing suggest that mechanisms like frames or objects would be appropriate for representation of local co-ordination devices. In a frame-based representation the relationships of e.g. a door with walls and other elements of the immediate context can be described as slots and facets linking the door frame with frames of walls, spaces and other elements. Such an implementation strategy has obvious advantages for the representation of local co-ordination devices, for example with respect to inter-changeability of elements by means of abstraction and inheritance. It is quite plausible that a single prototype would suffice for representation of all kinds of doors. This could facilitate manipulation of doors in computer-aided design, including automated substitution of one door type by another, if needed; due to spatial conflicts or a change in the designer's preference.

25.8 GLOBAL CO-ORDINATING DEVICES

Global co-ordinating devices generally appear in two forms. The first: sketches and diagrams explaining the general spatial articulation of a design. Such abstract representations —even if devised post factum— are commonly seen as embodiment of the driving forces in the development of the design. For our purposes they form a useful précis which can be placed at the top of a multi-level representation. The second form: the product of formal analysis. Usually applicable to more than one design, it is expressed in more abstract terms: grids, zoning schemes. Probably the most celebrated of such devices is the 5 x 3 grid proposed by Wittkower as the underlying grid of Palladian villas.[a]

This grid is universally accepted as the canonical formal expression of the intuitive perception of the Palladian villa's "triadic composition" of two symmetrical sequences of spaces laterally flanking the central series of spaces along the main axis.[b] As a result, the 5 x 3 grid forms the basis of most Palladian studies, including the Palladian shape grammar.[c] In it, the first stage invariably concludes with the definition of the 5 x 3 grid which serves further as a template for definition of spaces and positioning of building elements.

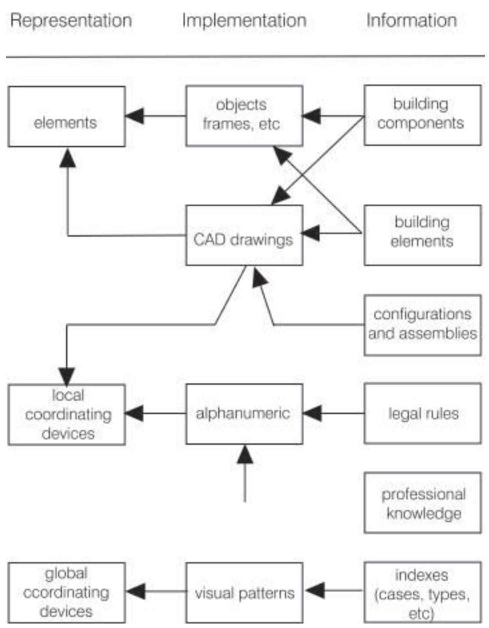
Global co-ordinating devices can be derived by visual abstraction eliminating the individual characteristics of elements and returning a skeleton, as in figure 245. This does not imply that these abstractions are accidental products of various, possibly unrelated design decisions. Another option is to treat devices like the Palladian 5 x 3 grid as prototypical patterns systematically repeated in variations. Such a view underlies most computational studies, even though there is no historical evidence that Palladio set out to exhaust the possibilities presented by a single pattern. The 5 x 3 grid appears to be an fusion of different preoccupations and influences, from notions of harmony to the traditional centralised arrangement of the local house type.[d]

25.9 MULTI-LEVEL DESIGN REPRESENTATIONS AND INFORMATION NETWORKS

Integration of all entities in holistic associative structures applies to design problems of limited size and complexity. As a problem expands to more elements, aspects and abstraction levels, atomistic associative representations grow beyond what is manageable for computation by computers and for direct comprehension by humans, even if compact implementation mechanisms like frames and objects are employed. In multi-level representations networks of architectural elements are complemented by local and global co-ordinating devices at different levels of abstraction. These devices integrate relationships in consistent and coherent local or global frameworks regulating positioning and properties of elements. Multi-level representations build on the natural abstraction of architectural representations; evident in the conventional sequence of drawings at different scales: 1:200, 1:100, 1:50, 1:10.

One main advantage of multi-level representations comprising elements, local and global co-ordinating devices is connectivity to external information sources. The increasing availability of design information on networks like the Internet makes connectivity a pre-requisite to integration of such networks in designing. The ability to instigate searches by means of intelligent, autonomous agents that collect appropriate information, to integrate this informa-

a Wittkower, R. (1952) *Architectural principles in the age of humanism*.
b Ackerman, J.S. (1977) *Palladio*.
c Stiny, G. and W.J. Mitchell (1978) *The Palladian grammar*.
d Ackerman, J.S. (1977).



249 Multi-level design representations and information retrieval

tion in design representations and to maintain a dynamic link with the original source of the information are already available on a limited or experimental basis.

Integration of hypermedia possibilities in drafting and modelling systems and addition of vector information in hypermedia interfaces to the Internet currently focus on dissemination of information on building elements and components. These are distributed as CAD documents to be downloaded from an Internet site and subsequently integrated in a design. Dynamic linking of a local document to the representation of a component or element on a remote system is also feasible.

With respect to integration of on-line information on elements a multi-level representation is similar to any analytical representation. The advantages of multiple co-ordinated levels on top of the networks of elements emerge when we consider integration of other kinds and forms of information. One such kind already being distributed through the Internet, but frequently escaping attention, are relationships and constraints that constitute local co-ordinating devices. This information is normally included in texts describing or analysing legal codes and regulations, as well as professional knowledge of the kind encountered in textbooks. Identification and extraction of relevant items from these documents is conceptually non-trivial but technically straightforward, given the hypermedia structure of current Internet interfaces.

These items can be linked to a design representation in the same way as elements, with the difference that elements are self-contained, while textual or mathematical information on rules and regulations constrains items and relationships between items. The explicit representation of local co-ordinating devices, either embedded in elements or as separate, superimposed entities, facilitates direct connection to external alpha-numeric values (figure 249). This permits precise control of input and constraint propagation in a design, for example for analysis and modification of specific aspects due to a change in the legal framework of the project. Co-ordinating devices are equally significant in guiding information retrieval. The constraints encapsulated in local co-ordinating devices often determine acceptability of an element to a particular situation.

As a result, network search routines can derive part of their parameters from the relevant local co-ordinating devices in the design, test the retrieved elements against the requirements in these devices and receive feedback on relevance of the search. Global co-ordinating devices can also be employed this way, especially for assemblies and sub-systems of elements. Such parts of a design are becoming increasingly available as examples of the application of principles, systems or elements.

The proliferation of ideas on case and precedent-based design could also increase the number of on-line configurations of elements. Their manipulation for retrieval and integration in new design can only be achieved by analytical means matching the complexity of the configurations. However, we may expect that most information on cases, prototypes and precedents will be at a high level of abstraction. This suggests that global co-ordinating devices can be used for indexing designs in a database and, hence, as query terms for retrieval of whole designs. The utility of current indexing schemes (usually on basis of a controlled vocabulary) demonstrates the advantages of such search intermediaries. Local, and especially global co-ordinating devices can enrich indexing with visual schemata which can be directly matched to the searcher's own graphic input.[a]

25.10 ANALYSIS AND REPRESENTATION

Well-defined design representations are a pre-requisite to analysis and evaluation of building behaviour and performance. With the increasing complexity of the built environment and rising requirements of environmental quality, analysis and evaluation of programmatic and functional aspects are becoming one of the highest priorities in architecture. Unfortunately, architectural analysis (and design) has been driven by normative models belonging to either of the following deontic approaches:

a Gross, M.D. (1995) *Indexing visual databases of design with diagrams*.

- *Proscriptive*: formal or functional rules that determine the acceptability of a design on the basis of non-violation of certain constraints. Formal architectural systems like classicism and modernism, as well as most building regulations are proscriptive systems.
- *Prescriptive*: systems that suggest that a pre-defined sequence of actions has to be followed in order to achieve acceptable results. Many computational design approaches are prescriptive in nature.

Dominance of a specific system or approach in a historical period has been instrumental for the evolution of architecture. It allowed in concentration of effort on concrete, usually partial problems within the framework of the system and hence supported innovation and improvement.

The eclectic spirit of recent and current architecture reduces the value of normative approaches, as it permits strange conjunctions, far-fetched associations and unconstrained transition from one system to another. In addition, the computer provides means for analyses and evaluations of a detailed and objective nature. These dispense with the necessity of abstraction and summarisation in rules and norms. This does not mean that abstraction is unwanted or unwarranted. On the contrary, abstraction is an obvious cognitive necessity, that emerges as soon as a system has reached a stable state. Consequently, one can expect emergence of new abstractions on the basis of the new detailed, accurate and precise analyses. It is quite probable that several older norms will be among the new abstractions.

The main characteristic of new forms of analysis is that they follow an approach we may call *descriptive*. They evaluate a design indirectly, by generating a description of a particular aspect comprising detailed measurable information on the projected behaviour and performance of the design. This description is normally closely correlated with formal representation of the design and therefore permits interactive manipulation, e.g. for trying different alternatives and variations. In short, the descriptive approach complements (rather than guides) human design creativity by means of feedback from which the designer can extract and fine-tune constraints.

In functional analyses it has become clear that most current norms and underlying principles have a very limited scope: control of minimal specifications by a lay authority. They are often obsolete as true performance measures and grossly insufficient as design guidance. The solution presented by the descriptive approach is substitution of obsolete abstractions by detailed information on functionality and performance, for example abandonment of Blondel's formula of stair sizes in favour of an ergonomic analysis of stair ascent and descent by means of simulation.[a] The analysis is performed in a multi-level system connecting normative levels to computational projections and to realistic simulations in a coherent structure, where assumptions of one level are subject of investigation at another level.[b]

25.11 AESTHETICS

The intuitive appreciation of aesthetic preference has been a hallmark of architectural design in practice. It has also been one main reason for conflict between architect and lay person, as the latter's appreciation of built form and space is less tempered by dominant architectural doctrines and more by the élite dictating good taste and 'vogue'. As vogue is often at odds with architectural history and criticism, architects have been reluctant to change what they consider to be part of their methodical background. The predominance of the intuitive approach agrees with many types of human inter-action with the built environment and its representations. This agreement adds an element of common sense to architectural analysis that may temper indifference to practical problems. However, common sense can be distorted or refuted by expert opinion and interpretation, especially if specific human experiences do not involve directly measurable performance criteria. Such distortions and refutations have contributed to the deep dichotomy between form and function in architecture and to the frequent

a Mitossi, V. and A. Koutamanis (1996) *Parametric design of stairs*.

b Koutamanis, A. (1995) *Multilevel analysis of fire escape routes in a virtual environment;* Koutamanis, A. (1996) *Elements and coordinating devices in architecture: An initial formulation*.

elevation of formal considerations to the highest priority in architectural design, either as a priori norms and canons or as direct and inescapable consequences of functional issues and problems.

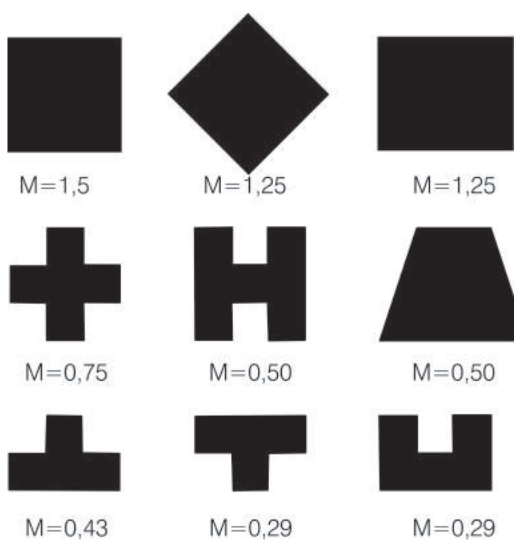
In the descriptive approach the principles of architectural aesthetics are drawn from perceptual and cognitive sources; and these principles are connected to architectural issues, strictly in this order. In other words, rather than starting with ordering, the existing architectural aesthetic norms and then proceeding to a search for cognitive relevance and justification, we apply general computational models of perception and cognition to architecture. This leads to an analysis that does not derive from a normative architectural model or system. Therefore, it does not exhibit any bias towards specific approaches but potentially accommodates all possible architectural systems. Different systems correspond to variations in the analysis with respect to the configuration of analytical devices, as well as to (parametric) differences within each device. The common basis of the different systems and of the corresponding analyses is an objective representation of the architectural object, i.e. a description not relating to a specific architectural formal system.

The distinction between the derivation of a description, its interpretation and finally its evaluation, is common to computational studies of vision but also of aesthetics.[a] Its particular value lies in that it stresses affinity between figural goodness in perception and aesthetic appreciation of built form. Figural goodness has been linked to aesthetic response by means of the relation between perceptual arousal and complexity.[b]

25.12 AESTHETIC MEASURES

The first significant attempt to quantity aesthetics was by the American mathematician George D. Birkhoff who, following, among others, Leibniz and Pythagorans, proposed that the aesthetic experience relies on principles of harmony, symmetry and proportion. Three successive phases:[c]

- arousal and effort of attention;
- the feeling of value or aesthetic measure which rewards the effort of attention; and finally
- the realisation that the perceived object is characterised by a certain aesthetic order.



250 The aesthetic measure of isolated polygonal forms according to Birkhoff [e]

Birkhoff states that the effort of attention is proportional to the complexity (*C*) of the perceived object and links complexity, the aesthetic measure (*M*) and aesthetic order (*O*) in the basic aesthetic formula:

$$M = O / C$$

Complexity is generally measured by the number of elements in the perceived object. For example, in isolated polygonal forms complexity is measured by the number of distinct straight lines containing at least one side of the polygon, similarly to the gratings of rectangular dissections.[d] The measurement of order varies with the specific class of objects to be evaluated but generally takes the form of the sum of weighted contributing elements:

$$O = ul + vm + wn + \dots$$

where *l, m, n, …* are the independent elements of order and *u, v, w, …* indices which may be positive, zero or negative, depending upon the effect of the corresponding element. Aesthetic order and consequently aesthetic measure are relative values which apply to specific classes of objects, so restricted that intuitive comparisons of the different objects becomes possible. There is no comparison between objects of different types.

Birkhoff suggests that order relates to associations with prior experience and acquired knowledge triggered by formal elements of order, that is: properties of the perceived object, like bilateral symmetry about a vertical axis or plane. Formal elements of order with a positive effect include repetition, similarity, contrast, equality, symmetry and balance. Ambiguity, undue repetition and unnecessary imperfection have a negative effect. For example, a rectangle

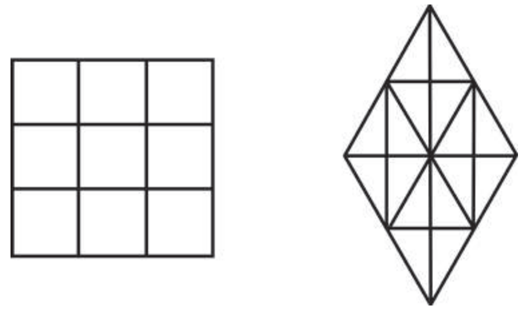
a Stiny, G. and J Gips (1978) *Algorithmic aesthetics. Computer models for criticism and design in the arts*.
b Berlyne, D.E. (1960) *Conflict, Arousal and Curiosity*; - (1971) *Aesthetics and psychobiology*.
c Birkhoff, G.D. (1933) *Aesthetic measure*.
d Steadman, J.P. (1983) *Architectural morphology*.
e Birkhoff, G.D. (1933).

not quite a square is unpleasantly ambiguous, according to Birkhoff. Also a square whose sides are aligned with the horizontal and vertical is superior to an unnecessarily imperfect square which has been rotated about its centre by 45 degrees "because it would be *so easy* to alter it (the rotated square) for the better" (p. 25).

In the example of isolated polygonal forms aesthetic order is measured by the formula

$$O = V + E + R + HV - F$$

where *V* stands for vertical symmetry, *E* for equilibrium, *R* for rotational symmetry, *HV* for the relation to a horizontal-vertical network (reference framework) and *F* for unsatisfactory form. "Unsatisfactory form" encompasses too small distances between vertices or parallel sides, angles too near to 0 or 180 degrees and other ambiguities, diversity of directions and lack of symmetry.



251 Examples of horizontal-vertical networks according to Birkhoff[a]

Ingrained aesthetic prejudices reduce applicability and reliability of Birkhoff's aesthetic measure. The highest values are achieved with symmetrical forms with the least number of parts. The square with sides aligned to the vertical and horizontal is the clear winner among polygonal forms, followed by the square rotated by 45 degrees and the rectangle with horizontal and vertical sides. Still, the aesthetic measure is important to our investigation for three basic reasons relating rather to the way the measure is calculated than the measure itself.

The first is that it equates beauty with order. While this does not hold for aesthetics in general, it is obviously relevant to prescriptive and proscriptive architectural formal systems where conformity to canons and rules, often explicitly and paradigmatically expressed, constitutes the usual measure of formal acceptability.

The second reason is factoring aesthetic order into discrete, independent formal elements each with a limited scope. The third reason is the rôles of order and complexity in the aesthetic measure and their affinity with information processing and the rôle of figural goodness in perception. This affinity was not lost on Birkhoff's epigones who linked aesthetic measures to information theory.[b] The applicability of Birkhoff's approach to architectural aesthetics is consequently restricted to:

- analysis of factors contributing to aesthetic appreciation and preference and
- evaluation of an object with respect to each of these factors.

25.13 CODING AND INFORMATION

Probably the greatest shortcoming of Birkhoff's approach is that it fails to take account of perception, that is, of processes by which an object elicits a pleasurable reaction. By linking aesthetics to perception we depart from the objectiveness of Birkhoff's measure and adopt an inter-subjective model of aesthetic appreciation stressing the cognitive similarities that exist between different persons and cultures.[c] Inter-subjectivity also allows to correlate different aesthetic approaches, i.e. different architectural formal systems.

Gestalt psychologists have formulated a number of principles (or 'laws'), like proximity, equality, closure and continuation, which underlie the derivation of a description from a percept by determining the grouping of its parts.[d] Probably the most important, certainly the most mysterious of the Gestalt principles of perceptual organisation is *Prägnanz* or *figural goodness* which refers to subjective feelings of simplicity, regularity, stability, balance, order, harmony and homogeneity arising when a figure is perceived. Figural goodness ultimately determines the best possible organisation of image parts under the prevailing conditions. As a result, it is normally equated to preference for the simplest structure. The view of perception as information processing has led to attempts to formulate figural goodness more precisely. Given capacity limitations of the perceptual system and the consequent necessity of minimisation, it has been assumed that the less information a figure contains (i.e. the more redun-

a Birkhoff, G.D. (1933) *Aesthetic measure*.
b Bense, M. (1954) *Aesthetica;* Moles, A. (1968) *Information theory and esthetic perception*.
c Scha, R. and R. Bod (1993) *Computationele esthetica*.
d Köhler, W. (1929) *Gestalt psychology;* Koffka, K. (1935) *Principles of Gestalt psychology;* Wertheimer, M. (1938) *Laws of organisation in perceptual forms*.

dant it is), the more efficiently it could be processed by the perceptual system and stored in memory.[a]

Arguably the most powerful model in this line of investigation was Leeuwenberg's coding or *structural information theory*.[b] According to Leeuwenberg a pattern is described in terms of an alphabet of atomic primitive types, like straight-line segments and angles at which the segments meet. This description (the *primitive code*) carries an amount of structural information (I) equal to the number of elements (i.e., instances of the primitives) it contains. The structural information of the primitive code is subsequently minimised by repeatedly and progressively transforming the primitive code on the basis of a limited number of coding operations:

- iteration, by which the patterns

  a a a a a a b b b b b b (I = 12) / *a b a b a b a b a b a b* (I = 12) become respectively 6 * [(*a*) (*b*)] (I = 3) / 6 * [(*a b*)] (I = 3)

- reversal, denoted by *r* […]:

  a b c = r [*c b a*] (I= 3) reversal allows the description of symmetrical patterns (Σ):

  a b c c b a = *a b c* r [*a b c*] = Σ [*a b c*] (I = 4)

  a b c b a = *a b c* r [*a b*] = Σ [*a b* (*c*)] (I = 4)

- distribution:

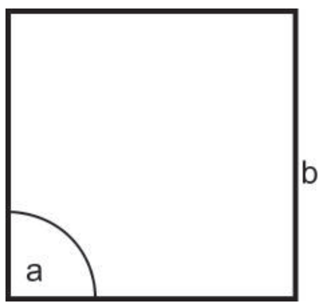
  a b a c = <(*a*)> <(*b*) (*c*)> (I = 3)

- continuation (⊂…⊃), which halts if another element or an already encoded element is encountered:

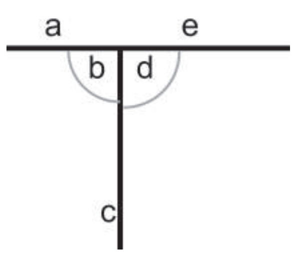
  a a a a a a a … *a* = ⊂*a*⊃ (I = 1)

The coding process returns the *end code*, a code whose structural information cannot be further reduced. The structural information (I) of a pattern is that of its end code.

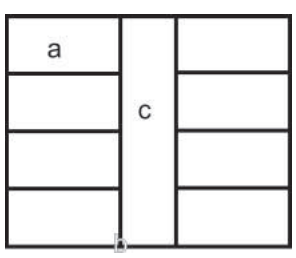
The structural information of a pattern is a powerful measure of its figural goodness. By equating a figure's goodness with the parametric complexity of the code required to generate it we can both derive the different descriptions an image affords and choose the one(s) that contain the least information.



252 Coding of square: abababab = ⊂a b⊃ (I = 2) (repeat a and b until reaching the starting point again, structural information is 2)



253 Coding of branching with bifurcation signs: a {b c} d e (after c return to end of a and proceed to following d)



254 Coding of a floor plan:
aaaabcbaaaa = 4 * [(a)] b c b 4 * [(a)]
= Σ [4 * [(a)] b (c)]
(I = 5)
(mirror 4 times a connected to c by b, structural information is 5)

25.14 ARCHITECTURAL PRIMITIVES

The main problem of theories of perceptual organisation, from Gestalt to structural information theory, is that they are developed and discussed within abstract domains of simple, mostly two-dimensional patterns and elementary primitives like dots and line segments. Such basic geometric forms should be treated with caution in evaluations of design aspects, as they refer to implementation mechanisms rather than to symbols of spatial representation. An extension to the three-dimensional forms of the built environment and to complex two-dimensional representations employed in architecture involves the problem of determining the primitives of these domains. Use of spaces and building elements as primitives demonstrates clearly the potential of structural information theory. In figure 254 coding of a floor plan on the basis of spaces yields a succinct and accurate description of spatial articulation.

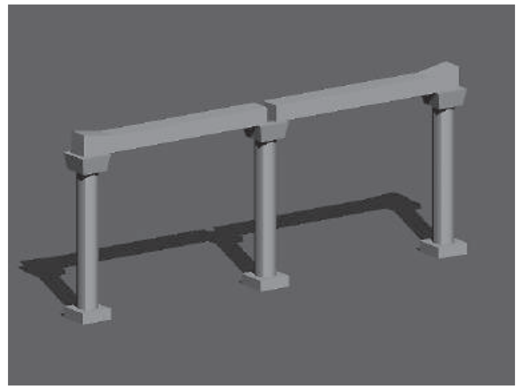
The end code is a symmetric tripartite configuration of two space groups flanking a central space.

Attempts to discover or define the primitives of architectural perception are impeded by confusion between the real built environment, its architectural representations and conventions underlying these representations. For this reason, we should make a sharp distinction between analysis and manipulation of representations and perception of and inter-action with the built

a Attneave, F. (1954) *Some informational aspects of visual perception;* Hochberg, J.E. and E. McAlister (1954) *A quantitative approach to figural 'goodness'*.

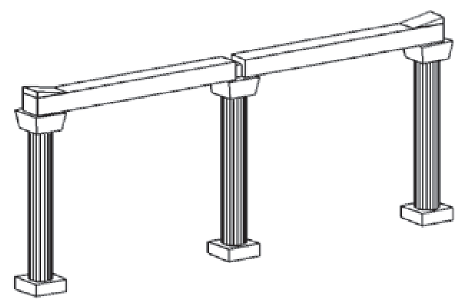
b Leeuwenberg, E.L.J. (1967) *Structural information of visual patterns. An efficient coding system in perception.*; – (1971) *A perceptual coding language for visual and auditory patterns*.

environment. The former rely firmly on architectural conventions and should be accordingly considered from the viewpoint of architectural knowledge. The adoption of building elements and spaces as the primitives of such representations offers pragmatic advantages that should not be neglected. On the other hand, a preferable starting point for the perception of built form and space are general computational models of perception and recognition, possibly enriched with constraints of architectural representation. These models provide a better understanding of perceptual and cognitive devices that also underlie architectural design and analysis. In addition to their direct applicability to the analysis and recognition of realistic architectural scenes they could ultimately also lead to improvements in existing architectural representations.



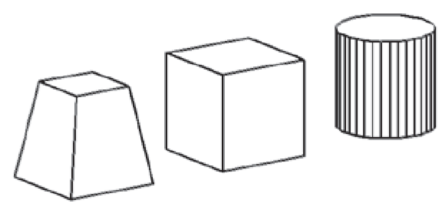
255 An architectural scene

Following low level processing, the first stage in recognition of a scene is invariably decomposition of its elements into simple parts, like the head, body, legs and tail of an animal. The manner of the decomposition into parts does not depend on completeness and familiarity. An unfamiliar, partly obscured animal or even a nonsensical shape are decomposed in a more or less the same way by all observers.[a] Detection of where parts begin and end is based on the *transversality principle* which states that whenever two shapes are combined their join is almost always marked by matched concavities.[b] Consequently segmentation of a form into parts usually occurs at regions of matched concavities, i.e. discontinuities at minima of negative curvature. The results of the segmentation are normally convex or singly concave forms.



256 A decomposition of figure 254 into geons

At first sight one might expect that there is an unlimited number of part types. However, with his *recognition-by-components* theory Biedermann proposed that these forms constitute a small basic repertory of general applicability, characterised by invariance to viewpoint and high resistance to noise. He calls the forms geons and suggests that they are only 24 in number.[c] Geons can be represented by generalised cones, i.e. volumes swept out by a variable cross section moving along an axis.[d] A scene is described by structured explicit representations comprising geons, their attributes and relations derived from only five edge properties.



257 The geons in figure 256

A combination of structural information theory and recognition-by-components provides a comprehensive basis for evaluation of figural goodness in architectural scenes. Coding geons according to structural information theory permits, moreover, grouping of a higher order than local binary relationships. This allows the development of multi-level representations which are less complex, better structured and ultimately more meaningful than atomistic relational representations.[e] In addition, the combination of the two theories makes it possible to establish general preference criteria for alternative descriptions on the basis of code compactness, which in turn relies on formal grouping principles.

The application of this combination to architectural scenes concentrates in first instance on definition of primitives and relationships. In that respect, the only deviation from the original theories concerns the relationship ignored in coding. In structural information theory this is horizontal alignment. In architectural scenes this changes to vertical alignment, in compliance with the general architectural bias for the vertical as canonical orientation. We presume that this bias refers both to a general reference frame reflecting the significance of the vertical in the real world (e.g. gravity) and to a specifically architectural reference frame which relates to the interpretation of general orientation preferences in architecture.

On this basis, the scene of figure 255, 256 and 258 can be coded as follows:

a b {c d e} f g {c d e} f g b {c d e} a (*I* = 17)

<({c d e})><(a b) (f g) (f g b) (a)> (*I* = 11)

The use of distribution in the second version of the code makes the grouping of the elements comprising the column explicit, as well as the repetition of the group in the scene. This reflects the translational symmetry of the scene (colonnade). The bi-lateral symmetry that char-

a Biederman, I. (1987) *Recognition by components: a theory of human image understanding*.

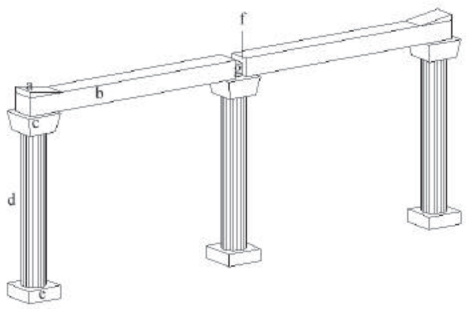
b Hoffman, D.D. and W. Richards (1985) *Parts of recognition*.

c Biederman, I. (1987); – (1995) *Visual object recognition*.

d Binford, T.O. (1971) *Visual perception by computer;* Brooks, R.A. (1981) *Symbolic reasoning among 3-D models and 2-D images*.

e Koutamanis, A. (1996) *Elements and coordinating devices in architecture: An initial formulation;* – (1997) *Multilevel representation of architectural designs*.

acterises the total scene is largely lost because of the integrity of the elements and groups in the scene. Bi-lateral symmetry would be discovered in the code, if line segments were used as primitives. This would have meant encoding the outline of the elements rather than the elements themselves and would permit splitting of a column into two symmetrical halves with respect to the vertical axis. However, the advantage of discovering and describing explicitly this accidental bi-lateral symmetry in a repetitive configuration like a colonnade does not counter-balance the corresponding multiplication of structural information in the primitive code and the initial detachment from the reality of the perceived integral components / geons.



258 Coding of figure 256

25.15 THE EVALUATION OF ARCHITECTURAL FORMAL GOODNESS

Recognition-by-components and structural information theory provide the basis for:

- recognising and representing the solid elements of an architectural scene;
- grouping the recognised elements in multiple alternative configurations;
- evaluating alternative configurations with respect to coding efficiency; and
- establishing preference for one or two dominant configurations which represent the intuitively acceptable or plausible interpretations of the scene.

These operations link the representation of the built environment to perception and figural goodness. The necessary deviations from established conventional architectural representations reflect the choice of general cognitive and perceptual theories as starting point of the investigation. It is proposed that architectural representations and in particular:

a) the use of outlines to denote solid entities and spaces and
b) the deterministic decomposition into known components

should be reconsidered with respect to the recognition-by-components theory and related vision research.

The addition of a memory component to structural information theory would facilitate transition from the basic level of the primitive and end codes to known configurations denoting familiar objects.

The representation of spaces remains a problem deserving particular attention and further research. The use of outlines, as in figure 254, is the obvious starting point, as it conforms to the way we read floor plans and other conventional representations; and to existing computational representations like rectangular arrangements and shape grammars. This would allow exploration of structural information theory and recognition-by-components in the application areas of these representations. From a cognitive point of view, however, the outline of a space in two or three dimensions might not be a relevant or meaningful representation. It has been proposed that surfaces could form a representation level that not only links higher with lower vision,[a] but also agrees with the Gibsonian perception of space in terms of surfaces which fill space.[b]

This view differs entirely from the mainstream Euclidean co-ordinated organisation of perceived space whereby the two dimensional retinal image is enriched with depth information derived primarily from binocular disparity. Perception of space in terms of surfaces stresses the biological and ecological relevance of these surfaces as containers of different actions and as subjects of their planning. One example of this relevance is locomotion for the ground surface and related generally horizontal surfaces.

Another issue requiring further study concerns the essentially bottom-up character of both recognition-by-components and structural information theory. The addition of a memory component to the system, i.e. a database of geon configurations corresponding to known, familiar entities, would facilitate processing of information at basic levels and permit rapid

a Nakayama, K., Z. He et al. (1995) *Visual surface representation: a critical link between lower-level and higher-level vision*.
b Gibson, J.J. (1966) *The senses considered as perceptual systems*.

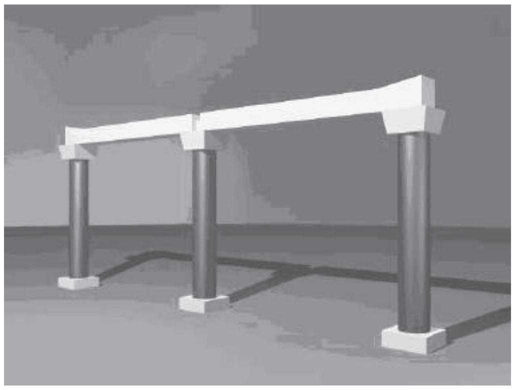
transition to higher levels of the representation. As these configurations would represent compact structures with respect to structural information, we assume that exposure to and recognition of similar or equivalent scenes leads to transformation of earlier experience into memories influencing our understanding and aesthetic evaluation.[a]

25.16 PROJECTING APPEARANCES

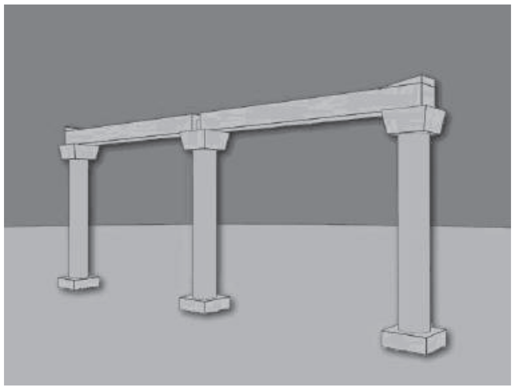
The difference between pictorial and other descriptions (e.g. textual) is commonly explained by resemblance. A picture represents a subject by the intended resemblance of its pictorial properties to the visual perception of its subject. Some interpretations of resemblance may lead to limited views, like assimilation of the experience of seeing a picture to the real life experience of seeing the picture's subject, which moreover are unrelated to the symbolic structure of a picture's content.[b] Nevertheless, resemblance remains an appropriate vehicle for investigating perceptual and cognitive issues in visual representation.

Architectural visualisation has been rather ambivalent with respect to the resemblance issue. On one hand, most basic design representations combine orthographic projection of canonical views with conventional symbolisation. On the other, axonometrics, isometrics, perspectives and especially the rendered ones consciously attempt to project or reconstruct a veridical visual experience. This ambivalence stresses correspondence of composition and projection in architecture to Euclidean and projective geometry. In both architecture and geometry a historical shift can be detected from measurement and accurate representation of a picture's subject to the picture itself.[c]

Proliferation of affordable computer tools for photo-realistic visualisation is placing even more emphasis on the architectural picture. The connectivity of these tools to the standard CAD documentation of design practice means that computer-rendered photo-realistic perspectives are often used instead of simpler images which would convey the same information, especially when the photo-realistic version includes too many assumptions concerning colour and material. It is ironic that some of the more interesting additions to computer visualisation include references to simpler rendering techniques from the past. For example, figure 260 has been rendered with the Illustrator 2 plug-in for 3D Studio MAX. In their attempt to reproduce the quality of colouring and backgrounds in comic books, such techniques are an alternative to the standard, almost photo-realistic renderings (figure 259). The abstraction of comic book imagery is arguably better suited to most stages of the design process, as well as to human recognition of built form.



259 Image produced with the standard (scanline) 3D Studio MAX renderer



260 Image rendered with the Illustrator 2 plug-in for 3D Studio MAX

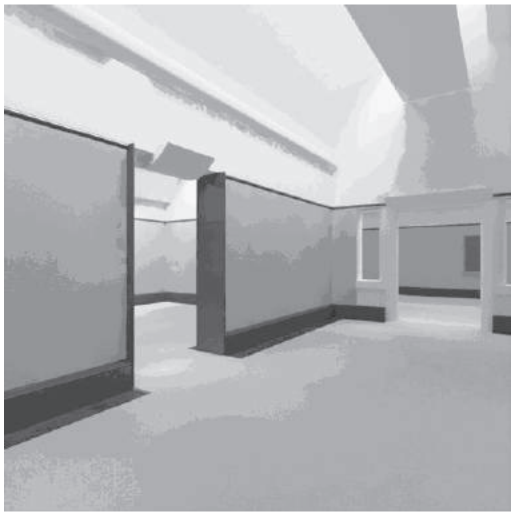
25.17 BEYOND INTUITION: SCIENTIFIC VISUALISATION

Design analysis was traditionally performed with normative rule-based systems geared to generative approaches. Numerous dissections of the design process have resulted in a multiplicity of models attempting to describe the steps a designer takes in the quest for a satisfactory solution. Most models also aspire to prescribe the optimal sequence of design actions. What they propagate is a form of *orthopraxy* (as opposed to the orthodoxy of formal systems such as Classicism and Modernism). Their underlying assumption is that if one follows the sequence of design stages prescribed in the model, one can arrive at a design *satisfying* the programmatic requirements.

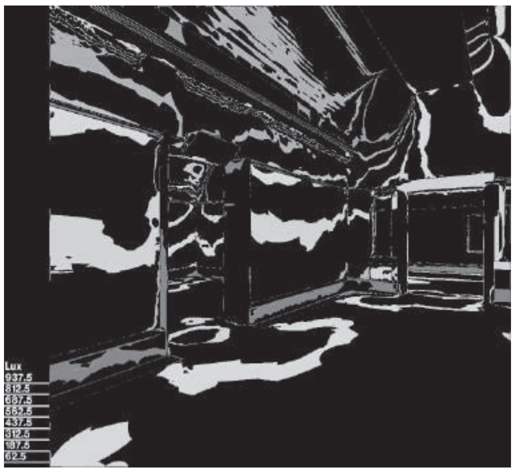
It is unfortunate that no such model to-date can match the intuitive performance and creativity of the human designer. Based on metaphors and similes, most models do little beyond explaining a few specific aspects of designing. Moreover, while they may improve the designer's awareness of actions and decisions, they seldom lead to the development of new, better tools for higher effectiveness and reliability in the face of today's complex design problems. Perhaps the main reason for the scarcity of such tools lies in a lack of interest in the analysis of design products.

Historically such an analysis was subservient to synthesis. Long before terms like functional and programmatic analysis were invented, buildings and design decisions were parsed

a Scha, R. and R. Bod (1993) *Computationele esthetica*.
b Goodman, N. (1976) *Languages of art: an approach to a theory of symbols;* Evans, G. and J. McDowell (1982) *The varieties of reference;* Lopes, D. (1996) *Understanding pictures*.
c Evans, R. (1995) *The projective cast, architecture and its three geometries*.



261 Photo-realistic light simulation (Radiance image by A.M.J. Post)



262 Light simulation: false colour intensity analysis in the space of figure 262 (by A.M.J. Post)

towards an identification of their causes and effects, and subsequently formalised into rules and stereotypical “good” solutions serving as the basis of most building regulations and design textbooks. Rules and stereotypes have a proscriptive function mostly. They attempt to offer design guidance by pointing out errors and inadequacies, i.e. what falls short of established norms.

Proscriptive approach also underlies computational studies focusing on the analysis of designs using the same or similar rules transformed into expert or knowledge-based systems. In these, a design is described piecemeal; which permits correlation of the relevant aspects or factors with the rules. The end product of the analysis is an acceptability test based on matching the constraints of the solution space. The added value of such systems lies in provision of feedback facilitating identification of possible failure causes.

Design analysis is moving towards a new paradigm, based rather on simulation than abstractions derived from legal or professional rules and norms. Recent developments in areas like scientific visualisation provide advanced computational tools for achieving rich detail and exactness, as well as feedback for design guidance. Close correlation of photo-realistic and analytical representations (figure 261 and 262) clarifies and demystifies the designer’s insights and intuitions. Moreover, the combination of intuitive and quantitative evaluation offers a platform of effective and reliable communication with other engineers who contribute to the design of specific aspects, as well as comprehensible presentation of projected building behaviour and performance.

25.18 DYNAMIC VISUALISATION

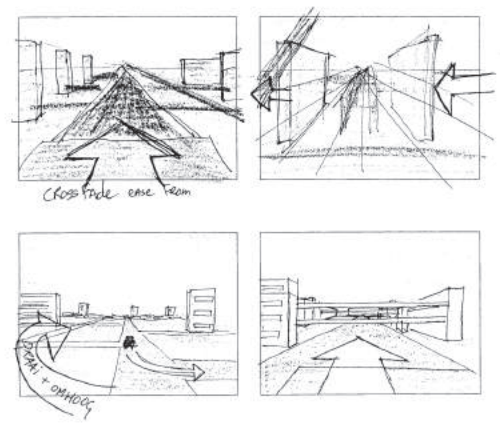
Dynamic visualisation is often presented as the pinnacle of architectural representation, the fullest form of visual realism. By including movement of one sort or another in a three dimensional representation the designer adds depth and time to the subject under controlled conditions, i.e. in the framework of a specific event or state. Since a dynamic description is a sequence of static, photo-realistic images, the results can be superior to other representations for visual inspection, analysis and communication.

As with photo-realism, a frequent argument for dynamic visualisation is the ease with which it can be produced out of three dimensional design representations. While this is true for simple, undemanding movements of the camera or in the scene, more complex subjects and presentation techniques require knowledge and skills beyond the scope of architecture. These are best found in filming. They range from camera positioning and movement to lighting and editing, mixing and visual effects. The technical aspects are largely integrated in the digital tools, but the architect must effectively step into the film director’s chair so as to coordinate, guide and manage the process.

Directing a dynamic description is a rôle that in principle fits the architect as specifier and co-ordinator of design and construction of a project and who does not necessarily have physical involvement in actual building. However, the fulfilment of the rôle necessitates substantial transfer of filming knowledge complementing the technical possibilities of digital dynamic visualisation. Ironically most of this knowledge refers to techniques for reproducing on film environments and events without actually having the camera there and then. Even when shooting on location artificial lighting and sets are used to enhance resemblance to the scene envisaged in the script. In the studio everything is not only artificial, but also opportunistically fragmented so as to minimise cost without loss of effectiveness and efficiency. The techniques involved in making a coherent and believable sequence of images from short takes of such fragments and illusions forms the core of the knowledge that has to be integrated in architectural visualisation. Several techniques have already been adopted in architectural design. Matting, for example, is widely used nowadays in making composite images from rendered perspectives of new designs and photographs of their prospective sites.

The main problem with filming techniques is that they run contrary to the holistic undercurrent of architectural design and CAD. The use of partial models for different aspects and abstraction levels does not agree with the idea of a single, complete and integral three-dimensional representation for the whole design. On the other hand, a multi-level modular representation is capable of accommodating the practicalities of dynamic architectural visualisation without sacrificing coherence and consistency of the representation.

Most filming techniques are born out of necessity. However, they are not restricted to compensating for practical limitations. They also offer the means for constraining and controlling a process. One such device is the storyboard, a series of annotated drawings, essentially similar to a comic strip (figure 263). The drawings depict the découpage of the film, i.e. its structure in terms of takes, camera positions and movements. The application of storyboarding in architectural visualisation on the basis of a modular co-ordinated representation adds a vertical co-ordinating device responsible for specific aspects arranged in a sequential way.



263 Storyboard extract (by I.R. van 't Hof)

26 THE EMPIRICAL CYCLE

HUGO PRIEMUS

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26.4 Systems analysis 250
26.5 Opening the shutters 252

26.1 SELF-CONTAINED APPROACH

For decades, a generally accepted research methodology existed in behavioural and technical sciences; taught for decades by faculties at institutions of scientific education. In all these educational programs, the letters M&T form a fixed component of the foundation course; methods and techniques of research are a part of every student's standard equipment, and certainly of every graduate.

Presently, the Faculty of Architecture at the TU Delft is looking for its own building methodology, its own design methodology. This does not happen with knowledge from, and reference to, the classical methodology of research at other faculties, nor does it happen together with faculties in other countries where people study architecture and learn design, nor together with other TUD sub-faculties in which building (Civil Engineering and Geosciences) and/or design (Industrial Design Engineering) play a central rôle, and not even together with the associated Faculty of Architecture at the TU Eindhoven. Is this wise? No. Is it effective? No. Are there good reasons for this self-contained, eccentric approach? No.

26.2 SCIENTIFIC FORUM

Let me take the methodology of the behavioural sciences for a starting point, as I learned it 35 years ago from Prof. Dr. A.D. de Groot, one of my supervisors. De Groot is author of the standard work, '*Methodologie. Grondslagen van onderzoek en denken in de gedragswetenschappen*' and was trained as a psychologist.[a] Many followed him, like Baarda & De Goede and Swanborn, each in his own way.[b] Some decidedly more modern authors also concur with the approach presented by De Groot, who strongly emphasises the rôle of the scientific forum. The nomological network of every science (discipline) is constantly in motion, thanks to new empirical data, new insights, new questions, new answers. Discussions within the forum, i.e. the international community of leading researchers (peers) in the field, constantly determine which insights and theories are considered 'true', or labelled untenable. In this process, international associations and/or networks of researchers in the respective domain play a crucial rôle, like international conferences and workshops, along with international journals.

This is a problem for architectural research. The CIB (Conseil International du Bâtiment) is not orientated towards design, the UIA (Union Internationale des Architectes) is not orientated towards practicing of academic science, and there are not many international scientific activities in the field of design. While there are indeed respected international scientific journals, like *Environment & Planning Ed. B (Planning and design)* and *Built Environment*, no designer from Delft has published in them since Olim's day.

Research methodology is first and foremost a habitus: an active willingness to write down insights, justify them, make them verifiable to others, make oneself vulnerable, seek out critics, and allow others to take a look behind the scenes. This is the function of debate in the scientific forum, epitomised in presentations and discussions during international conferences, and in articles and commentaries in academic journals. The faculty is familiar with this phenomenon, for example in the form of the successful Ph.D. conference of architectural schools, organised in Delft several years ago (with Herman van Wegen and Theo van der Voordt as driving forces), or the conferences launched by Arie Graafland. But, on the whole, design research gets unsatisfactory marks. The architectural intervention seems to have been a very local renovation until now.

a Groot, A.D. de (1961) *Methodologie: grondslagen van onderzoek en denken in de gedragswetenschappen*. English translation: (1969) *Methodology: foundations of interference and research in the behavioural sciences*.

b See e.g. Swanborn, P.G. (1991) *Basisboek Sociaal Onderzoek;* Baarda, D.B. and M.P.M. de Goede (2001) *Basisboek methoden en technieken*.

26.3 EMPIRICAL CYCLE

The empirical cycle can be used as basic scheme for a logical-methodological consideration of research, thinking, and reasoning in empirical science. The cycle according to De Groot:

- *Phase 1: Observation*: collecting and grouping empirical factual material; forming hypotheses;
- *Phase 2: Induction*: formulating hypotheses;
- *Phase 3: Deduction*: deriving particular consequences from the hypotheses, in the form of verifiable predictions;
- *Phase 4: Testing*: of the hypothesis, or hypotheses, based on the possible results of predictions in new empirical material;
- *Phase 5: Evaluation*: of the test results with regard to the proposed hypothesis/hypotheses and/or theory/theories, and to possible new, related research.

Phase 1 can be classified almost completely under the psychological induction process. It is assumed that a researcher rarely collects material without some "viewpoint". He chooses, selects, or abstracts from certain data or aspects, groups and registers according to certain criteria. Throughout, at least certain implied hypotheses have inevitably already been decided upon.

In Phase 2 these hypotheses are specified. A hypothesis only becomes a hypothesis if it has been formulated so that particular consequences and specifically concrete, verifiable predictions can be derived from it (operationalisation), then to be tested. This takes place in Phase 3: from a general hypothesis a concrete prediction is derived, one which is strictly verifiable.

Testing hypotheses (Phase 4) has to do with a general connection presumed to exist in, or apply to, a collection of elements considered to be non-identical. On this basis, the researcher makes his prediction regarding cases not yet researched. The results of this test are evaluated in Phase 5. What is the value of the test results? Do they support the hypothesis? Must the hypothesis be dismissed? And what happens to the theory to which the hypothesis is connected? Can it be maintained? Does the theory have to be adjusted? Or completely dismissed?

For Phases 3, 4, and 5, i.e. deduction, testing, and evaluation, there are many statistical techniques and means of calculating probability. These three phases seem to be far removed from the culture of architectural design. Design more closely resembles the processes that take place in Phases 1 and 2: the observation and the "devising" of hypotheses. But, it would be strange to conclude that design could be adequately described using De Groot's empirical cycle. And this was never De Groot's presumption. What is important, is that a designer making a design for a building ensures that his design (which can be compared to a hypothesis) be verifiable. This can, for example, take on the form of a Post Occupancy Evaluation: an evaluation of the building's use. Before an architect makes a design, it is advisable that he learns from prior experience. He needs to become acquainted with previously executed evaluation research, and to be able to interpret the results of this research well. When he has completed his design, he must be able to declare that the design can stand up to the test of experiences and evaluations from comparable buildings that have already been built (precedents).

Even if the designer wants to give maximal space to his creativity, he can be supported by research methods like systems analysis.

26.4 SYSTEMS ANALYSIS

Systems analysis is a craft developed in the United States (for instance by researchers from the Rand Corporation), that was gradually introduced to Europe. This approach is popular with the sub-faculty of Technische Bestuurskunde (Systems Engineering, Policy Analysis and Management). The standard work is the *'Handbook of systems analysis. Overview of uses,* *procedures, applications, and practice'* edited by Hugh J. Miser and Edward S. Quade. What follows has been extracted from Chapter 4, *'The Methodology of Systems Analysis: An Introduction and Overview'* by W. Findeisen and E.S. Quade. These authors make use of the diagram alomgside (see figure 264).

Distinctions are made between the following components:

- Formulating the problem;
- Identifying, designing, and screening possible (solution) alternatives;
- Pre-calculating future contexts of "states of the world";
- Constructing and using models for predicting results;
- Comparing and classifying the (solution) alternatives.

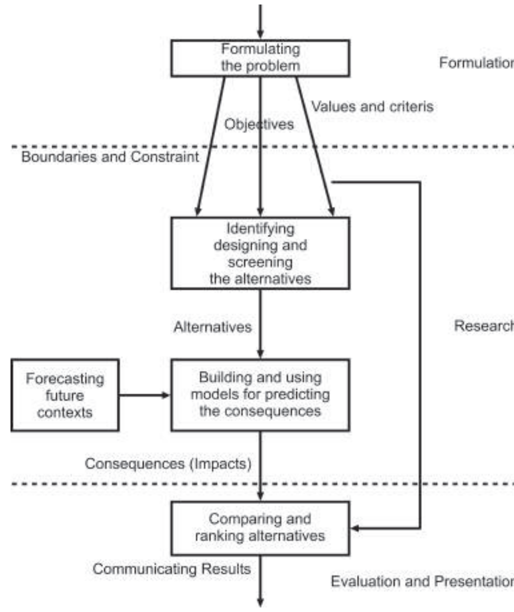
Systems analysis is specifically orientated towards the future. The procedure begins with the formulation of a problem. Without a problem, there is no need to think up solutions. The goals are specified, along with the values and criteria, as well as the borders and limits. One can only talk of a problem when a goal has been introduced, along with the obstacles related to reaching this goal. For a designer, this can be a programme of requirements for a building, as well as budgetary pre-conditions. The problem is that what is desired is a building that has not yet been built, one for which the design must first be made.

Step 2 involves identifying, designing, and screening alternative solutions. Here, designing as a solution-orientated strategy is the primary concern. What is interesting is that Findeisen & Quade fail to mention a single word about any specific solution, but instead discuss alternatives. In general, there are many roads to Rome, and only later will it become apparent which road best meets the requirements. In this second phase, there is ample space for fantasy and creativity. As long as an alternative can be verified according to the pre-determined requirements, this is the criterion that determines whether or not an alternative "complies" with this stage.

In Step 3, we take a look into the future and investigate how the world will look in 10 or 20 years, or even further along. What demographic and economic changes are to be expected? In the Netherlands, we can fall back upon the body of work of the *Centraal Bureau voor de Statistiek* (Statistics Netherlands) (population prognoses), the *Centraal Planbureau* (Netherlands Bureau for Economic Policy Analysis) (economic prognoses), and the *Rijksinstituut voor Volksgezondheid en Milieu* (National Institute of Public Health and the Environment) (environmental prognoses). We do not need to choose between these three calculations. Sometimes it is more useful to construct a sensitivity analysis: how adequate is a certain (solution) alternative under various presumptions about the future?

The results, per alternative, are pre-calculated in Step 4, using models that are constructed and then applied. This is an art that hardly anyone within the Faculty of Architecture possesses, and thus professional help would need to be called in here. During Phase 4, we investigate how each alternative would actually turn out concretely, under various presumptions. A given solution might, for example, achieve good results under economically favourable conditions, but may fall short when interest rates increase or if economic growth stagnates.

With the help of the criteria specified in Step 1, the alternatives are compared and classified during Step 5. This can take place based on various presumptions. Ultimately a choice must be made. This means dealing with uncertainty, since no one knows precisely what the future will bring. The policy of the decision-makers plays a major rôle here. Are they trying to reduce risks? Or aiming for extraordinary results? What priorities are they setting with regard to how the building will be used?



264 System analysis: procedure (according to Findeisen & Quade).[a]

a Findeisen, W. and E.S. Quade (1985) *The methology of systems analysis: an introduction and overview,* p.123.

Systems analysis is an extremely suitable tool for helping designers. It forces the designer to consider criteria, values, and goals, that have been specified in advance necessarily. It introduces the desirability of thinking in alternatives, of scanning the future. Alternatives are evaluated *ex ante*. The balancing of various alternatives becomes discussable, and in part even quantifiable. Discussion between various designers, each of whom believes in his or her own design principles, will be removed from the realm of nagging and mutual condemnation. This allows both long-term and short-term discussions of uncertainties, and supports policy considerations of the final decision-maker. In short: an ideal tool for the architectural engineer.

26.5 OPENING THE SHUTTERS

If the Faculty of Architecture takes the search for a methodological foundation seriously, it should continue to build on long-standing, carefully developed, generally applied research methods and techniques. This is the language spoken in scientific education and research, the language of the NWO *(Netherlands Organisation for Scientific Research)* and the STW (Technology Foundation), as well as the one of the international scientific forum. This basic methodology must be offered in the foundation course, so that architectural education can be considered scientific education.

These methods should be employed in architectural research, and the ill-will and bungling which currently exist in the faculty with regard to empirical research (exceptions excluded) must be cast aside. Every designer must be able to evaluate critically the results of empirical research. Toward this end he must be thoroughly familiar with the methods and techniques used.

A complication in the discipline of architecture is heterogeneity. Each building, every location is unique. The formation of theories implies that one is striving for generalisation. In a domain where heterogeneity holds the trump card, there is a tendency to emphasise the uniqueness of the object considered. The tension between uniqueness and generalisation is interesting, but certainly not fatal. The same tension is familiar in psychology: each person is completely unique; yet it remains possible (and wise) to make generally applicable statements about human behaviour in certain situations and circumstances.

If the faculty wants to concentrate more on design in addition to the induction phase, and wants to offer a better methodological substantiation for design, its practitioners should be required to steep themselves in systems analysis, a craft pre-eminently useful to designers. Systems analysis reasons in a problem-orientated way, and stimulates the researcher or designer to think of alternative solutions in evaluating and balancing these alternatives. One must be explicit about the criteria by which these alternatives will be tested. This introduces goals and values to the order of the day. These interesting currents of discovery blowing into the world of methods and techniques need not exist exclusively in the corridors of the Faculty of Architecture in Delft, but rather should encourage communication between researchers., teachers, and students from other faculties and other universities, both domestically and abroad. These currents are an invitation to participate in international conferences, and in the circuits of refereed scientific journals, authors, as well as reviewers. Currently there is not much evidence of such an open, external orientation. The shutters of the Architectural Building seemed to be closed in regard to issues of research methods and techniques. Would it not be a good idea to open these shutters wide for once?

27 FORECASTING AND PROBLEM SPOTTING

TAEKE DE JONG
HUGO PRIEMUS

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Scientific forecasting and problem spotting calls for models of a reality within which one tries to make predictions. In this Chapter some crucial concepts will be treated that are involved during modelling. They are treated in large-scale examples, since examples like these feature fewer boundary conditions complicating forecasting than examples on a small scale. Forecasting the size of the world's population is after all, easier than the one of an individual household. As long as one believes in the human freedom of choice, forecasting on the level of the individual is even impossible. Only with large numbers may a meaningful relation be established, up to now, of one event A with the possible results W (chance = A/W, see page 219). That is the reason why 'chance' is a central model in a science aiming at generalising statements for the future. These are needed to come to grips with the future and its problems.

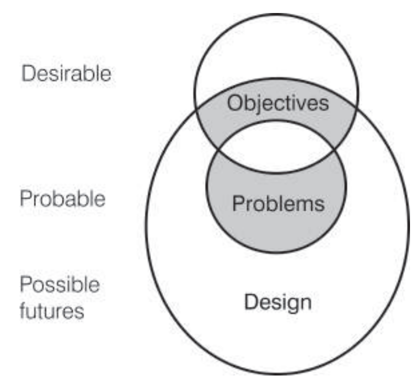
Forecasting study cannot do without this concept of chance. At increasing numbers, the choices of individuals group themselves around an increasing stable, 'reliable', mean against which the deviation of individual cases may be measured. From a range reliable numbers or averages as a function of time, a trend can be read, that may be generalised eventually to a mathematical formula (curve fitting). If this trend is undesirable, we have spotted a problem. A sequence of numbers X may be related to any other sequence of numbers Y -not only to a time axis- if in the swarm of graphical dots a line can be drawn, the line of regression.

Such a line of regression can be arrived at by many different (non-declarative) mathematical formulae. In forecasting study relations of this type are important in order to arrive at statements like 'If x then y'. That does not say yet that x causes y. However, the effort to explain a statistical coherence remains important.

Departing from such a causal explanation one can sometimes arrive at a mathematical formula. Only who can explain, can predict with authority?[a]

27.1 PROBLEM SPOTTING

The set of all problems is a set of probable, but not desirable futures. An applied empirical study starts with formulating such an undesirable probability (problem statement or formulation). A problem is probable and so can be forecast (signalled). Problem signalling already pre-supposes two predictions: the prediction of wishes and of probabilities. So the problem statement is not the real beginning, but part of an empirical cycle[b], in which problem statement, forecast, new problem statement produce one another



265 Futures and their modalities

The aim, or objective (several aims), of the study points from that problem to the more desirable future. The future aimed at, from which the aim of the study has been derived, is per definition desirable and not probable (one does not aim at realising tomorrow's sunrise) but possible; as far as we can see. Since an aim is not probable, it can not be forecast. So it must be chosen, posed, often even designed. An aim is an abstract pre-design of an alternative deemed possible for the present situation and its probable development (zero variant). These abstract concepts refer for everyone to comparable situations that usually remain implicit in order 'not to exchange aim and means', regardless of the level of abstractness, such as increased safety or accessibility. However, what is termed 'aim' and 'means' depends on the level of abstraction. The acquisition of a government subsidy can be an aim for a community, but for the country it is a means for a higher aim.

a Even if a problem exists in life-size and even if we are for one hundred percent convinced that it is a problem, it is nevertheless more appropriate from a scholarly viewpoint to talk about 'probability'; for we are dealing in principle with the future. Even the etymology of the word 'problem' (Greek *'proballein'*, 'to throw ahead') pre-supposes this. When the threatening event, experienced as a problem, has taken place it has passed its problem status. Next there will probably be many different problems, but they require new problem formulations.

b Groot, A.D. de (1961) *Methodologie: grondslagen van onderzoek en denken in de gedragswetenschappen*. English translation: (1969) *Methodology: foundations of interference and research in the behavioural sciences*.

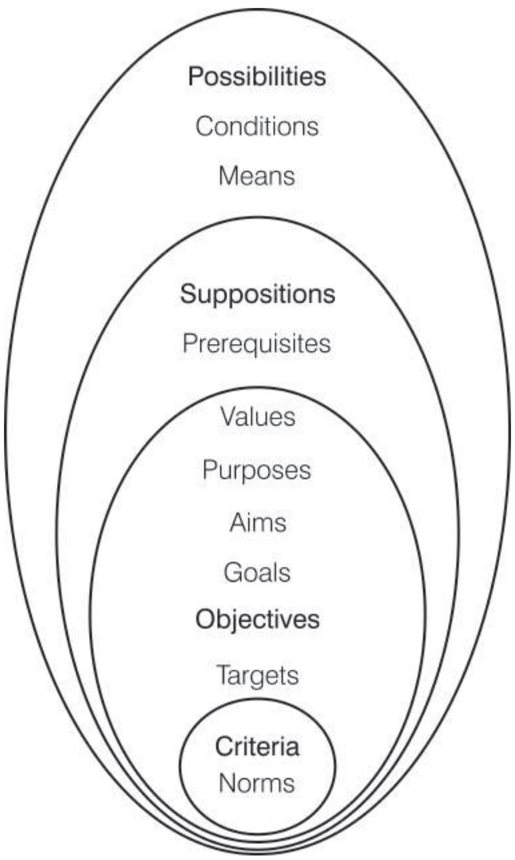
27.2 SYSTEMS' CONSIDERATION

From the problem formulation, further demarcations may be derived allowing separation of a dynamically variable and researchable system from a context that is largely independent (systems analysis).[a] This also results in the constraints within which the system must function. In addition, the problem formulation generates clues how system and context should be reduced to a composite of researchable, valid and reliable (see page 92) variables (operationalisation) with the mutual relations (modelling in functions). The functions are going to be mutually related, so that the output of one function becomes the input of one or more subsequent functions (systems modelling).

An inter-action with a causality in two directions often occurs. These relations within and between functions pre-suppose previous empirical study (empirical cycle) that demonstrated such a relation or that seems probable by reflection. Within the demarcation, the constraints and from the objective, the alternatives are designed (means, pre-supposed solutions, encroachments, with the rôle of hypotheses[b] that can be checked). These alternatives must be weighed (assessment). Against which values or criteria? Between vague values and precise criteria, a wide range of gradation exists.

27.3 CRITERIA OF ASSESSMENT

It is impossible to imagine criteria and norms without objectives underlying them and objectives without the basis of individual or social values. On the other hand, it is possible to imagine values that have not been worked out into objectives as yet and objectives not yet associated with norms. These concepts operationalise subsequently increasingly concretely a desirable future, so far as is possible, so that it may be tested. Along these lines public safety is an important social value that may be made workable (operational) in more specific objectives, like better lighted public spaces, worked out into norms for lighting that can be controlled. The English language is rich in terms for objectives in the various stages between what is strategic and what is operational. In the diagram alongside they have been put in a conditional sequence.



266 From possibility to norm

However, one is well advised to remember that values have been founded on a body of implicit conditions, pre-suppositions and imaginations: culture. As far as they are related to truth, these are the conditions from classical logic (if…then, if…if, then and only if, see page 189). They yield a consistency check with regard to the logic of the discourse, not yet with regard to causal correctness of the premises in the wider area of probabilities. An example: "If public illumination ameliorates security, then illumination will make this insecure, not illuminated place more secure." However, the premise that illumination ameliorates security is always a context-sensitive causal pre-supposition. Moreover, in each and every objective also technical conditions are hidden, allowing checking. They are often implicit (lamps exist, the electricity needed is available and affordable, the neighbours do not experience illumination as cumbersome, implementation is feasible in the municipal council). They delimit what is possible and feasible from what is impossible and are creating room for what is improbable (conditions).[c]

All these conditions, values and their development into objectives and criteria, should be part of the problem formulation. But, then the problem formulation would encompass the study as a whole; it is more advisable to make a rough sketch first to be added to and updated during the study, in periodical consultation with the initiator. The central objective of this is a reduction in which underlying values, conditions, suppositions, means, conditions and possibilities of the problem investigation have been omitted.

However, the investigator is obliged to identify as many of the prevailing values as possible and to make them explicit in order to operationalise them in criteria for the assessment of alternatives. In the classical scheme of Findeisen and Quade (see page 250), this part of the systems' analytical study is rendered with: 'Identifying, designing and screening the alternatives'.

154 Findeisen, W. and E.S. Quade (1985) *The methology of systems analysis: an introduction and overview*.
155 An architectural design may be seen as a hypothesis; in that case it is an object of an evaluating study before or after execution (ex ante or ex post).
156 This is latin for mee-giften.

27.4 ESTABLISHING ALTERNATIVES

One line of text might make a hypothesis, but not a full-fledged design. The systems' analysis requires different hypotheses (alternatives) to test the completeness of the system. Alternatives may emerge for which the system has not an answer yet.

The classical model of the empirical cycle lacks an instruction for establishing hypotheses. The establishing of hypotheses is 'free'. Scientifically spoken, anything can be argued or drawn up, until the hypothesis is refuted[a] or replaced by a better alternative.

But, if the hypothesis is an architectural design, establishing the hypothesis is an important part of the study by design, highlighted in other Chapters of this book. In the process of building generating an alternative for the present situation (zero-variant) may entail 95% of the studying effort. Often just one alternative exists, to be varied upon at best during the process. There is then less time available for problem analysis. In architectural design many more (detailed) decisions are taken than for which the objective can be directive; often even a tacitly presupposed type of building or neighbourhood.

The decisions as to shape and structure outside of the objective - usually regarded as of secondary importance - require insight into a combinatorial explosion of possibilities (see page 208). In order to reduce them, the designer uses not only the problem formulation and the objective (the site and the programme of requirements), but also existing examples (precedents, design study) and types (typological study). The designer is guided by a global concept that allows more aims than formulated in words by the initiator. A sustainable building must be able to serve in a different context after selling it subsequent owners ('robustness' of the design). If the designer varies the context taken to be obvious in the problem analysis as well (study by design[b]), the design may lead to a review of the perception of the problem; and, by the same token, of the systems analysis. This feed-back arrow is missing in the schema of Findeisen *et al.* on page 250.

27.5 EVOKING SYSTEM BEHAVIOUR

Calculating the consequences of an action, alternative, design or hypothesis (evaluating study ex ante, see page 159) requires prediction. Each prediction pre-supposes a context, within which the proposed action functions. Change in context (perspective) changes the consequences of the action.

The sensitivity of the prediction for these changes cannot be avoided in applied study by pre-supposing 'other things being equal' (*ceteris paribus*). One has to highlight the consequences of several actions in several perspectives in order to achieve a more general insight into 'system behaviour' under different circumstances and actions. The construction of these perspectives (future contexts) with unexpected aspects and decision moments (scenarios) will be dealt with in a following paragraph. Scenarios play a rôle within the predictions in the system itself as providers of external exogenous variables (parameters) in the equations on which the systems' model has been built. By changing these parameters in the equations per scenario, additional consequences may emerge.

27.6 GENERALISATIONS

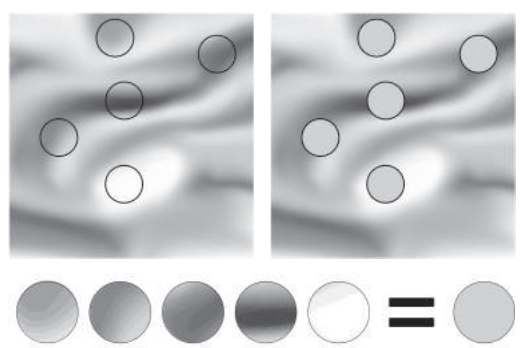
The possibility of forecasting and prediction depends on external generalisations from previous experiences (empiry). Not only external ones apply, also internal. Particularly the use of the average as the most important form of statistical generalisation and its extrapolation in time, meets with opposition from scientific disciplines supposed to deal with a large variety in objects and contexts: especially ecology, organisational science[c] and designing. In ecology this reduction to an average by the analysis of ecosystems is known as the 'mean-field assumption' (see following diagram).This reduction can smooth local variations in such a way, that the character of the object and its context evaporate. At the same time the survey of the specific possibilities of the site ceases to exist. The statistical measure for deviation cannot replace this variation.

a Popper, K.R. (1963) *Conjectures and refutations: the growth of scientific knowledge*. Partly translated in Dutch: (1978) *De groei van kennis*.

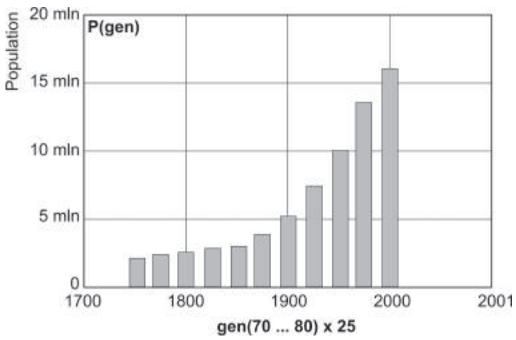
b In the scheme on page 14 this distinction is made with typological study (testing the same design in various contexts) and design study (testing various design variants in the same context).

c Riemsdijk, M.J. van (1999) *Dilemma's in de bedrijfskundige wetenschap*.

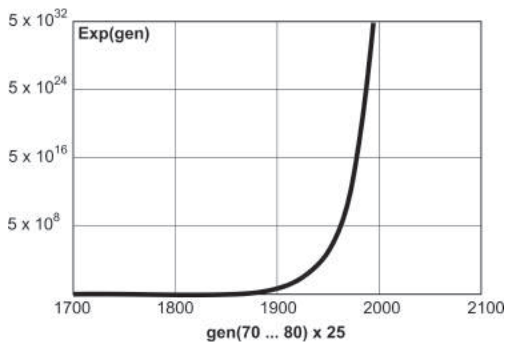
d Derived from Law, R., U. Dieckmann et al. (2000) *Introduction*. p. 4.



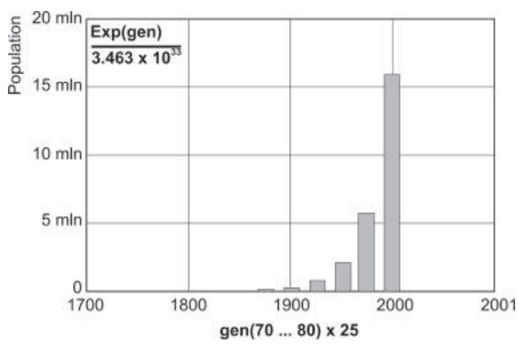
267 Reduction to the average[d]



268 Actual growth of the population in the Netherlands



269 f(Gen) = exp(Gen)



270 The same with parameter

In evolutionary ecology[b] especially a few cases of exception, outside of the 95% area, determine the future course of the ecological process, since these rarities may lead in particular to the emergence of new species and systems. This suggests mathematical chaos functions, featuring, by means of iteration an unpredictable course; they are very sensitive to the minutest variations at first input. Rounding-off strategies in different computer brands may even lead to the circumstance that one and the same formula yields a different outcome on two distinct machines.

All this does not derive from the fact that forecasts, or less explicit, expectations, are the base of acting. By way of an example, we select the growth function of a population.

27.7 CURVE FITTING

The population of the Netherlands has grown during the last three centuries as follows.

The bars in this graph comprise class intervals of 25 years (interpreted here as generations) with the population figures levelled out over the period. First of all, the progress recalls an exponential function f(x) = exp(x); after the industrial revolution reaching the country around 1800.

In figure 269 a part of this function from the beginning of the Christian era has been drawn for the last 10 generations (1750 – 2000 AD). For x Gen (generations) has been substituted.

This function leads from 0 persons in year 0, to an unlikely high population figure of f (80)= 5.541 x 10^{34} in the year 2000 (generation 80). If we divide this number by the current population size of 16 million Dutch (wo)men f (80)/ 16 = 3.463 x 10^{33}, the first parameter materialises in the model that as denominator reduces the last generations (80) to 16 million (figure 230). This does not reflect reality well. The mathematical representation of population growth is differing too much from actual data.

27.8 CAUSAL PRE-SUPPOSITIONS

Each generation comprises the number of parents, times their average number of children, plus the parents themselves. In each following generation children become parents, and parents grandparents. They die; and the grandparents should be subtracted from the following generation. This can be rendered in an iterating formula.[c]

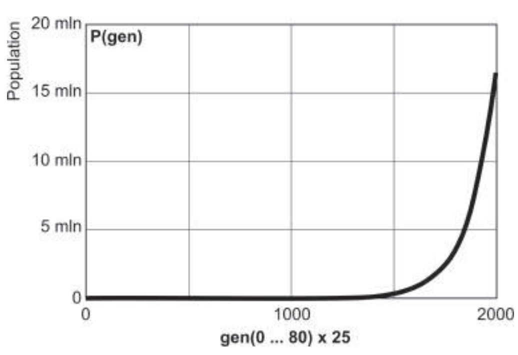
$$Pop_{Gen+1} := Children \times Pop_{Gen} + ParentsPop_{Gen} - GrandparentsPop_{Gen-1}$$

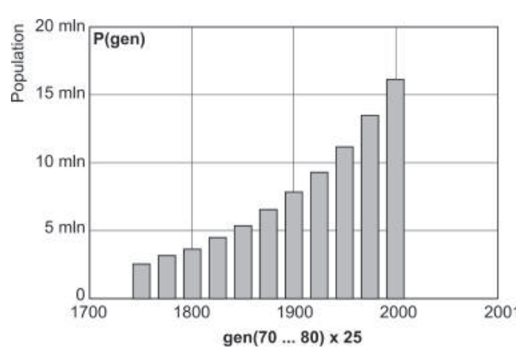
For this graph we had to fix the first generation to two people in the year 0 and the number of children to a parental couple through all these centuries to 1.034 on average.

This formula cannot be extrapolated readily to earlier years. However, the slice since 1750 starts to resemble reality. A function allowing more extrapolation would require for the aim of this discourse too many additional parameters from earlier contexts (for instance: a parameter working out negatively for the emergence of epidemics of plague at certain population densities and a medieval state-of-the-art of medical science).

271 The exponential growth of a population

272 Slice of figure 271





b Pianka, E.R. (1994) *Evolutionary ecology*.

c Such a formula takes its own output as an input for a next round. It contains instead of the = sign a sign := (gets) and before the getting sign an index that is one step larger than after the getting sign.

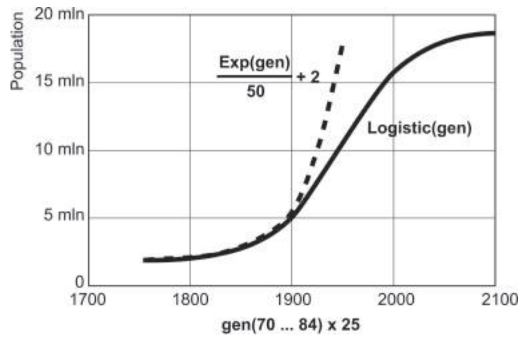
27.9 CONSTRAINTS

If a population approximates its limits, growth lessens. For such a phenomenon mathematics offers the logistical curve.

In a non-iterative form, the formula for exponential growth has an exponent, compare page 229; the logistical curve is an extension with two parameters, the first of which initiates the restriction.

$\text{Exponential}_{Gen} = \text{Pop}_0 \times \text{Children}_{Gen}$
$\text{Logistical}_{Gen} = \text{Pop}_0 \times \text{Children}_{Gen} \times \text{restriction} / (\text{Children}_{Gen} + \text{Children}^a)$

The second parameter, exponent 'a' in the denominator of the logistical function, regulates the speed of growth.



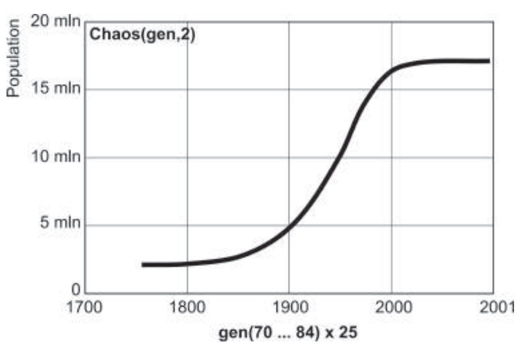
273 The logistical curve

27.10 SENSITIVITY

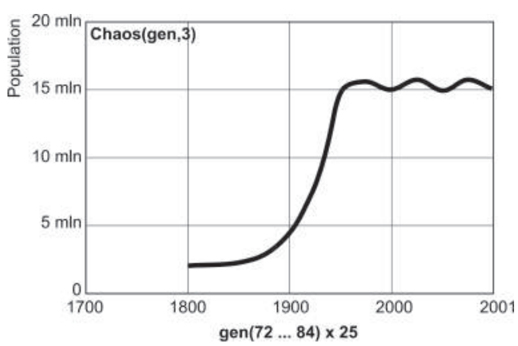
The iterating functions leading to fractal geometry and chaos theory have proven that systems may vary vastly by minimal differences in input and parameters (sensitivity).[a] The following function (chaos function) resembles at the value a = 2 of parameter a the logistical curve.

For an initial value $X_0 = 0.0016$ this function is congruent with the growth of the Dutch population. In order to match it, one must multiply by 30 and to get it to the right height one should add 2. Then, it 'forecasts' a stability following the year 2000.

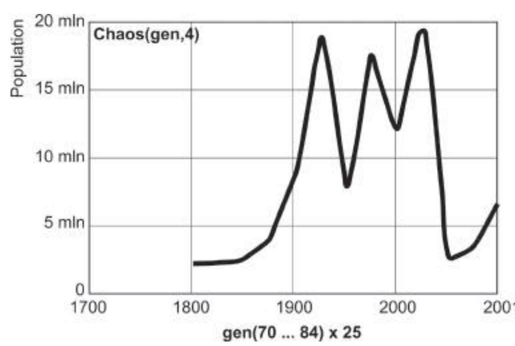
However, if one chooses for parameter a = 3, the function starts to flutter. At a = 4 the function becomes chaotic. And at a = 4.1, an entirely different graph results.



274 a = 274



275 a = 3



276 a = 4

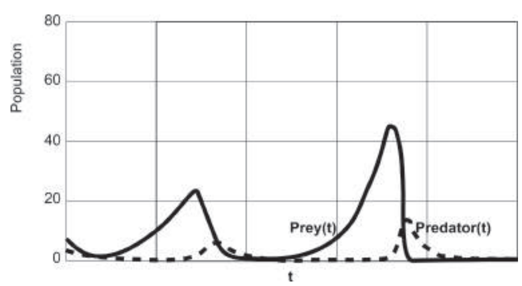
$x_{Gen\text{-}1} = ax_{Gen} - ax_{Gen}^2 \; \text{Chaos}_{Gen} = 30x_{Gen} + 2$

27.11 PARAMETERS

A quiet constraint like lack of space results in a quiet smoothing, while wars and epidemics result in wild fluctuations. Rather more causally, fluctuations like that are simulated by the Lotka-Volterra function for preys (like people) and their predators (for instance the plague bacteria or other people in their guise of enemy).

In the case of this function time is input (here at a scale implying nothing particularly from 1 to 150), with the densities of prey and predator for output. They are inter-changing as 'causation' of rising and declining. For the densities initial amounts should be stated; as well as the value of the four parameters.

These regulate the waxing of prey if there are no predators, the percentage of animals of prey caught, the death and emigration of predators and increase by consumption of prey. If the value of parameters of this type cannot be ascertained by empirical research, it is possible to arrive at a state that proves to match time series of the past (calibration). In both cases it imports to check the sensitivity of the model to parametrical selection and to report on it. Varying parameters as to their effect on the graph is no punishment anymore, considering current computer capability, and sometimes provides outright sensations. But the larger



277 Lotka-Volterra function[b]

a Broer, H.W. and F. Verhulst (1992) *Dynamische systemen en chaos, een revolutie vanuit de wiskunde;* Broer, H.W., J. van de Craats et al. (1995) *Het einde van de voorspelbaarheid*?

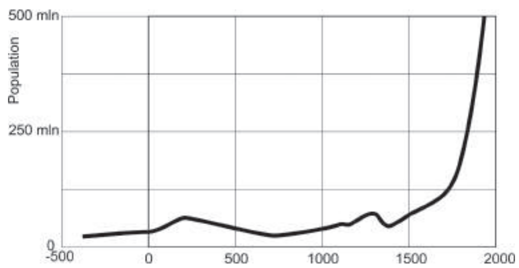
b Mack, Dr. T.P., Associate Professor Department of Entomology, modelled predator-prey dynamics in Mathcad (Alabama) Auburn University.

the number of parameters one may use as ‘buttons on the piece of equipment’, the more difficult it becomes to determine the influence of each button separately on the result. The influence of one parameter can change drastically, if one turns other dials. If one has 6 buttons at one’s disposal, each with at least 10 positions, like in the Lotka-Volterra function, the minimal number of combinations, $6^{10,}$ is already not to be surveyed. Which of the resulting 6^{10} functions should one choose for the model desired?

With the explosion in terms of number of combinations (see page 208) of the tuning of parameters the fringes of realistic modelling are attained often in sciences markedly sensitive to context; as there are architecture, ecology and organisational science.

27.12 EXTERNAL FACTORS

In the case of the Lotka-Volterra function the predator, initially regarded as external factor, has been assimilated within the model (internalisation). While the predator was directly dependent on the availability of prey, they formed together a ‘system’ that might be modelled with inter-changing causality. Obviously, internalisation cannot digest everything. Quite a few external influences just have to be stated, or to be varied for some scenarios.



278 Population development in Europe

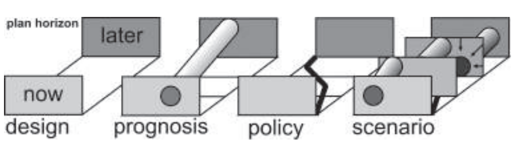
The presumable course of Europe’s population[a] shows the consequences of a lot of unidentified external factors, although it is known, that the plague, The Black Death, raged at the end of the Middle Ages.

It seems as if an internal drive to exponential growth is always stinted, until all brakes vanish in modern times. Which factors have been responsible for that: spatial, ecological, technical, economical, cultural, managerial? Pre-suppositions concerning size and nature of immigration and emigration are crucially important for real-life population prognoses.

Determining the parameters for fluctuations like these, and establishing the functions by which they operate, requires more data and more detailed analyses. They may result in data files of parameters that may be consulted through the systems’ model during calculation with ‘if.. then..’ statements. They require knowledge of the influence of spatial, ecological, technical, economical and managerial developments in time series.[b]

Among all influences in the case of population forecasts, for instance, the crucial factor of the average number of children per household (fertility) determines the factor of reproduction. The shorter the time-span of validity of the forecasting, the longer the time series on which the forecast is based (founding period); and the more substantial the explanation enabled by the independent variables, the more reliable the forecast. The Statistics Netherlands (CBS) publishes a population prognosis on an annual basis (in *Monthly Statistics for the Population*).

27.13 SCENARIOS



279 Possible, probable, desirable, image of future and scenario

Although scenarios are made for many reasons (insight, strategy, management inside and interaction between organisations) they are considered here as purveyors of exogenous variables in order to serve systematically forecasting and problem spotting study.

A scenario is not a calculated (prognosis) or an assumed probable future (perspective). A scenario is a time series projected into the future within which managerial, cultural, economical, technical and spatial influences (stemming from actors in these different sectors) are varied consistently and plausibly. It is the description of a possible future, partly designed, that corresponds partly to prognoses and perspectives and that may contain partly policy decisions.

In figure 279, rendered in a sequence lacking intention, it is represented that the design tries to project one whole image on the planning horizon, while the prognosis delimits there an area that might be put to work in a scholarly way. In this area in which the subsequent

a Slicher van Bath, B.H. (1976) *De agrarische geschiedenis van West-Europa 500-1850*. English translation: (1966) *The agrarian history of Western Europe, A.D. 500-1850*.

b For interesting time series, see ‘x years of time series’ of Statistics Netherlands (CBS), published every five years. The ‘x’ in this title was successively ‘85’, ‘90’, ’95’ and ‘100’. See e.g. Centraal Bureau voor de Statistiek (1989) *1899 -1989 negentig jaren statistiek in tijdreeksen*.

consequences are depicted, there just might be another area than where preceding causes are dwelling. The prognosis departing from a plan, not from the current situation (evaluation ex ante, effect analysis, see page 149), is taking its bearings on uncertain pre-suppositions regarding context (boundary conditions; the tiny arrows in the last drawing).

Policy punctuates a path with limiting values when a part of the policy goal should have been reached (target figure). In principle, a scenario compromises all components although one component may take the lead. If the starting point is the design with a final stage and from there the reasoning is backwards (back-casting), one talk about a prospective scenario, in other cases of a projective scenario. If prognosis stands central, the parlance is ‘trend-scenario’; and if, on the contrary, unexpected events play a more important rôle, ‘surprise-scenario’. If policy objectives pre-dominate, normative- scenario is the word. In the policy time-path decision moments may materialise that can give the scenario a twist. In order to survey consequences of such decisions, policy scenarios or exploring scenarios are made, with alternative scenarios branching out tree-wise.

27.14 SECTOR SCENARIOS

Scenarios exist emphasising particularly political, cultural, economical, technical, ecological (demographical) or spatial sectors for a driving force. Their yardsticks, research methods and variables differ greatly, which makes them difficult to combine and integrate. Sector scenarios like that are made, for instance, by the Netherlands Bureau for Economic Policy Analysis (CPB, economical scenarios), the Social and Cultural Planning Office the Netherlands (SCP, cultural scenarios), the National Institute for Public Health and the Environment (RIVM, ecological and nature scenarios) and the Consultancy Service for Traffic and Transport (AVV, mobility scenarios).

Often, they are cross-wise combined to more integral scenarios, contrasting with one another (extreme scenarios or contrast scenarios). This contrast is called for to make effective sensitivity analyses in systematic study of the future.

With the 6 sectors mentioned previously, 15 quartets of this type can be produced.[a] One may also contrast, for instance, policy (steering <> following) with culture (tradition orientated <> experiment orientated), technique (combining <> specialising), ecology (homogeneous <> heterogeneous) or space (deconcentration <> concentration).

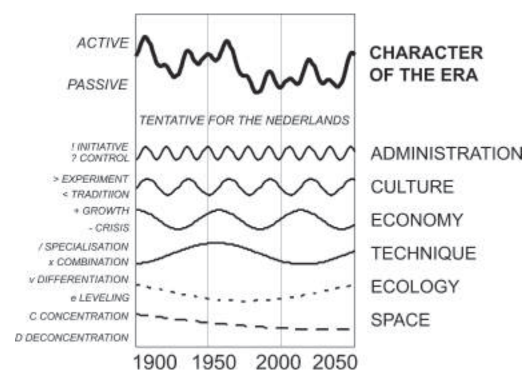
CPB scenarios vary with the relative strength of European economy in the competition with American or South-East Asiatic clusters, or with the effectiveness of co-operation within Europe itself: Global Competition, Divided Europe and European Co-ordination.

However, these sectors have different time horizons and dynamics; this hinders contrasting them mutually. The figure alongside illustrates this as a series of trend-scenarios fluctuating between the extremes.

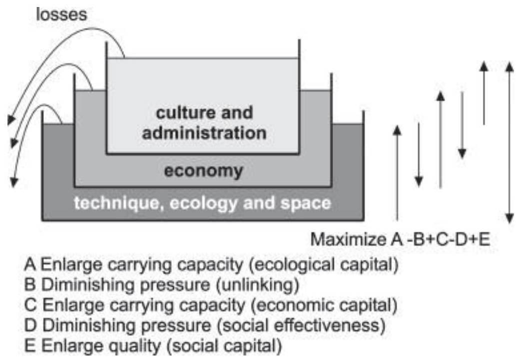
In this figure the face of the time is an addition of rather dubious sector trends, depending on the notion that each action calls for an anti-thetical (Hegelian) reaction; so that, between two extremes, an oscillating movement emerges; here depicted as a clean sinus curve. Now imagine that these sinuses have been calibrated on the century now passed; while it has been shown that policy fluctuates by seven years between guiding and following, and culture is shifting within 14 years between traditional and experimental orientation; and so forth

What name should be given to the extremes of their super-position? ‘Active’ and ‘passive’ are the, scarcely meaningful, terms chosen here. Balancing developments in different sectors is usually left to politics. Nevertheless, it establishes a scientific challenge that gets attention especially in ecological policy: how does one weigh environmental interest against economical and spatial priorities?

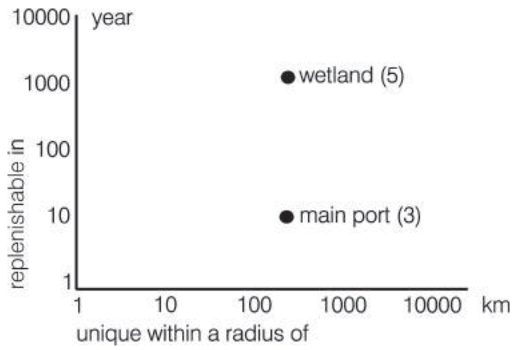
280 Cross-wise integration of sector scenarios



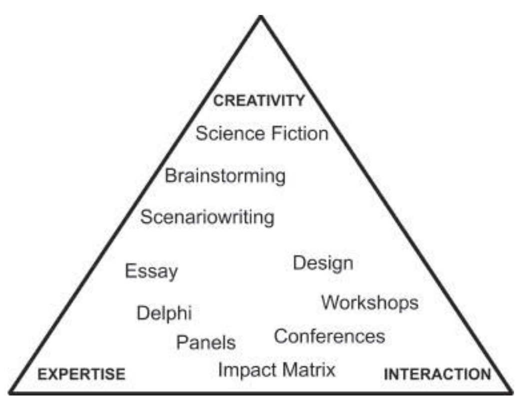
a Combinations of 2 out of 6 = 15. Enter in Excel: =COMBINATIES(6;2) and the result is 15. See page 190.



282 Balancing between sectors



283 Technical balancing of projects



284 Foresight triangle

27.15 POLICY BALANCING

Between the sectors one can pre-suppose conditionality. Technique has its ecological conditions: without food, water or materials, technique would not exist. There are technical conditions for the economy: without dikes in our low delta lands, the economy would not exist either, etc. This leads to a model of sustaining capacity. In it, the sectors do not pre-suppose one another causally (a certain technique leads to a certain economy), but conditionally (a certain technique makes different economies possible).

One may also try to make a more technical judgment, for instance between nature areas and airports. The product of their scarcity in a wide surroundings and the viability to generate them within a particular time, provides a yardstick for comparison.

This diagram pre-supposes that in The Netherlands a main port is unique in a radius of some 300 km, but may be built in 10 years. Wetlands are unique on the same scale, but are only replenishable in a period of 1000 years. The logarithm of the product (the sum of the zero's) is respectively approximately 3 and 5.

27.16 SCENARIO DEVELOPMENT

With emphasis on possibilities, likelihoods or desirables, the builders of a scenario should have access to a wider field of creativity, expertise or actor-orientation than with more causally driven planning. The means in figure 284 are at disposal.[a]

For further explanation we refer to the literature. An example of the Delphi Method is discussed in this book on page 491. The method comprises questioning a group of experts and confronting it quickly with the outcome, so that they might consider again. Next they are interviewed again, so that they may get an idea to what extent the group is going to support views. This usually leads to a convergence of ideas. The cycle may be repeated several times, until the outlines of a scenario manifest themselves.

27.17 LIMITATION SHOWS THE MASTER

Just very large-scale examples have been given here, since they feature a relatively small number of exogenous variables: by the same token they supply, with a grain to match, broader models which are, however, more appropriate for explanation. In the daily practice of study the scenarios of CPB, SCP, RIVM and AVV are usually regarded as given entities; that applies also for the population prognosis of the CBS. With a fitting amount of modesty study then tries to state something in the range of the first 5 years, 10 years at the most. Such a 'forecast' generates subsequently annual verification of the results (monitoring).

Perhaps the impression is created here that the study challenges the position of the Creator Himself. In the examples as practice generates them, much more is already determined or given. One just looks at a limited number of variables on a rather narrow time-horizon. As already mentioned in the start of paragraph 27.13, usually scenarios are not meant to be forecasts, but tools to facilitate and improve social and political discussion and decision making.

a Meulen, Van der; Cameron et al. (1996), quoted by Geels, F. (1997) *Met de blik vooruit - op weg naar socio-technische scenario's*.