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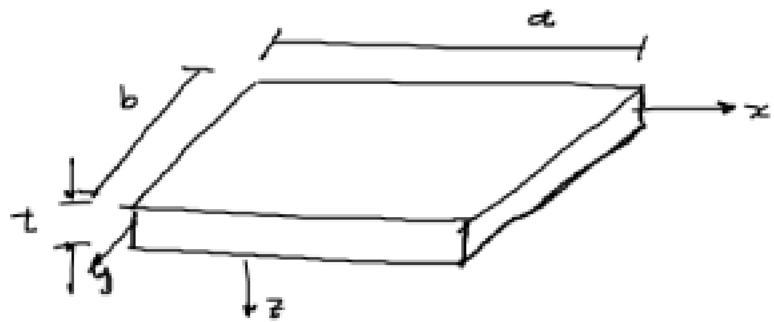
Plate theory:-

$\frac{t}{a} << 1 \qquad \frac{t}{b} \leq 1$



flat reference surface (plane)

mid-plane as a reference surface.

(x, y) to describe points on the midplane.

z. location in thickness direction.

Purpose of plate theory:-

To eliminate the dependence on z in the equations.

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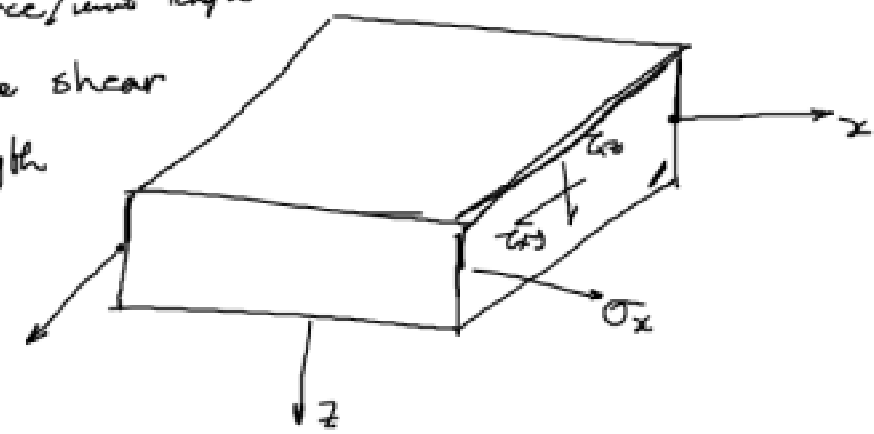
Eqm equations of plates

N_x: inplane normal force /L x direction

N_{xy}: inplane shear force/unit length

Q_x: is out of plane shear

force per unit length



Force per unit length of the mid plane

$$F_x = \iint \sigma_x \, dz \, dy = \int \underbrace{\left(\int_{-t/2}^{t/2} \sigma_x \, dz \right)}_{N_x} dy, \quad N_x = \int_{-t/2}^{t/2} \sigma_x \, dz$$

$$F_y = \iint \tau_{xy} \, dz \, dy = \int \underbrace{\int_{-t/2}^{t/2} \tau_{xy} \, dz}_{N_y} \, dy, \quad N_y = \int_{-t/2}^{t/2} \tau_{xy} \, dz$$

$$F_z = \iint \tau_{xz} \, dz \, dy = \int \underbrace{\int_{-t/2}^{t/2} \tau_{xz} \, dz}_{Q_x} \, dy, \quad Q_x = \int_{-t/2}^{t/2} \tau_{xz} \, dz$$

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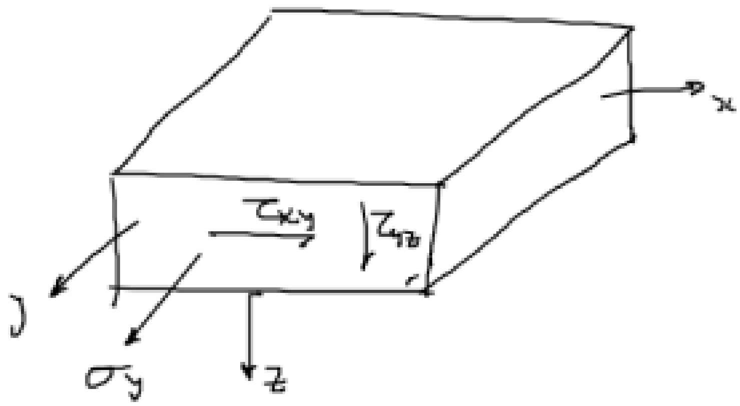
$$N_y = \int_{-t/2}^{t/2} \sigma_y \, dz$$

$$N_{xy} = \int_{-t/2}^{t/2} \tau_{xy} \, dz$$

$$Q_y = \int_{-t/2}^{t/2} \tau_{yz} \, dz$$



Five force resultants:-

N_x N_y N_{xy}

Q_x Q_y

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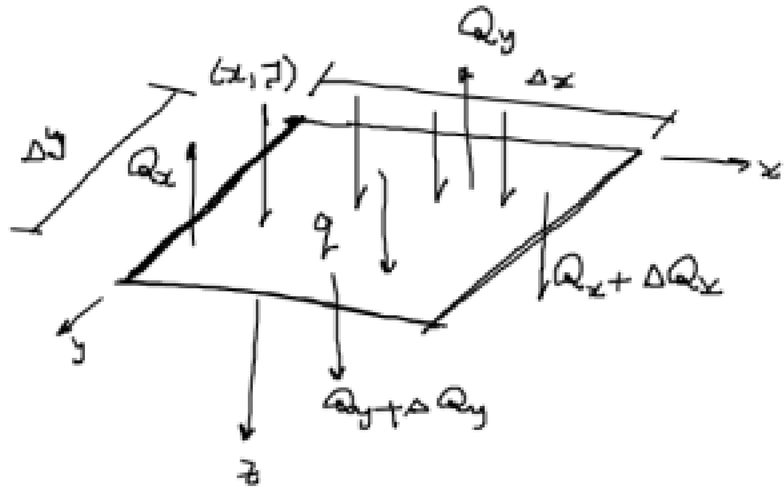
q: lateral pressure

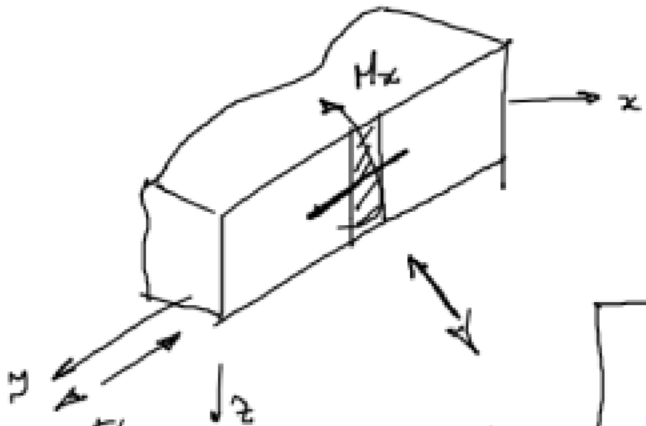
$q(x, y)$

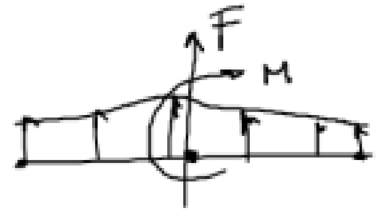


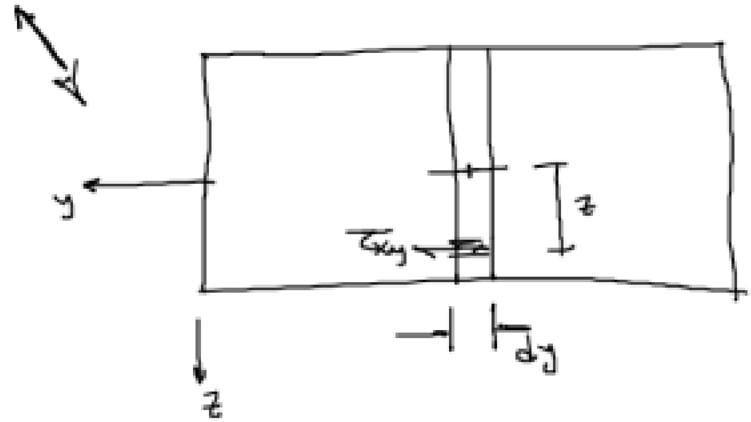
$$q\,\Delta x\,\Delta y + (Q_x + \Delta Q_x)\,\Delta y - Q_x\,\Delta y + (Q_y + \Delta Q_y)\,\Delta x - Q_y\,\Delta x = 0$$

$$\frac{\Delta Q_x}{\Delta x} + \frac{\Delta Q_y}{\Delta y} + q = 0$$

$$\boxed{\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0}$$







$$M_{xy} = -\int_{-t/2}^{t/2} (\tau_{xy}\, dz)\, z$$

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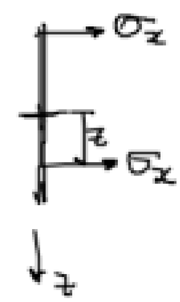

$$M_x = \int_{-t/2}^{t/2} (\sigma_x\, dz)\, z$$

$$= \int_{-t/2}^{t/2} \sigma_x\, z\, dz$$

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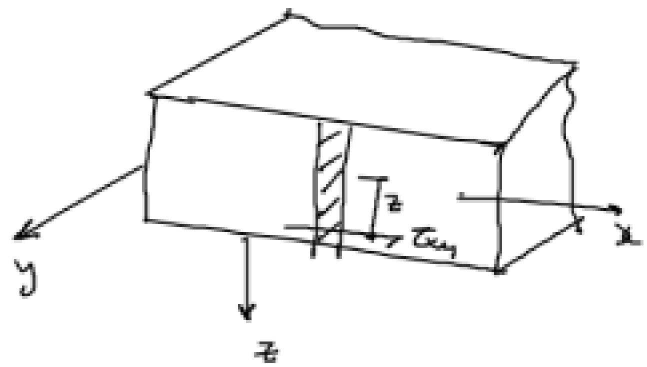
M_x : bending moment / unit length

M_{xy} : twisting moment / unit length.

$$M_y = \int_{-t/2}^{t/2} \sigma_y \, z \, dz$$

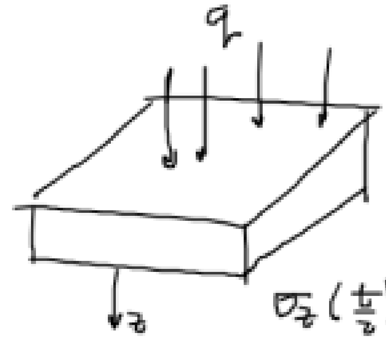


$$\left.\begin{aligned} M_{yx} &= \int_{-t/2}^{t/2} \tau_{xy} \, z \, dz \\ M_{xy} &= -\int_{-t/2}^{t/2} \tau_{xy} \, z \, dz \end{aligned}\right\}$$

$$M_{yx} = -M_{xy}$$

$$M_x = \int_{-t/2}^{t/2} \sigma_x \, z \, dz$$

Derive Eq- equations



$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$(z) \longrightarrow \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0$$

$$\underline{\frac{\partial \tau_{xz}}{\partial x}} + \underline{\frac{\partial \tau_{yz}}{\partial y}} + \frac{\partial \sigma_z}{\partial z} = 0 \longleftarrow$$

$$\sigma_z\left(\tfrac{t}{2}\right) = 0$$

$$\sigma_z\left(-\tfrac{t}{2}\right) = -q$$

$$\underline{\frac{\partial}{\partial x}\left(\int_{-t/2}^{t/2} \tau_{xz}\, dz\right)} + \frac{\partial}{\partial y}\left(\int_{-t/2}^{t/2} \tau_{yz}\, dz\right) + \Big[\sigma_z\Big]_{-t/2}^{t/2} = 0$$

$$Q_{x,x} + Q_{y,y} + q = 0$$

$$\underbrace{\frac{\partial}{\partial x}\int_{-t/2}^{t/2} \tau_{xy}\, z\, dz}\; + \frac{\partial}{\partial y}\int_{-t/2}^{t/2} \sigma_y\, z\, dz + \int_{-t/2}^{t/2} z\, \frac{\partial \tau_{yz}}{\partial z}\, dz = 0$$

$$M_{yx,x} + M_{y,y} + \underbrace{z\,\tau_{yz}\Big|_{-t/2}^{t/2}}_{0} - \underbrace{\int_{-t/2}^{t/2} \tau_{yz}\, dz}_{Q_y} = 0$$

$$\underline{M_{yx,x} + M_{y,y} = Q_y}$$

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Eq. equations

$$\left.\begin{aligned} M_{x,x} + M_{yx,y} &= Q_x \\ M_{yx,y} + M_{y,y} &= Q_y \\ Q_{x,x} + Q_{y,y} + q &= 0 \end{aligned}\right\}$$

$$\left.\begin{array}{l} (M_x, M_y, M_{yx}) \\ (Q_x, Q_y) \end{array}\right] \quad 5 \text{ unknowns}$$