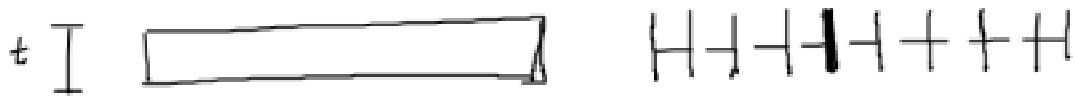
Classical Plate theory:-

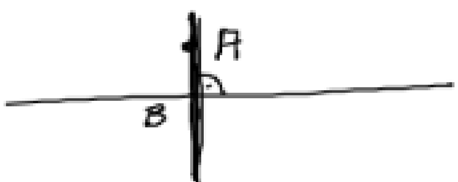


mid surface:- $\bar{u}(x,y)$ $\bar{v}(x,y)$ $\bar{w}(x,y)$

(x, y, z) !? $(x+u, y+v, z+w)$

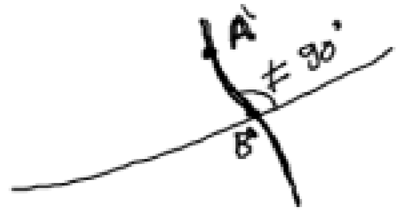
undeformed deformed

(x, y, z) connected $(x, y, 0)$



undeformed

Kirchhoff



deformed

1- stretch or shrink ✗
2- it may deform ✗
3- it may not remain normal to the surface. ✗

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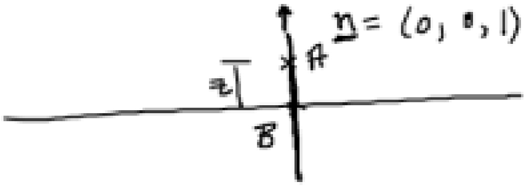

$$A: \quad (x, y, z) \qquad\qquad \underbrace{(x, y, \bar{w}) + z\,\underline{n}'}_{!?} \quad : \quad A'$$

$$B: \quad (x, y, 0) \longrightarrow (x, y, \bar{w}) \quad : \quad B'$$

$$\bar{w}(x, y) \quad , \quad \underline{n}'(x, y)$$

$$(1, 0, \bar{w}_{,x}) \quad , \quad (0, 1, \bar{w}_{,y})$$

$$\underline{n}' = \frac{(-\bar{w}_{,x}, -\bar{w}_{,y}, 1)}{\sqrt{1 + \bar{w}_{,x}^2 + \bar{w}_{,y}^2}}$$

$$\underline{n}' \approx (-\bar{w}_{,x}, -\bar{w}_{,y}, 1)$$

$$(x, y, z) \longrightarrow (x - z\bar{w}_{,x}, y - z\bar{w}_{,y}, z + \bar{w})$$

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$$(x, y, z) \longrightarrow (x - z\bar{w}_{,x},\ y - z\bar{w}_{,y},\ z + \bar{w})$$

$$\left.\begin{aligned} u &= -z\, w_{,x} \\ v &= -z\, w_{,y} \end{aligned}\right\}$$ Displacements

$$\boxed{w = w(x, y)}$$

Strains:-

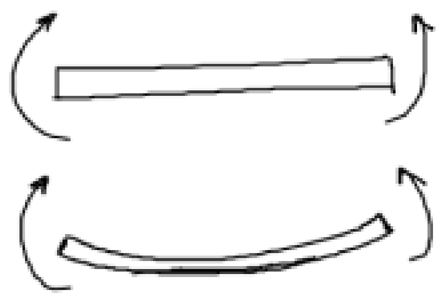
$$\varepsilon_x = -z\, w_{,xx} \qquad \varepsilon_y = -z\, w_{,yy} \qquad \gamma_{xy} = -2z\, w_{,xy}$$

$$\varepsilon_z = 0 \qquad \gamma_{xz} = -w_{,x} + w_{,x} = 0 \qquad \gamma_{yz} = -w_{,y} + w_{,y} = 0$$

* state of plane stress

$$\tau_{xz} = 0 \qquad \tau_{yz} = 0$$

$$\sigma_z = 0$$



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$$W = - \iiint \sigma_x \, \delta\varepsilon_x + \sigma_y \, \delta\varepsilon_y + \tau_{xy} \, \delta\gamma_{xy} \; dx \, dy \, dz$$

$$\varepsilon_x = +z \, (-w_{,xx}) \qquad \sigma_x = \underbrace{\frac{E}{1-\nu^2} \left(-w_{,xx} - \nu \, w_{,yy}\right)} \, z$$

$$\varepsilon_y = z \, (-w_{,yy}) \qquad \sigma_y = \frac{E}{1-\nu^2} \left(-w_{,yy} - \nu \, w_{,xx}\right) z$$

$$\gamma_{xy} = z \, (-2w_{,xy}) \qquad \tau_{xy} = \frac{E}{2(1+\nu)} \left(-2 \, w_{,xy}\right] z$$

$$\underline{(M_x, \; M_y, \; M_{yx})}$$

$$(Q_x, \; Q_y)$$

$$Q_{x,x} + Q_{y,y} + q = 0$$

$$M_x = \int_{-t/2}^{t/2} \sigma_x \, z \, dz = \frac{E}{1-\nu^2} \left(-w_{,xx} - \nu \, w_{,yy}\right) \underbrace{\int_{-t/2}^{t/2} z^2 \, dz}_{\frac{t^3}{12}}$$

$$M_x = \underbrace{\frac{Et^3}{12(1-\nu^2)}}_{D:\ \text{bending stiffness}} \left(-w_{,xx} - \nu \, w_{,yy}\right)$$

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$$M_x = D(-w_{,xx} - \nu\, w_{,yy})$$

$$M_y = D(-w_{,yy} - \nu\, w_{,xx})$$

$$M_{yx} = D\frac{(1-\nu)}{2}(-2w_{,xy})$$

$$\begin{pmatrix} M_x \\ M_y \\ M_{yx} \end{pmatrix} = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{pmatrix} -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{pmatrix}$$

$$Q_{x,x} + Q_{y,y} + q = 0$$

$$M_{x,x} + M_{yx,y} = Q_x$$

$$M_{yx,y} + M_{y,y} = Q_y$$

$$M_{x,xx} + 2M_{yx,xy} + M_{y,yy} + q = 0$$

$$\boxed{-D[w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy}] + q = 0}$$

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$$\left.\begin{aligned} \sigma_x &= \frac{12}{t^3} M_x \, z \\ \sigma_y &= \frac{12}{t^3} M_y \, z \\ \tau_{xy} &= \frac{12}{t^3} M_{yx} \, z \end{aligned}\right\}$$

Solution procedure:

1) Solve the Classical Plate equation for $w(x, y)$
2) Calculate (M_x, M_y, M_{xy})
3) Calculate stress dist. $(\sigma_x, \sigma_y, \tau_{xy})$