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Buckling of plates

moderate rotation

$$u = -z\, w_{,x} \qquad v = -z\, w_{,y} \qquad w = w(x,y)$$

$$2\phi_x = w_{,y} - v_{,z} = w_{,y} - (-w_{,y}) = 2\, w_{,y}$$

$$2\,\phi_y = u_{,z} - w_{,x} = -w_{,x} - w_{,x} = -2\, w_{,x}$$

$$2\,\phi_z = v_{,x} - u_{,y} = -z\,w_{,xy} - (-z\, w_{,xy}) = 0$$

$$\phi_x = w_{,y} \qquad \phi_y = -w_{,x} \qquad \phi_z = 0$$

$$\varepsilon_x = u_{,x} + \frac{1}{2}\left(\phi_y^2 + \phi_z^2\right) = u_{,x} + \frac{1}{2} w_{,x}^2 = \underline{-z\,w_{,xx}} + \underline{\frac{1}{2} w_{,x}^2}$$

$$\varepsilon_y = v_{,y} + \frac{1}{2}\left(\phi_x^2 + \phi_z^2\right) = v_{,y} + \frac{1}{2} w_{,y}^2 = \underline{-z\,w_{,yy}} + \underline{\frac{1}{2} w_{,y}^2}$$

$$\gamma_{xy} = u_{,y} + v_{,x} - \phi_x\,\phi_y = \underline{-2\,z\,w_{,xy}} + \underline{w_{,x}\,w_{,y}}$$

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$$W_{in} = - \iiint \sigma_x \, \delta \varepsilon_x + \sigma_y \, \delta \varepsilon_y + \tau_{xy} \, \delta \gamma_{xy} \, dz \, dx \, dy$$

$$W_{in} = (W_{in})_{bending} + (W_{in})_{stretching}$$

$$(W_{in})_s = - \iint \left(\int_{-t/2}^{t/2} \sigma_x \, dz \right) \delta \frac{w_{,x}^2}{2} + \left(\int_{-t/2}^{t/2} \sigma_y \, dz \right) \delta \frac{w_{,y}^2}{2} + \left(\int_{-t/2}^{t/2} \tau_{xy} \, dz \right) \delta (w_{,x} w_{,y}) \, dx \, dy$$

$$(W_{in})_s = - \iint \left[N_x \, \delta \frac{w_{,x}^2}{2} + N_y \, \delta \frac{w_{,y}^2}{2} + N_{xy} \, \delta (w_{,x} w_{,y}) \right] dx \, dy$$

$$(W_{in})_{inplane} = - \frac{1}{2} \iint N_x \, \delta w_{,x}^2 + N_y \, \delta w_{,y}^2 + 2 N_x N_y \, \delta (w_{,x} w_{,y}) \, dx \, dy$$

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$$\varepsilon_x = \underline{-z\, w_{,xx}} + \underline{\frac{w_{,x}^2}{2}}$$

$$(W_{in})_{inplane} = -\frac{1}{2}\iint \delta \left(N_x\, w_{,x}^2 + N_y\, w_{,y}^2 + 2N_{xy}\, w_{,x}\, w_{,y} \right) dx\, dy$$

$$(W_{in})_{inplane} = -\ \delta V_i$$

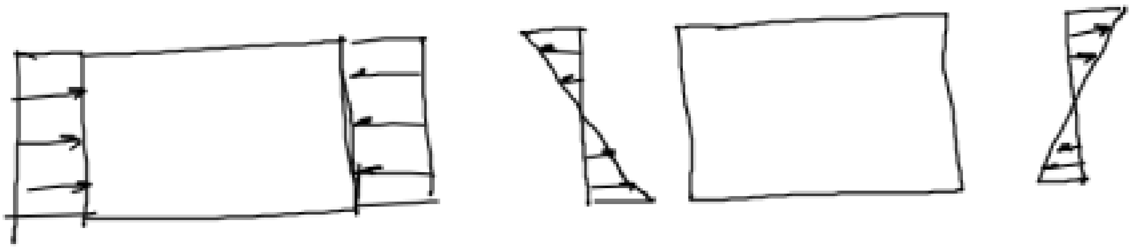
$$\underline{V_i = \frac{1}{2}\iint \left(N_x\, w_{,x}^2 + N_y\, w_{,y}^2 + 2\, N_{xy}\, w_{,x}\, w_{,y} \right) dx\, dy}$$

$$V_i = \frac{1}{2} \iint \begin{pmatrix} w_{,x} & w_{,y} \end{pmatrix} \begin{pmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{pmatrix} \begin{pmatrix} w_{,x} \\ w_{,y} \end{pmatrix} dx\, dy$$

Eqm equations:-

$$\delta\left(\underline{U + V_e} + \underline{\underline{V_i}}\right) = 0$$

$$D\left[w_{,xxxx} + 2\,w_{,xxyy} + w_{,yyyy}\right] -$$
$$\left[(N_x w_{,x})_{,x} + (N_y w_{,y})_{,y} + (N_{xy} w_{,x})_{,y} + (N_{xy} w_{,y})_{,x}\right] = q$$



$$\boxed{D\left[w_{,xxxx} + 2\,w_{,xxyy} + w_{,yyyy}\right] - N_x w_{,xx} - N_y w_{,yy} - 2N_{xy} w_{,xy} = q}$$

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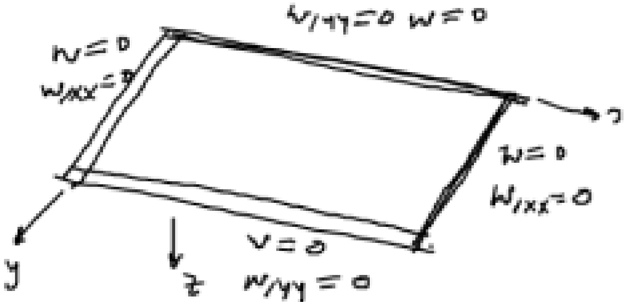
Navier Solution:-

$N_{xy} = 0$

$w = A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$



$$\left\{D\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2 + N_x\left(\frac{m\pi}{a}\right)^2 + N_y\left(\frac{n\pi}{b}\right)^2\right\} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} = q$$

$$q = \underline{a_{mn}} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$A_{mn} = \frac{a_{mn}}{D\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2 + N_x\left(\frac{m\pi}{a}\right)^2 + N_y\left(\frac{n\pi}{b}\right)^2}$$

$$q = \sum_{m,n} \underline{a_{mn}} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$w = \sum_{m,n} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

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$$\left.\begin{aligned}
\sigma_x &= \frac{N_x}{t} - \frac{12}{t^3} M_x z \\
\sigma_y &= \frac{N_y}{t} - \frac{12}{t^3} M_y z \\
\tau_{xy} &= \frac{N_{xy}}{t} - \frac{12}{t^3} M_{yx} z
\end{aligned}\right]$$

Buckling:-

$$A_{mn} = \frac{a_{mn}}{D\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2 + N_x\left(\frac{m\pi}{a}\right)^2 + N_y\left(\frac{n\pi}{b}\right)^2}$$

if $N_x \geq 0 \quad N_y \geq 0$

$$(-N_x)\left(\frac{m\pi}{a}\right)^2 + (-N_y)\left(\frac{n\pi}{b}\right)^2 = D\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2$$

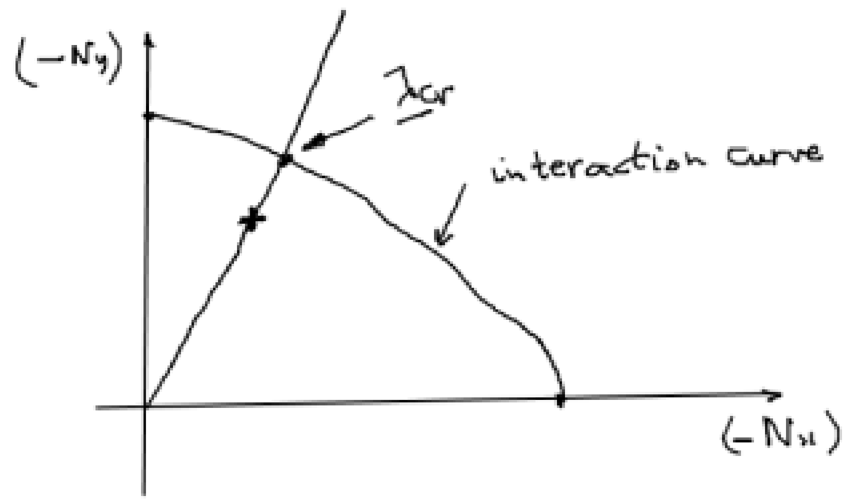
Condition for buckling.

proportional loading

$N_x = -\lambda$

$N_y = -2\lambda$

λ: load multiplier



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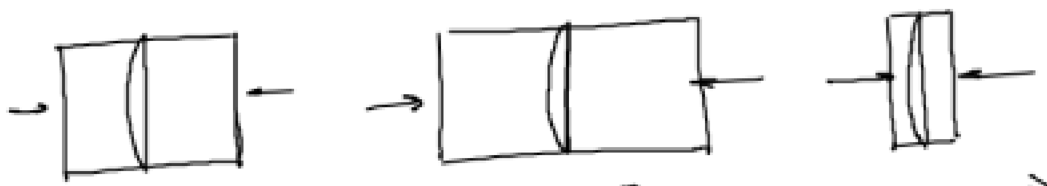
Uniaxial Compression:-

$$N_y = 0 \qquad -N_x\left(\frac{m\pi}{a}\right)^2 = D\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2$$

$$-N_x = \underbrace{\frac{\pi^2 D}{b^2}}\,\underbrace{\left[m\frac{b}{a} + \frac{n^2}{m}\frac{a}{b}\right]^2}_{K(m,n,\frac{a}{b})}$$



$$(N_x)_{cr} = K_{cr}\frac{\pi^2 D}{b^2}, \quad K_{cr} = \min_{m,n} K$$

$$K = \left[\frac{m}{r} + \frac{n^2}{m}r\right]^2, \quad \underline{n = 1}$$

$$K = \left(\frac{m}{r} + \frac{r}{m}\right)^2$$

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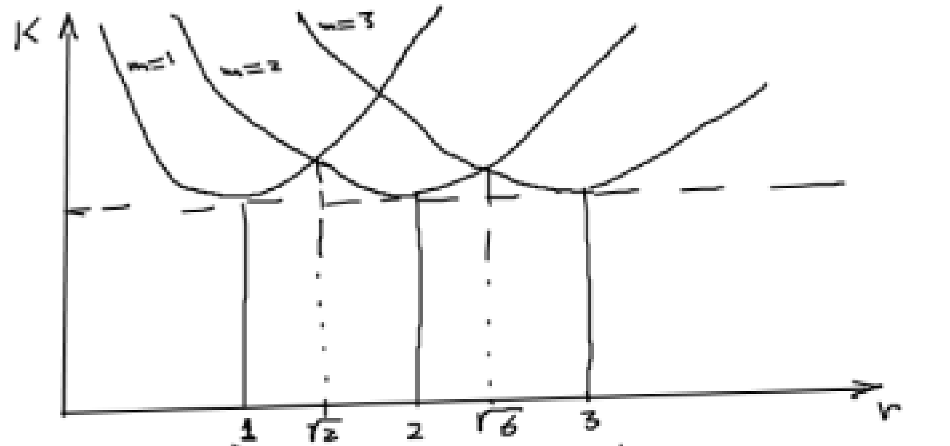


$$K = \left(\frac{m}{r} + \frac{r}{m}\right)^2 \qquad -\frac{m}{r^2} + \frac{1}{m} = 0 \Rightarrow r = m$$

$$\boxed{K_{min} = 4}$$

$$\frac{m}{r} + \frac{r}{m} = \frac{m+1}{r} + \frac{r}{m+1}$$

$$\cancel{\frac{m}{r}} + \frac{r}{m} = \cancel{\frac{m}{r}} + \frac{1}{r} + \frac{r}{m+1}$$

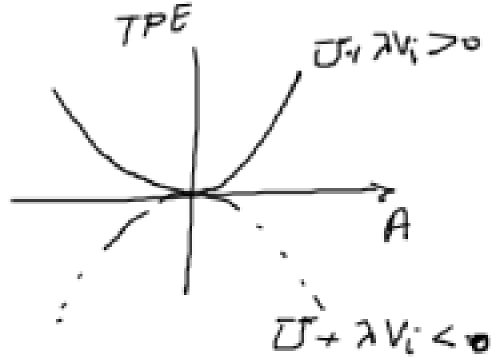
$$\frac{1}{r} = r\left[\frac{1}{m} - \frac{1}{m+1}\right] = \frac{r}{m(m+1)}$$

$$r = \sqrt{m(m+1)}$$

Buckling using energy

$$U + \underline{\lambda} V_i (N_x, N_y, N_{xy}, w)$$

$$\lambda V_i = \frac{1}{2} \iint \lambda N_x w_{,x}^2 + \lambda N_y w_{,y}^2 + 2\lambda N_{xy} w_{,x} w_{,y} \, dx\,dy$$



$$\delta (U + \underline{\lambda} V_i) = 0$$

$$TPE = A^2 (U + \lambda V_i)$$

$$A (U + \lambda V_i) = 0$$

$$\boxed{U + \underline{\lambda} V_i = 0}$$

$$\lambda_{cr} = \frac{U}{-V_i}$$

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$$V_i = \frac{1}{2} \iint \begin{pmatrix} w_{,x} & w_{,y} \end{pmatrix} \underbrace{\begin{pmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{pmatrix}}_{\text{positive semi definite}} \begin{pmatrix} w_{,x} \\ w_{,y} \end{pmatrix} dx\, dy$$

if principal "loads" are non-negative no buckling can occur.