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Stress - strain relation

Constitutive Law (relation)

$$\underline{\sigma} = \frac{\partial \psi(\underline{\varepsilon})}{\partial \underline{\varepsilon}}$$

$$\sigma_x = \frac{\partial \psi}{\partial \varepsilon_x} \qquad \sigma_y = \frac{\partial \psi}{\partial \varepsilon_y}$$

$$\frac{\partial \sigma_x}{\partial \varepsilon_y} = \frac{\partial^2 \psi}{\partial \varepsilon_y \partial \varepsilon_x} \qquad \frac{\partial \sigma_y}{\partial \varepsilon_x} = \frac{\partial^2 \psi}{\partial \varepsilon_x \partial \varepsilon_y}$$

$$=$$

$$\left(\frac{\partial \sigma_x}{\partial \varepsilon_y} = \frac{\partial \sigma_y}{\partial \varepsilon_x} \right)$$

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Case of zero initial stress

$$0 = \left.\frac{\partial \psi}{\partial \varepsilon_x}\right|_{\underline{\varepsilon}=\underline{0}} \qquad 0 = \left.\frac{\partial \psi}{\partial \varepsilon_y}\right|_{\underline{\varepsilon}=\underline{0}} \; , \; \ldots\ldots$$

$$\psi(\underline{\varepsilon}=\underline{0}) = 0$$

$$\psi(\underline{\varepsilon}) = \underbrace{\psi(\underline{0})}_{0} + \underbrace{\left.\frac{\partial \psi}{\partial \underline{\varepsilon}}\right|^{t}_{\underline{\varepsilon}=\underline{0}}}_{\underline{0}} \underline{\varepsilon} + \frac{1}{2}\underline{\varepsilon}^{t}\,\underline{\underline{C}}\,\underline{\varepsilon} + H.O.T$$

$$\psi(\underline{\varepsilon}) \sim \frac{1}{2}\underline{\varepsilon}^{t}\,\underline{\underline{C}}\,\underline{\varepsilon}$$

$\underline{\underline{C}}$: is a 6×6 symmetric matrix

$\underline{\underline{C}}$: the material stiffness matrix
elasticity matrix

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$$\boxed{\underline{\sigma} = \underline{\underline{C}}\ \underline{\varepsilon}}$$

$$\Psi = \frac{1}{2}\,\underline{\varepsilon}^{t}\,\underline{\underline{C}}\,\underline{\varepsilon}$$

$$\Psi > 0 \quad \forall\ \underline{\varepsilon} \neq \underline{0}$$

$\underline{\underline{C}}$ is symmetric & positive definite

<u>21</u> elasticities

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Isotropic material

$$\underline{\sigma} = \underline{\underline{T}}_\sigma \, \underline{\sigma}'$$

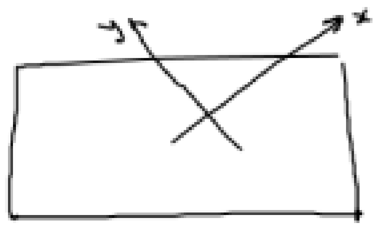
$$\underline{\varepsilon} = \underline{\underline{T}}_\varepsilon \, \underline{\varepsilon}'$$

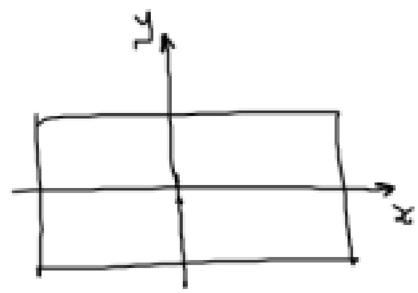
$$\underline{\sigma} = \underline{\underline{C}} \, \underline{\varepsilon}$$

$$\underline{\underline{T}}_\sigma \, \underline{\sigma}' = \underline{\underline{C}} \, \underline{\underline{T}}_\varepsilon \, \underline{\varepsilon}'$$

$$\underline{\sigma}' = \left(\underline{\underline{T}}_\sigma^{-1} \underline{\underline{C}} \, \underline{\underline{T}}_\varepsilon \right) \underline{\varepsilon}'$$

$$\underline{\underline{C}}' = \underline{\underline{T}}_\sigma^{-1} \underline{\underline{C}} \, \underline{\underline{T}}_\varepsilon \quad \#$$





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$$\underline{\underline{C}}' = \underline{\underline{C}} \quad \text{(condition of isotropy)}$$

$$\left.\begin{array}{l}\text{Young's modulus } (E) \\ \text{Poisson's ratio } (\nu)\end{array}\right\} \Longrightarrow \quad \begin{array}{l}\text{shear modulus } (G) \\ G = \dfrac{E}{2(1+\nu)}\end{array}$$

$$\varepsilon_x = \frac{\sigma_x}{E} + \left(-\nu \frac{\sigma_y}{E}\right) + \left(-\nu \frac{\sigma_z}{E}\right)$$

$$\varepsilon_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \left(-\nu \frac{\sigma_z}{E}\right)$$

$$\varepsilon_z = -\nu \frac{\sigma_x}{E} + \left(-\nu \frac{\sigma_y}{E}\right) + \frac{\sigma_z}{E}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G} \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

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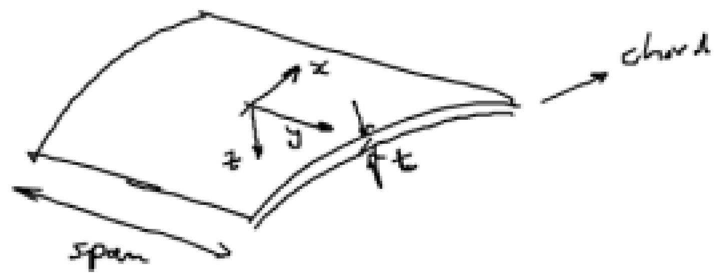
Plane stress:-

$\sigma_z = 0 \quad \tau_{xz} = 0 \quad \tau_{yz} = 0$



$$\gamma_{xz} = \frac{\tau_{xz}}{G} = 0$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = 0$$

$$\left.\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \\ \varepsilon_y &= -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} \\ \gamma_{xy} &= \frac{\tau_{xy}}{G} \end{aligned}\right\}$$

$$\epsilon_z \neq 0$$

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

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Strain energy expression for plane stress:

$$W_{in} = -\int \left(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy} + \underline{\sigma_z} \delta\varepsilon_z + \underline{\tau_{yz}} \delta\gamma_{yz} + \underline{\tau_{zx}} \delta\gamma_{zx} \right) dV$$

$$\boxed{W_{in} = -\int \sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy} \quad dV}$$

$$W_{in} = -\int \delta\Psi \, dV$$

$$\Psi = \frac{1}{2}\int \frac{E}{1-\nu^2}\left(\varepsilon_x^2 + \varepsilon_y^2 + 2\nu\,\varepsilon_x \varepsilon_y + \left(\frac{1-\nu}{2}\right)\gamma_{xy}^2 \right) dV$$