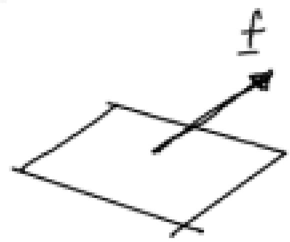
Public Page

Stress:

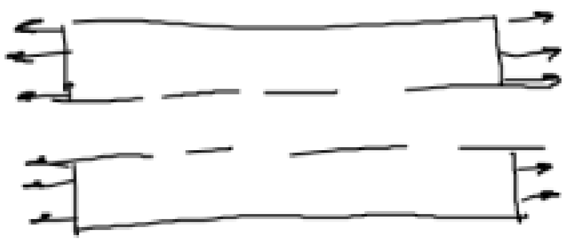
stress ⇒ force per unit area

~~stress~~ traction

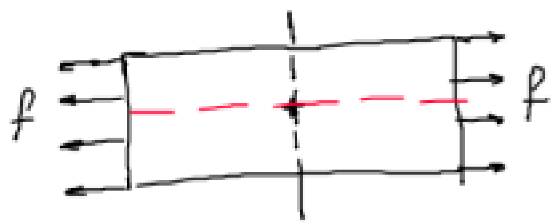


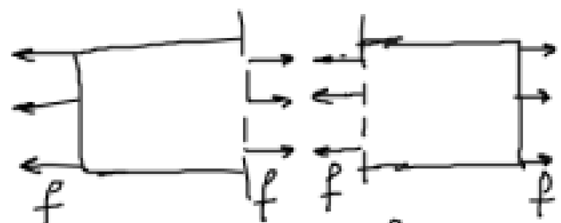
$$\underline{t} = \frac{\underline{f}}{A}$$

$$d\underline{f} = \underline{t}\, dA$$



$t = 0$





$$t = \frac{f}{A}$$

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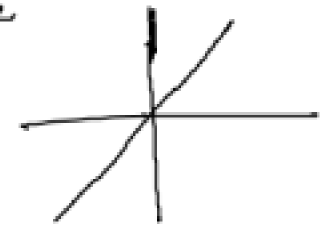
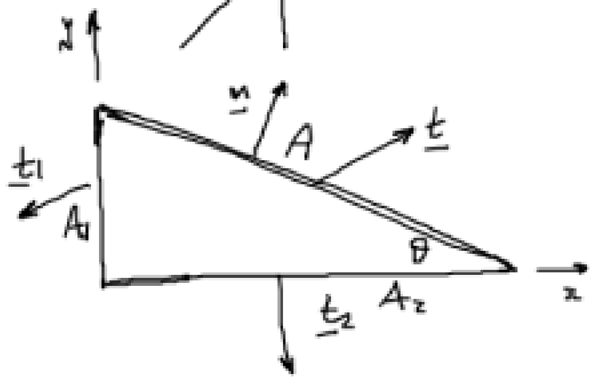
The <u>orientation</u> of the cut we make at a point

$\underline{n}$ (unit vector, normal to the plane of the cut)



Traction at a given point

$$\underline{t}\ (\underline{n})$$



$$\underline{t}\,A + \underline{t}_1\,A_1 + \underline{t}_2\,A_2 = \underline{0}$$

$$\underline{t} = (-\underline{t}_1)\,\frac{A_1}{A} + (-\underline{t}_2)\,\frac{A_2}{A}$$

$$\underline{t} = \sin\theta\,(-\underline{t}_1) + \cos\theta(-\underline{t}_2) \quad , \quad \underline{n} = (\sin\theta, \cos\theta)$$

$$\underline{t} = (-\underline{t}_1)\,n_x + (-\underline{t}_2)\,n_y$$

$$\boxed{\underline{t} = \underline{\underline{\sigma}}\ \underline{n}}$$

$\underline{\sigma}$: stress

second order tensor

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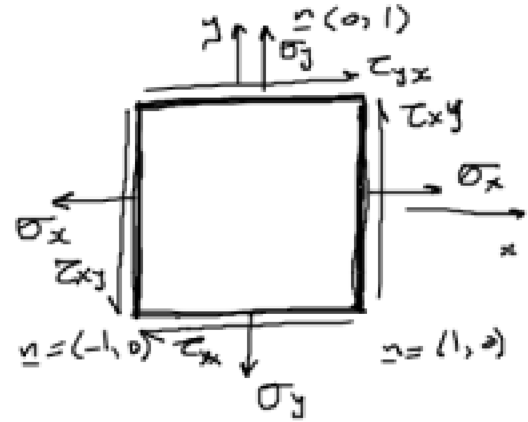
$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_x & \tau_{yx} \\ \tau_{xy} & \sigma_y \end{pmatrix}$$

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix}$$



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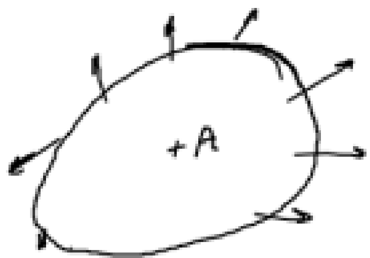
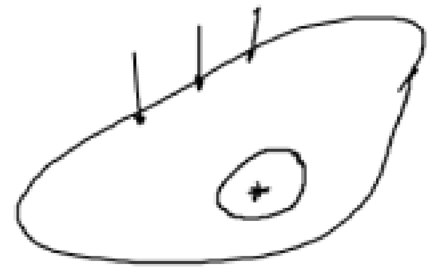
$\underline{b}$ force per unit volume





p (part of the body)

∂p (surface of the part)

$$\underline{f}_c = \int_{\partial p} \underline{t} \, dA \qquad \underline{f}_b = \int_p \underline{b} \, dV$$

$$\underline{f}_c + \underline{f}_b = \underline{0}$$

$$\underline{\int_{\partial p} \underline{\underline{\sigma}} \cdot \underline{n} \, dA} + \int_p \underline{b} \, dV = \underline{0}$$

$$\int_p \underline{\nabla} \cdot \underline{\underline{\sigma}} \, dV + \int_p \underline{b} \, dV = \underline{0} \implies \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{b} = \underline{0}$$

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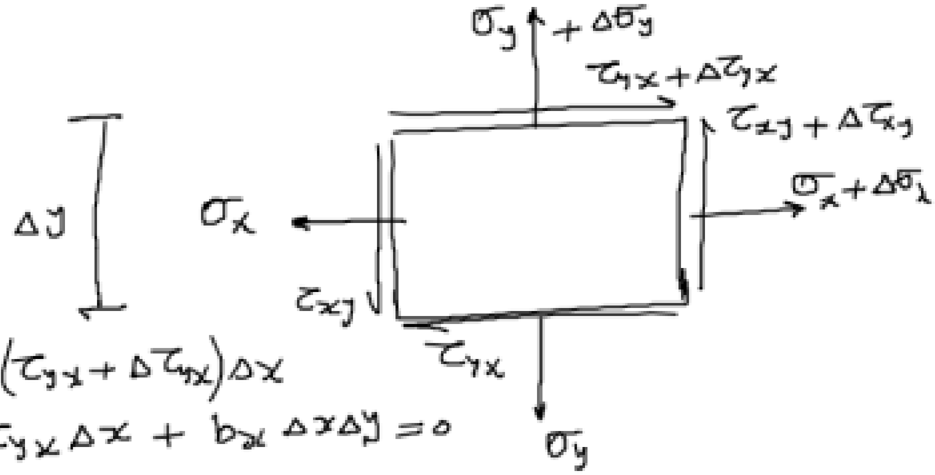

$$(\sigma_x + \Delta\sigma_x)\,\Delta y + (\tau_{yx} + \Delta\tau_{yx})\,\Delta x - \sigma_x\,\Delta y - \tau_{yx}\,\Delta x + b_x\,\Delta x\,\Delta y = 0$$

$$\Delta\sigma_x\,\Delta y + \Delta\tau_{yx}\,\Delta x + b_x\,\Delta x\,\Delta y = 0$$

$$\frac{\Delta\sigma_x}{\Delta x} + \frac{\Delta\tau_{yx}}{\Delta y} + b_x = 0$$

$$\left.\begin{aligned}\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + b_x &= 0\\ \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + b_y &= 0\end{aligned}\right\}$$

$$\underline{\underline{\nabla \cdot \underline{\underline{\sigma}} + \underline{b} = \underline{0}}}$$

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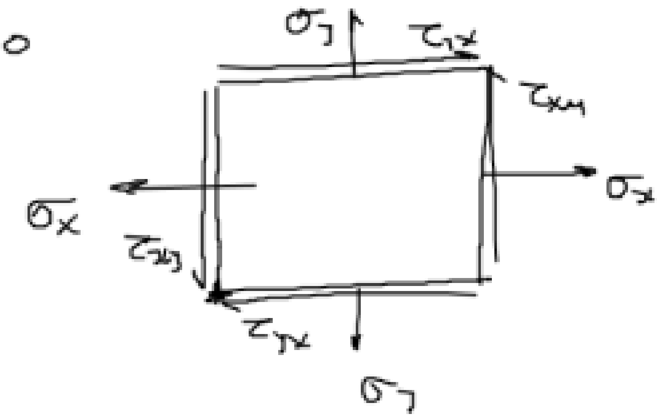
$$(\tau_{xy}\,\Delta y)\Delta x - (\tau_{yx}\,\Delta x)\,\Delta y = 0$$

$$\tau_{xy} - \tau_{yx} = 0$$

$$\tau_{yz} - \tau_{zy} = 0$$

$$\tau_{zx} - \tau_{xz} = 0$$



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6 strain

6 stress

3 disp.

6 strain. disp.

3 eq^{m}

Material Law

Constitutive Law

$$6 \,|\, \underline{\sigma} = f(\underline{\varepsilon})$$

elastic body

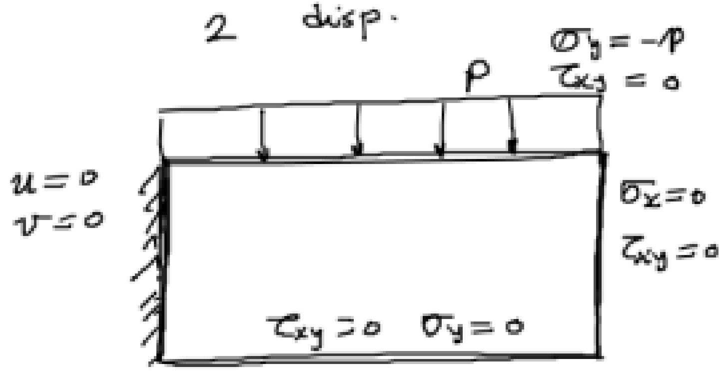
* displacements as primary unknowns.

* stresses as primary unknowns

3 strain

3 stress

2 disp.



1- deformation ?

2- stress distribution ?

3- stability ?

Compatibility → 1 (2D), 3 (3D)

Eq^{m} → 2 (2D), 3 (3D)