AE4520: Advanced Structural Analysis

Some Quick Math.

Mostafa Abdalla

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$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & \cdots \\ A_{21} & A_{22} & A_{23} & \cdots \\ \vdots & \vdots & \ddots & \\ \vdots & \vdots & & \ddots \end{bmatrix}_{n \times n} \qquad A^T_{ij} = A_{ji}$$

$$A^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} & \cdots \\ A_{12} & A_{22} & A_{32} & \cdots \\ \vdots & \vdots & \ddots & \\ \vdots & \vdots & & \ddots \end{bmatrix}_{n \times n}$$

$A = A^T \implies$ symmetric

$A = -A^T \implies$ Anti, skew symmetric

$A = B + C \qquad B = \frac{1}{2}(A + A^T)$

$A^T = B - C \qquad C = \frac{1}{2}(A - A^T)$

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

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quadratic forms

$$Q(x) = x^t A x$$

if A is skew symmetric ?

$$Q = x^T A x = x^T A^T x = -x^T A x = -Q$$

$$\Longrightarrow \quad Q \equiv 0$$

Q is positive definite iff $Q(x) > 0 \quad \forall x \neq 0$

Q " semi definite iff $Q(x) \geq 0 \quad \forall x \neq 0$

A is pd iff $x^t A x > 0 \quad \forall x \neq 0$

A is psd iff $x^t A x \geq 0 \quad \forall x \neq 0$

Learning Objectives:

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Recall

- Symmetric and Skew symmetric parts of a matrix
- Positive and Semi-positive definite matrices
- Transforming components of a vector
- Rotation matrices, orthogonality
- Eigenvalue problem
- Transforming components of a tensor

TU Delft

Challenge the future 2

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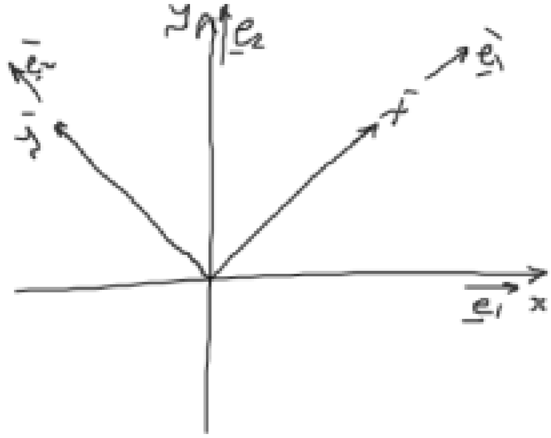

$$\underline{v} = v_1 \underline{e}_1 + v_2 \underline{e}_2$$

$$\underline{v} = v_1' \underline{e}_1' + v_2' \underline{e}_2'$$

$$v_1 \underline{e}_1 + v_2 \underline{e}_2 = v_1' \underline{e}_1' + v_2' \underline{e}_2'$$

$$v_1 = v_1'(\underline{e}_1' \cdot \underline{e}_1) + v_2' (\underline{e}_2' \cdot \underline{e}_1)$$

$$v_2 = v_1' (\underline{e}_1' \cdot \underline{e}_2) + v_2' (\underline{e}_2' \cdot \underline{e}_2)$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \underline{\underline{R}} \begin{pmatrix} v_1' \\ v_2' \end{pmatrix} \longrightarrow \#1$$

rotation matrix

$$\underline{\underline{R}}^t \underline{\underline{R}} = \underline{\underline{I}} \qquad \underline{\underline{R}}^t = \underline{\underline{R}}^{-1}$$

$$|\underline{\underline{R}}|\,|\underline{\underline{R}}| = 1 \implies |\underline{\underline{R}}| = \pm 1$$

$$\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \underline{\underline{R}}^T \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \longrightarrow \#2$$

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$\underline{\underline{B}} = \underline{\underline{I}}$, standard eigenvalue problem

$$\underline{\underline{R}} = [\underline{x}_1 \mid \underline{x}_2 \mid \cdots \mid \underline{x}_n]$$

$$Q = \underline{x}^t \underline{\underline{A}} \, \underline{x} = (\underline{\underline{R}} \underline{x}')^t \underline{\underline{A}} (\underline{\underline{R}} \underline{x}') = \underline{x}'^t (\underline{\underline{R}}^t \underline{\underline{A}} \, \underline{\underline{R}}) \underline{x}'$$

$$Q = \underline{x}'^t \underline{\underline{A}}' \underline{x}' =$$

$$\Longrightarrow \quad \underline{\underline{A}}' = \underline{\underline{R}}^t \underline{\underline{A}} \, \underline{\underline{R}}$$

in eigen-basis

$\underline{\underline{A}}'$ is diagonal.

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Rayleigh Quotient:-

$$\rho = \frac{x^t A x}{x^t B x} \qquad B \text{ is pd}$$

$$(\underline{A} - \lambda \underline{B}) \underline{x} = \underline{0}$$

$\lambda \Rightarrow$ eigenvalues , $\underline{x} \Rightarrow$ eigen vectors.

1 – there are $\underline{n}$ eigenvalues

2 – " " " eigenvectors

3– $\underline{x}_i^t \underline{A} \underline{x}_j = 0$, $i \neq j$

$\underline{x}_i^t \underline{B} \underline{x}_j = 0$, $i \neq j$

4– $\underline{x}_i^t \underline{B} \underline{x}_i = 1$