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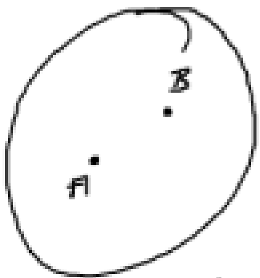
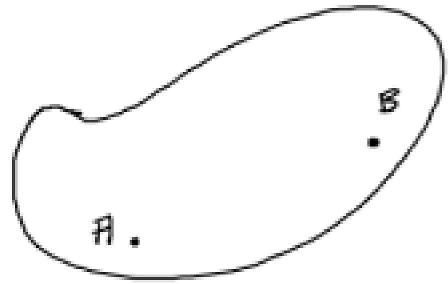
Deformation Gradient



undeformed



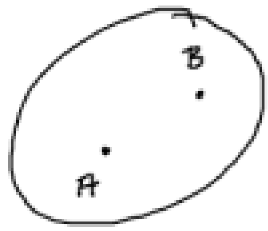
deformed

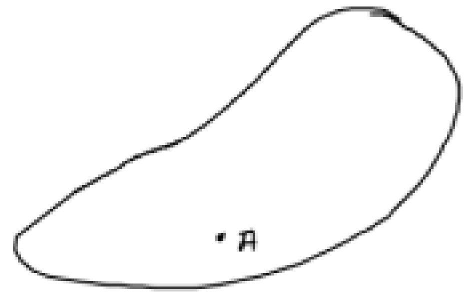
$$\underline{r}_A = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \underline{r}_B = \begin{pmatrix} x + dx \\ y + dy \\ z + dz \end{pmatrix}$$

$$\underline{r}'_A = \begin{pmatrix} x + u_A \\ y + v_A \\ z + w_A \end{pmatrix} \qquad \underline{r}'_B = \begin{pmatrix} x + dx + u_B \\ y + dy + v_B \\ z + dz + w_B \end{pmatrix}$$

$$\underline{e} = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} \qquad \underline{e}' = \begin{pmatrix} dx + du \\ dy + dv \\ dz + dw \end{pmatrix}$$

$$\underline{e} \longrightarrow \underline{e}'$$





$\underline{u}(x, y, z)$

$v(x, y, z)$

$w(x, y, z)$

$$du = u(x + dx, y + dy, z + dz) - u(x, y, z)$$

$$du = u_{,x}\, dx + u_{,y}\, dy + u_{,z}\, dz$$

displacement gradient

$$\underline{\underline{H}} = \begin{bmatrix} u_{,x} & u_{,y} & u_{,z} \\ v_{,x} & v_{,y} & v_{,z} \\ w_{,x} & w_{,y} & w_{,z} \end{bmatrix}$$

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$$\underline{e}' = \begin{pmatrix} dx + du \\ dy + dv \\ dz + dw \end{pmatrix} \qquad \underline{e} = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

$$\underline{e}' = \underline{e} + \underline{\underline{H}}\,\underline{e}$$

$$\underset{\text{deformed}}{\underline{e}'} = \underbrace{(\underline{\underline{I}} + \underline{\underline{H}})}_{\underline{\underline{F}}}\,\underset{\text{undeformed}}{\underline{e}}$$

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$$\ell^2 = \|\underline{e}\|^2 = \underline{e}^t \underline{e}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1^2 + 2^2 = 5$$

$$\|\underline{e}'\|^2 = \underline{e}'^t \underline{e}' = (\underline{\underline{F}}\,\underline{e})^t (\underline{\underline{F}}\,\underline{e})$$

$$\ell'^2 = \underline{e}^t (\underline{\underline{F}}^t \underline{\underline{F}})\, \underline{e}$$

$$\underline{\underline{C}} = \underline{\underline{F}}^t \underline{\underline{F}}$$ Cauchy tensor

$$r = \left(\frac{\underline{e}^t \underline{\underline{C}}\, \underline{e}}{\underline{e}^t \underline{e}} \right)$$

$$(\underline{\underline{C}} - \lambda^2 \underline{\underline{I}})\, \underline{e} = \underline{0}$$

in principal strain directions

$$\underline{\underline{C}} = \begin{bmatrix} \lambda_1^2 & & 0 \\ & \lambda_2^2 & \\ 0 & & \lambda_3^2 \end{bmatrix} \qquad \underline{\underline{U}} = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{bmatrix}$$

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Stretch tensor $\underline{\underline{U}}$

$$\underline{\underline{U}}^2 = \underline{\underline{C}}$$

$$A = F U^{-1} \qquad A^t A = U^{-1} F^t F U^{-1} = \underline{\underline{U}}^{-1} \underline{\underline{C}}\, \underline{\underline{U}}^{-1} = \underline{\underline{I}}$$

$$R = F U^{-1}$$

$$\underline{\underline{F}} = \underline{\underline{R}} \quad \underline{\underline{U}}$$

rigid rotation   a stretch

Polar decomposition

$$\underline{\underline{I}} + \underline{\underline{H}} = \underline{\underline{R}} \quad \underline{\underline{U}}$$

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$$\underline{\underline{U}} = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{bmatrix} \qquad \underline{\underline{E}} = \begin{bmatrix} \lambda_1 - 1 & & 0 \\ & \lambda_2 - 1 & \\ 0 & & \lambda_3 - 1 \end{bmatrix}$$

(1) $$\boxed{\underline{\underline{E}} = \underline{\underline{U}} - \underline{\underline{I}}}$$

$$\underline{\underline{C}} = \begin{bmatrix} \lambda_1^2 & & 0 \\ & \lambda_2^2 & \\ 0 & & \lambda_3^2 \end{bmatrix} \qquad \underline{\underline{E}}_g = \begin{bmatrix} \frac{(\lambda_1^2 - 1)}{2} & & 0 \\ & \frac{\lambda_2^2 - 1}{2} & \\ 0 & & \frac{\lambda_3^2 - 1}{2} \end{bmatrix}$$

(2) $$\boxed{\underline{\underline{E}}_g = \frac{1}{2}\left(\underline{\underline{C}} - \underline{\underline{I}}\right)}$$

$$\underline{\underline{C}} = \underline{\underline{F}}^t \underline{\underline{F}}, \qquad \underline{\underline{F}} = \underline{\underline{I}} + \underline{\underline{H}}$$

$$\boxed{\underline{\underline{E}}_g = \frac{1}{2}\left(\underline{\underline{H}} + \underline{\underline{H}}^t\right) + \frac{1}{2}\underline{\underline{H}}^t \underline{\underline{H}}}$$