

Public Page

Advanced Structural Analysis AE4520

# Advanced Structural Analysis AE4520

Mostafa Abdalla



Challenge the future 1

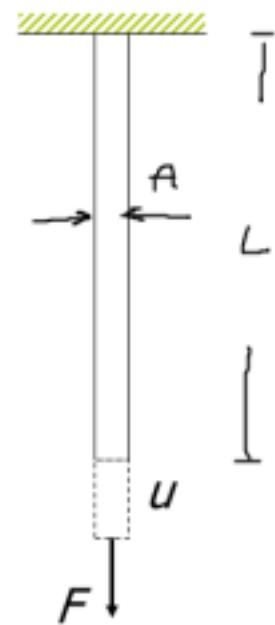
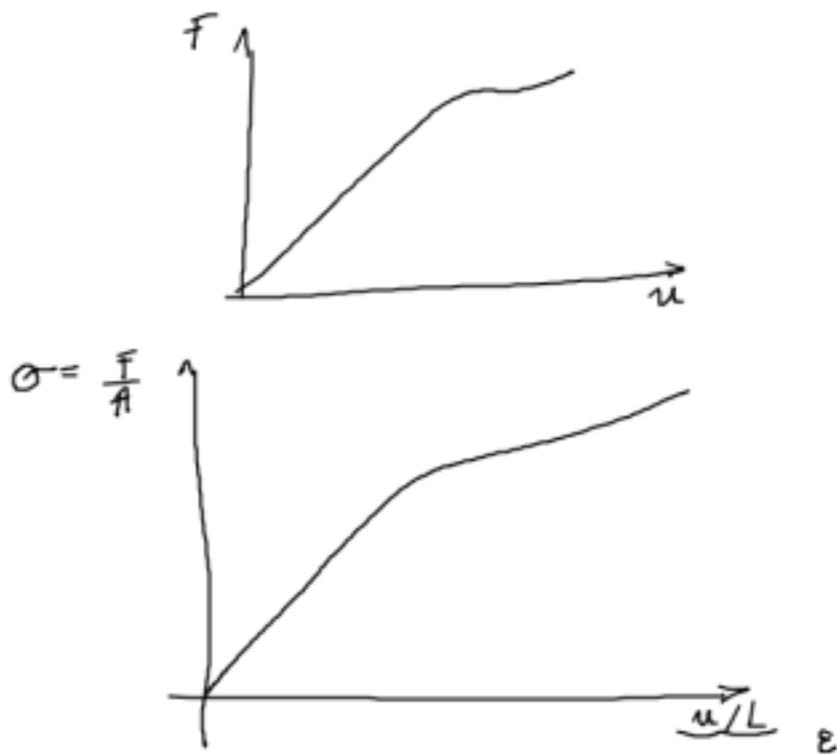
## Learning Objectives

# Learning Objectives

- Understand the definition of stress and strain
- Understand the relation between strain and displacement in bars
- Understand the possibility of different definitions of strain
- Understand the effect of rigid rotation on strain definition

## The uniaxial test

## The uniaxial test



## Definitions

# Definitions

- Stress
  - Force per unit area
- Strain
  - Displacement per unit length
  - Change in length divided by original length

The inclined bar

## The inclined bar

$$\ell^z = \sqrt{x^2 + z^2}$$

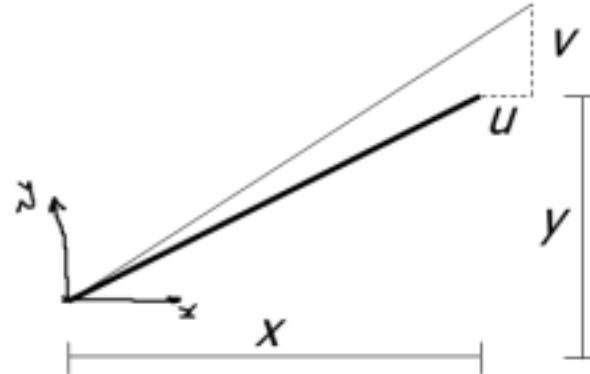
$$(x, z)$$

$$(x+u, z+v)$$

$$\ell^z = \sqrt{(x+u)^2 + (z+v)^2}$$

$$\varepsilon = \frac{\ell - \ell_0}{\ell_0}$$

$$\ell^z = \ell_0 + \sqrt{2xu + 2zu} + \sqrt{u^2 + v^2}$$



## Displacements and strains

# Displacements and strains

$$\varepsilon = \frac{u}{l_0}$$

$x$  —————  $x + \Delta x$

$$x + u(x) \qquad \qquad x + \Delta x + u(x + \Delta x)$$

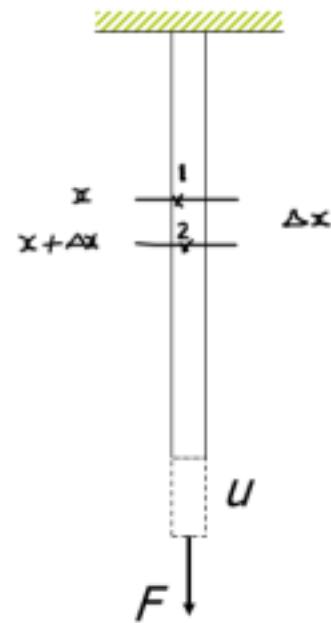
$$l_0 = \Delta x$$

$$l = \Delta x + u(x + \Delta x) - u(x)$$

$$l = \Delta x \left( 1 + \frac{u(x + \Delta x) - u(x)}{\Delta x} \right)$$

$$l = \Delta x \left( 1 + \frac{du}{dx} \right)$$

$$\varepsilon = \frac{du}{dx}$$



## The inclined bar revisited

## The inclined bar revisited

$$(s \cos \theta, s \sin \theta) \quad ((s+\Delta s) \cos \theta, (s+\Delta s) \sin \theta)$$

$$(s \cos \theta + u(s), s \sin \theta + v(s))$$

$$(s+\Delta s) \cos \theta + u(s+\Delta s), (s+\Delta s) \sin \theta + v(s+\Delta s))$$

$$\vec{r}_{12} = \Delta s (\cos \theta, \sin \theta)$$

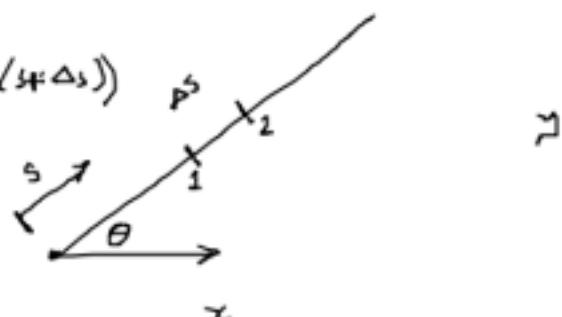
$$\vec{r}'_{12} = \Delta s \left( \cos \theta + \frac{du}{ds}, \sin \theta + \frac{dv}{ds} \right)$$

$$|\vec{r}_{12}| = \Delta s$$

$$|\vec{r}'_{12}| = \Delta s \sqrt{(\cos \theta + u')^2 + (\sin \theta + v')^2}$$

$$\varepsilon = \sqrt{(\cos \theta + u')^2 + (\sin \theta + v')^2} - l$$

$$u = s \cos \theta \quad v = s \sin \theta \quad \varepsilon = s'$$



Multiple Definitions of ??strain??

## Multiple Definitions of “strain”

$$\Delta \varepsilon_t = \frac{\Delta u}{l_0 + u}$$

$$\frac{du}{du} = \frac{1}{l_0 + u}$$

$$\varepsilon_t = \ln l_0 + u + c$$

$$\delta = \ln l_0 + c$$

$$\varepsilon_t = \ln l_0 + u - \ln l_0 = \ln 1 + \frac{u}{l_0}$$

$$\varepsilon_t = \ln(1 + \epsilon)$$

regular (conventional)  
engineering strain



## Green Strain

## Green Strain

$$\varepsilon_g = \frac{l^2 - l_0^2}{2 l_0^2} = \frac{(l - l_0)(l + l_0)}{2 l_0}$$

$$\varepsilon_g = \frac{1}{2} \varepsilon (2 + \varepsilon) = \varepsilon + \frac{1}{2} \varepsilon^2 \quad \leftarrow$$

$$\varepsilon_t = \ln(1 + \varepsilon) = \varepsilon + \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} + \dots$$

$$\varepsilon_g = \frac{1}{2} \left[ (\cos\theta + u)^2 + (\sin\theta + v)^2 - 1 \right]$$

$$\varepsilon_g = \cos\theta u + \sin\theta v + \frac{1}{2} (u^2 + v^2)$$

$$\varepsilon \approx \cos\theta u + \sin\theta v$$

## Summary

# Summary

- Stress is **force per unit area**
- Strain is **change in length divided by original length**
- Strain depends on the **rate of change of displacements**
  - **Rate of change wrt space not time!**
- Strain-displacement relation is always linear if no rotations are involved
- Multiple strain definitions are possible
- Green strain is **always** quadratic in displacements
- If we neglect quadratic terms we get **approximate linear** strain displacement relations