Exam CIE-4821-09
Traffic Flow Theory and Simulation

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The exam has 5 questions. 53 points can be obtained, which are specified per question and subquestion. Some questions might require more time than others, so *use your time wisely*! The total time available for this exam is 3 hours.

Remarks:

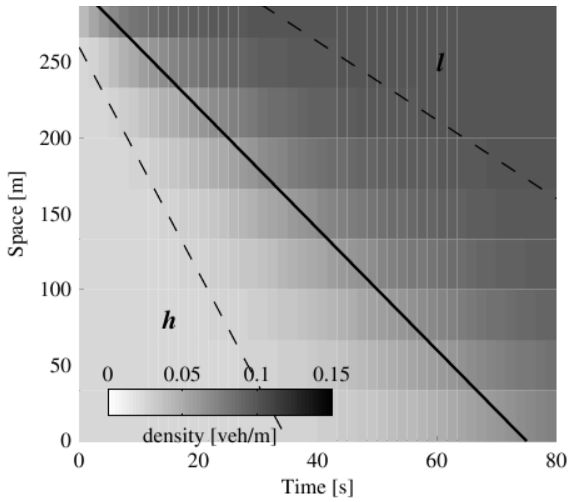
- Allowed: calculator (but no smartphones...), self-made equation sheet (1 double sided A4 max)
- Put labels at all your graph axes.
- If a *sketch* is asked, there is no need for an exact drawing. Do make sure, though, that it is clear whether points lie higher or lower or on one line, and that this is correct.
- Your answer will be judged on the good elements in there, but for all wrong answers points will be deducted.
- For some questions, an indicative number of words is given as guidance for the required level of detail. Your answer may be shorter or longer.
- Make sure you provide the calculus procedure as well as the result in order to get the maximum points.

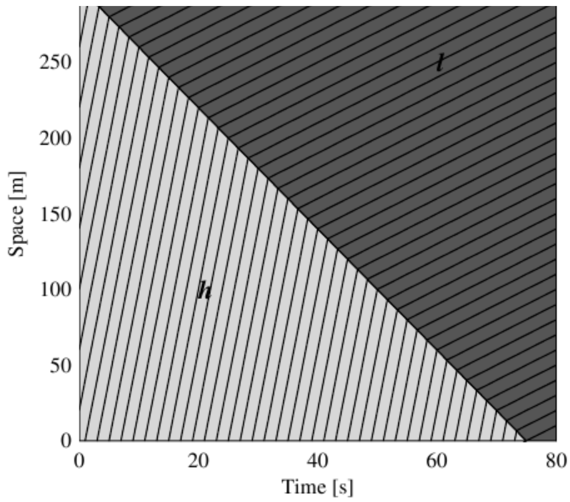
Question	Points
1	6
2	6
3	19
4	12
5	10
Total:	53

1. Short open questions

Total for Question 1: 6

Below, two simulation results of a homogeneous congested area spilling back upstream are plotted. You see a high density, congested area, indicated with *h* downstream of x=0 at t=0, and a low density, uncongested area, indicated by *l*. The driving direction is up. One of the figures plots the results of a simulation in Eulearian coordinates and one of a simulation in Lagrangian coordinates.





(a) **Explain which of the two is Eulerian and which one Lagrangian.** (1)

> **Solution:** The left one is Eulerian. Is shows numerical diffusion. Moreover, the grid structure in time and space is clearly visible, whereas in Lagrangian coordinates the grids follow a fixed time and the vehicles (trajectories are visible in the right figure) (1 point for one of the explanations)

(b) **Give the two main advantages of using Lagrangian coordinates.** (2)

> **Solution:** The representation of traffic flow equations is more accurate (no numerical diffusion) (1); also, the calculations are more efficient since it is not needed to consider both the upstream and downstream cell as with Eulerian coordinates (e.g., CTM) (1)

(c) **Explain what a pce value is. (30 words)** (1)

> **Solution:** The relative amount of space (in time! -0.5 if not mentioned) that a non-passenger car vehicle occupies on the road

(d) **How does the pce value of trucks depend on the speed, qualitatively (i.e., does it increase, decrease, ...). Give your reasoning** (2)

Solution: Truck are longer, which influences the pce value. If the pce is the ratio of the length + net headway, assuming the same net headway the pce value increases with deceasing speed.

2. Multi-leader car-following models

Total for Question 2: 6

Consider the IDM car-following model, prescribing the following acceleration:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a_0 \left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right) \tag{1}$$

In this equation, v is the speed of the following vehicle, v_0 a reference speed, s the distance headway and s^* a reference distance headway as function of speed v and the difference in speed with the leader, Δv:

$$s^*(v, \Delta v) = s_0 + v\,T + \frac{v\,\Delta v}{2\sqrt{a\,b}} \tag{2}$$

In this equation s_0 is a desired distance at standstill, and a is a reference acceleration and b is the maximum comfortable braking.

(a) **Explain in words the working of this car-following model; i.e. comment on the acceleration. Only comment on equation 1. (indication: 50 words)** (2)

> **Solution:** Vehicles accelerate in principle with acceleration a, but this reduces when they approach their desired speed (1) or if they approach their desired distance (1). The desired distance increases with speed.

(b) **Explain in words what is meant with multiple-leader car-following models. Only comment on the multi-leader part. (indication: 50 words)** (1)

> **Solution:** In regular cf-models, the drivers react on actions by their leader. In multiple-leader models, drivers take several leaders into account in determining their action

(c) **Reformulate the IDM model into a multiple-leader car-following model by including two leaders. Give your reasoning, and, for all points, formulate those reasonings into equations.** (3)

> **Solution:** Calculate for both leaders the desired distance:
>
> $$s_1^*(v, \Delta v) = s_0 + v\,T + \frac{v\,\Delta v}{2\sqrt{a\,b}} \tag{3}$$
>
> and
>
> $$s_2^*(v, \Delta v) = s_0 + 2 * v\,T + \frac{v\,\Delta v}{2\sqrt{a\,b}} \tag{4}$$
>
> Note that the desired distance to the second leader has twice the speed component.(1) Each of these will lead to an acceleration:
>
> $$a_1 = a_0 \left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s_1^*(v, \Delta v)}{s}\right)^2\right) \tag{5}$$

$$a_2 = a_0 \left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s_2^*(v, \Delta v)}{x_2 - x_0}\right)^2\right) \qquad (6)$$

(1) (Note the desired distance to the second leader is compared with the distance between the second leader and current vehicle) These have to be combined into one acceleration (0.5). For safety, we choose the minimum acceleration rather than for instance the mean:

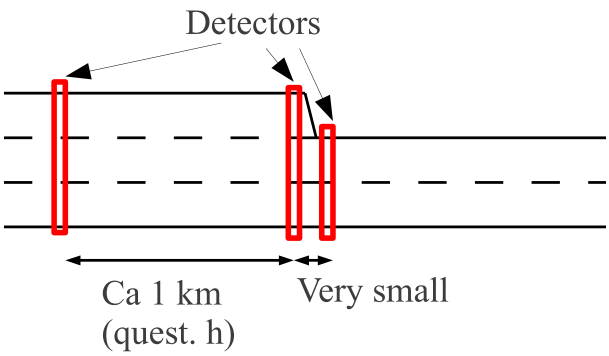
$$\frac{\mathrm{d}v}{\mathrm{d}t} = min\{a_1, a_2\} \qquad (7)$$

(0.5)

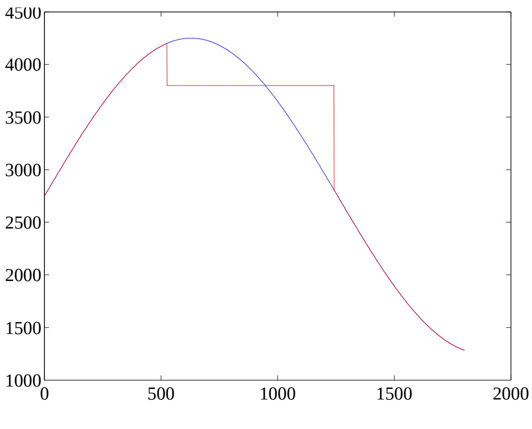
3. Measuring the speed at a cross section

Total for Question 3: 19

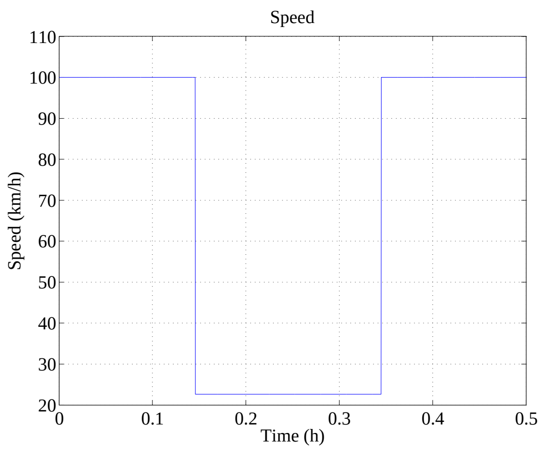
Consider the following road layout with a lane drop from 3 to 2 lanes.



The demandm as well as the flow at the downstream detector, is given in the figure below. The speed at the detector, located just upstream (you may assume no spacing between the bottleneck and the detector) of the bottleneck, is as follows.



(a) Flow and demand



(b) Speed

In the question, you might need values from the graph. Slightly misreading the graph is not a problem, but *mention the values you directly read from the graph and where you find these*.

(a) **In figure a with the flow and the demand, which line is the demand and which line the flow. Argue why** (1)

Solution: The edgy line is the flow, since the demand will never exceed the flow

(b) **Give the free flow capacity (and the reasoning how you find it)** (2)

Solution: The free flow capacity can be found by the flow at the upstream detector before congestion sets in (1), so 4200 veh/h (read from the graph at the time congestion sets in)(1)

(c) **Give the queue discharge rate. (and the reasoning how you find it)** (2)

Solution: The queue discharge rate can be found by the flow after congestion has set in (1), so 3800 veh/h (1)

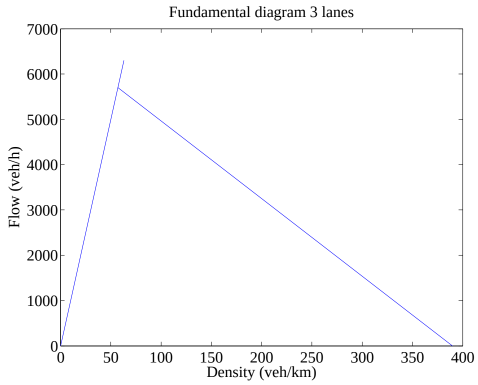
Assume the fundamental diagram per lane is the same for all lanes, at all locations in this setting.

(d) **Explain why can this situation not be described with a triangular fundamental diagram.** (1)

Solution: The free flow capacity is higher than the queue discharge rate, i.e., there is a capacity drop (1 for either explanation)

(e) **Draw the simplest fundamental diagram possible for the three lane section (aggregated over all lanes). Explain how you find the values for the relevant points, and give calculate them.** (5)

Solution: The FD should have a capacity drop (see previous questions), so the simplest is to assume an inverse lambda shaped FD (0.5 pt either by naming or in drawing) The free flow capacity is found in question a: 3x2100 veh/h. Also the free flow speed is known (from the speed figure): 100 km/h (0.5 for both combined). This gives the free flow branch (0.5 point for drawing). The density matching the queue outflow rate can be found on the free flow branch, by looking up the density for the queue outflow rate (0.5). Kc=qc/vfree=1900/100=19 veh/km. (multiply by 3 for the 3-lane section (1)). Furthermore, it is known that with a flow of 2 lanes x 1900 veh/h/lane (the flow through the bottleneck), the speed is 22,6 km/h (read from graph) (1 pt), which gives a density of 3800/22.6=168 veh/km (0.5 pt). Now, the congested branch can be constructed. We find the wave speed w=$\Delta q/\Delta k$=((3*1900)-(2*1900))/(3*19 -168)=-17 km/h. The jam density is found by kj=Kc-qc/w=3*(19-1900/-17.1)=390 veh/km (0.5 pt).

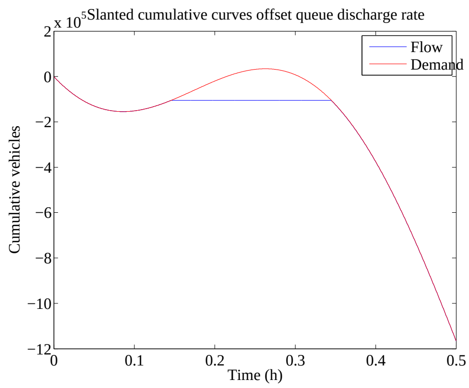


Now the delay is analysed by slanted cumulative curves.

(f) **Explain the offset you choose.** (1)

Solution: The most useful offset is the maximum flow of the road which is maintained for a while, i.e. the queue discharge rate (1 pt). (Alternative: the free flow capacity: 1/2)

(g) **Sketch the slanted cumulative curve of the flow and the demand for the situation at hand.** (2)



Solution:

(h) **Indicate in the figure how you can determine the delay.** (1)

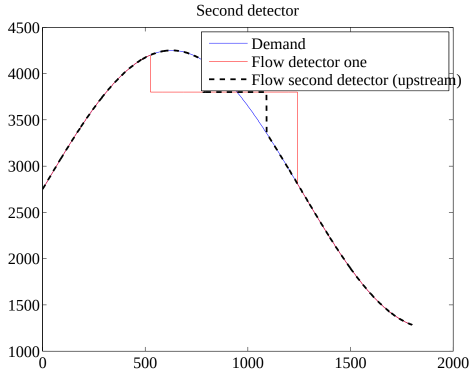
Solution: This is the area between the curves

Now a second detector is constructed approximately 1 km upstream of the bottleneck.

(i) **Sketch the demand as function of time (i.e., copy the second figure of this question, indicate the times of speed change in the graph - no points given) and in the** (2)

same graph, sketch the resulting flow at this second detector. Pay attention to the times at which the flow changes compared to the time of the speed changes.

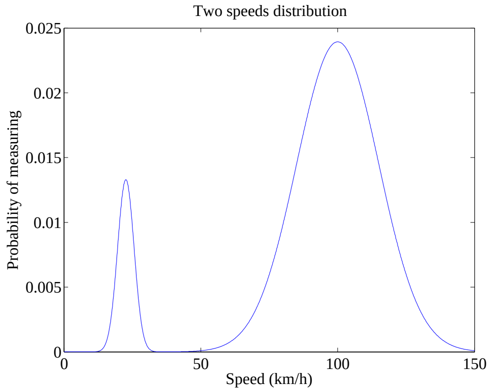
Solution: It takes a while before the congestion spills back to the second detector. Similarly, congestion resolves from the tail, so the flow starts following the demand already earlier in time



We now relax the assumption of all traffic states at the fundamental diagram to a more realistic situation (i.e., including demand and supply variations). In this situation, We measure the one-minute aggregated (harmonically averaged) speeds during one month.

(j) **Sketch a probability distribution or histogram of these speeds. Explain the shape** (2)

Solution: If we do not relax the assumption of the fundamental diagram, there are only two speeds: the free flow speed and the speed for the state in the congested branch where the flow matches the queue discharge rate for the 2 lane stretch. (1) The frequency at which they are measured depends on the relative time of congestion (typically, 2 peaks at each 1 hour of congestion, so approximately 10% of the time congestion) (0.5). Relaxing the assumption of the fundamental diagram now, we do not find two exact values of the speed, but a distribution centered around two values:



(0.5)

4. Moving bottleneck

Total for Question 4: 12

Consider a two-lane motorway. Assume a triangular fundamental diagram with a free flow speed of 80 km/h, a critical density of 25 veh/h/lane and a jam density of 150 veh/km/lane. The inflow is stationary at 2000 veh/h.

(a) **Draw the fundamental diagram and calculate the capacity of the road stretch** (2)

> **Solution:** Capacity is maximum flow, q=ku=25*80=2000 veh/h/lane. For a two lane section, this is 4000 veh/h. (1 for points) plus 1 for graph

Consider a wide vehicle (special transport) driving slowly is entering the road. There are no overtaking possibilities. This creates congestion, *of which the tail happens to stay at the same position.*

(b) **Calculate the flow in the jam. (Hint: you can use the speed of the shockwave at the tail of the queue)** (1)

> **Solution:** Since the tail is at the same place, the flow in the queue should be equal to the inflow (1), so 2000 veh/h.

(c) **Calculate the density in the jam.** (2)

> **Solution:** The fundamental diagram is described by q=C(1-((k-kc)/(kj-kc))); use kj=300;C=4000;kc=50; (1) Use the fundamental diagram to find the density at q=2000 veh/h. (0.5) Solving q=1000 veh/h gives k=175 veh/km (0.5)

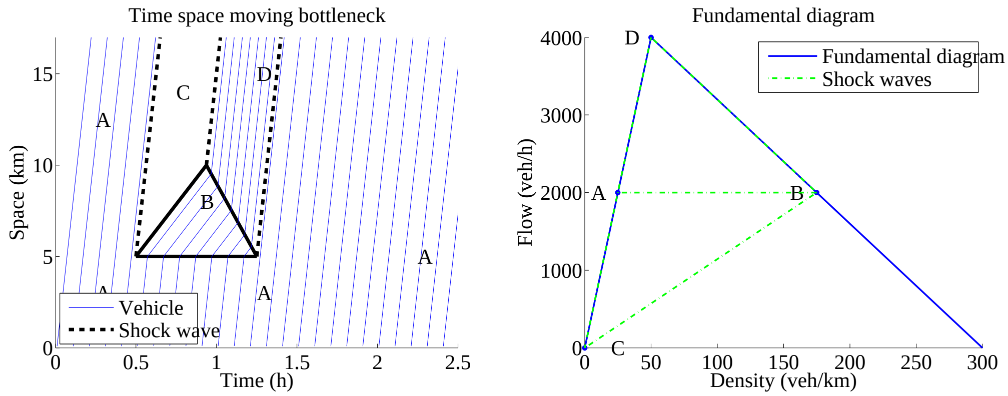
(d) **Calculate the speed of the special transport.** (2)

> **Solution:** The speed of the vehicle is equal to the speed in the congested state (1). Using q=ku (0.5), this gives v=2000/175=11,4 km/h (0.5)

After 5 km, the truck leaves the road.

(e) **Construct the space-time diagram of this situation, from before the moment the truck enters the road to after the moment the traffic situation is stationary. Explain how you find the speed of the solving of congestion; you may re-use the figure you created in a.** (4)

> **Solution:** The truck drives at 11,4 km/h, and forms the moving bottleneck for 5 km. This can be plotted in the space time diagram (1). The tail of the queue remains stationary (1).



For the solution of the traffic jam, construct the line in the FD between the jam point and the capacity point (1). Finishing the figure: 1 point.

(f) **At the maximum queue length, how many vehicles are in congestion** (1)

Solution: The queue has at maximum a length of 5 km, and the density is 175 veh/km – so 5*175= 875 veh.

5. Multi-lane traffic flow

Total for Question 5: 10

(a) **Give the names of two regimes according to Daganzo's theory of slugs and rabbits (only names are required).** (1)

Solution: Two-pipe regime and one-pipe regime

Below, you find a Google Earth image of the A1 motorway near Bathmen. Consider the right to left (east to west) direction

(b) **To which of the regimes of question a) does the situation in the figure resemble most. Explain why** (2)

Solution: Two-pipe regime (1). Trucks drive in one lane, and passenger cars in the other, at higher speeds (based on the density) (1).

There are 4 trucks and 7 passenger cars (including one delivery van) in the image. The fraction of passenger cars in the density is hence 7/11.

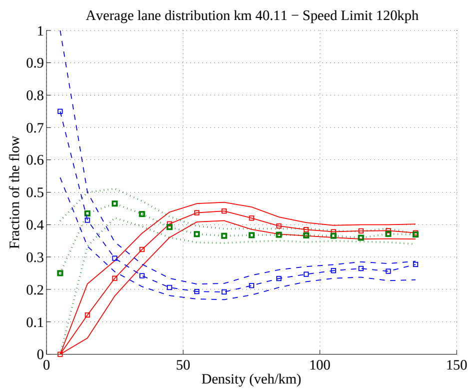
(c) **What would you expect from the fraction of passenger cars in the flow: is this higher, lower or equal to 7/11. Motivate your answer** (2)

Solution: The speed of the passenger cars is higher, so they will pass the stationary detectors more "often" (1). Hence, the fraction of the flow will be higher (1).

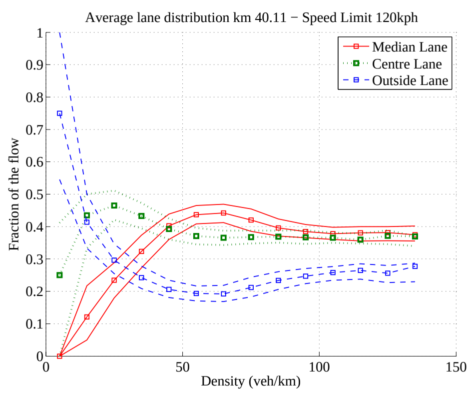
(d) **Calculate the fraction of the is passenger cars in the flow. Assume reasonable speeds for the different vehicle types in your answer – state your assumptions clearly.** (4)

Solution: Requested is the fraction of the flow of passenger cars, (class 1): $\rho = q_1/q_{tot} = q_1/(q_1+q_2) = (k_1 v_1)/(k_1 v_1 + k_2 v_2)$. (1 point for q=kv per class)
From the image we know k2=4/7*k1 (0.5 point), so $\rho = (k_1 v_1)/(k_1 v_1 + 4/7 k_1 v_2) = v_1/(v_1 + 4/7 v_2)$ (0.5 point) in which v indicates the speeds. Assuming v_1=120 km/h and v_2=80 km/h (1 point – assuming equal speeds maximizes the nr of points for this question to 1), we find: $\rho = 120/(120 + 4/780) = \ldots$ (1)

Below, a figure of a lane flow distribution for a three lane road in the Netherlands is given.



(e) **In the figure, which color matches which lane. Argue why** (1)



Solution: If it is really quiet on the road, people keep right (hence blue is right). When it gets busier, more people go towards the middle lane, and then towards the left lane. (0.5 for the right reasoning, 0.5 for drawing the right conclusions; no points for only conclusions; 0.5 if middle and median lane exchanged.)