Exam CIE-4821-09
Traffic Flow Theory and Simulation

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The exam has 6 questions. 58 points can be obtained, which are specified per question and subquestion. Some questions might require more time than others, so *use your time wisely*! (The question "Moving bottleneck with different speeds" might require time). The total time available for this exam is 3 hours.

Remarks:

- Allowed: calculator (but no smartphones...), self-made equation sheet (1 double sided A4 max)
- Put labels at all your graph axes.
- If a *sketch* is asked, there is no need for an exact drawing. Do make sure, though, that it is clear whether points lie higher or lower or on one line, and that this is correct.
- Your answer will be judged on the good elements in there, but for all wrong answers points will be de ducted.
- For some questions, an indicative number of words is given as guidance for the required level of detail. Your answer may be shorter or longer.
- Make sure you provide the calculus procedure as well as the result in order to get the maximum points.

Question	Points
1	9
2	6
3	13
4	10
5	4
6	16
Total:	58

1. Short open questions

Total for Question 1: 9

(a) **Explain why Kerner claims that in his three phase theory there is no fundamental diagram (indication: 25 words)** (1)

> **Solution:** The congested branch is not a line, but an area (1)

(b) **What is a Macroscopic Fundamental Diagram (indication: 25 words)?** (1)

> **Solution:** An MFD relates the *average* density in an area, the accumulation, to the *average* flow in the area

(c) **What are similarities and differences of a Macroscopic Fundamental Diagram compared to a normal fundamental diagram? Explain based on the underlying phenomena (indication: 75 words).** (2)

> **Solution:** The MFD is flattened at the top compared to the regular FD (1). This is due to averaging effects: the top can only be reached if *all* links are operating at capacity. If some have a over-critical density and some an under-critical, the average density can be critical, but the flow is less than at capacity (1).

(d) **Explain in words the how the flow in from one cell to the next is calculated according to the Cell Transmission Model (indication: 100 words; equations can be useful, but need to be explained).** (2)

> **Solution:** The CTM is a numerical scheme to calculate traffic flows in a macroscopic way (0.5), based on demand of the upstream cell and supply of the downstream cell(0.5). The flow is the minimum of both (0.5); demand is increasing with density up to the critical density and remains constant afterwards, supply starts at capacity and starts decreasing from capacity (0.5)

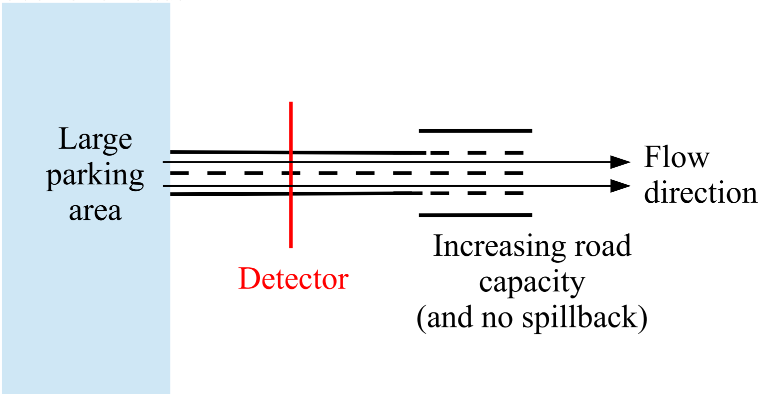
(e) **What do we mean with Lagrangian coordinates in macroscopic traffic flow simulation? Which variables are used? Why is this an advantage over other systems? (indication: total 100 words)** (3)

> **Solution:** Lagrangian coordinates are used in macroscopic traffic flow description. Instead of fixing coordinates at moments in space, the coordinates go with the traffic (1). Variables used in this system are hence time, *vehicle number* (1) and speed. This is an advantage since the representation of traffic flow equations is more accurate (no numerical diffusion) (1)

2. Leaving the parking lot

Total for Question 2: 6

Consider the situation where many cars are gathered at one place, and they can only leave over one road.



The above figure shows a simplified representation of the road layout. There is a detector at the red line, in the two-lane road stretch. Downstream of the detector the road widens and there are no further downstream bottlenecks.

(a) **Explain what the capacity drop is (be precise in your wording!).** (1)

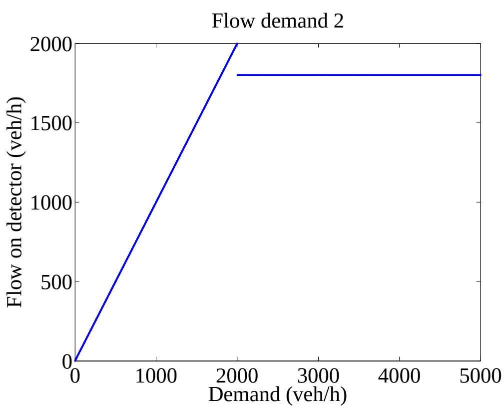
> **Solution:** The maximum flow on the road before the onset of congestion is higher than the flow of vehicles driving out of congestion. (0.5) This phenomenon of drivers keeping a larger headway after a standstill is called the capacity drop. (0.5)

(b) **How large is the capacity drop? Give a typical interval bound.** (1)

> **Solution:** Approximately a few to sometimes 30% is claimed

(c) **Sketch the traffic flow at the detector (indicated in the figure) as function of the demand (in veh/h, ranging from 0 to three times the road capacity) from the parking lot.** (2)

Solution:



(1 for shape + 1 for capacity

drop):

(d) **Explain the general shape you draw in the previous question.** (1)

Solution: the flow equals the demand (0.5), but is never higher than the capacity, and it will not decrease (no over-critical densities) (0.5). Capacity drop explained if present.

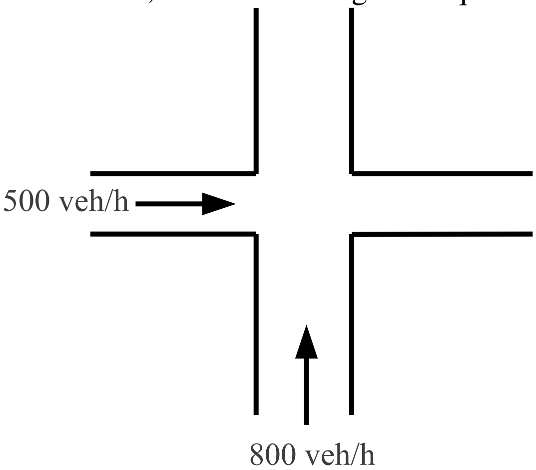
(e) **Give some rough estimates for values in your graph.** (1)

Solution: Capacity is approximately 2000 veh/h/lane (1500-2500: 0.5 pt, possibly lower if urban roads are assumed and explained), so 4000 veh/h for the roadway (0.5)

3. Traffic lights

Total for Question 3: 13

Consider a junction with a traffic light with equal green time g, and a clearance time of 2 seconds (i.e., the time needed to clear the junction; during this time, both directions are red). If the traffic light turns green, the first vehicle needs 3 seconds to cross the line. Afterwards, vehicles waiting in the queue will follow this vehicle with a 2 second headway.



(a) **What is the fraction of time traffic is flowing over the stop line per direction as function of the cycle time c?** (2)

> **Solution:** Effective green: $(g - 3)$ seconds per direction (0.5). The cylce time is $c = 2(g + 2)$ (0.5). Thus,$g = (c/2) - 2$ (0.5). Relative green time: $2\frac{g-3}{c} = 2\frac{c/2-5}{c}$ (0.5) $= 1 - \frac{10}{c}$.

(b) **What is the maximum flow per direction as function of the cycle time? In your equation, what are the units for the variables?** (3)

> **Solution:** Relative green time per direction is half that of the total relative green time: $\frac{1}{2} - \frac{5}{c}$ (1). The flow when effective green is 3600(sec/h)/2(s/veh)=1800 vehicles per hour, or 1/2 vehicle per second (either flow value: 1 point). The flow per direction is thus $1/2(\frac{1}{2} - \frac{5}{c})$ with flow in veh/s and c in seconds or $1800(\frac{1}{2} - \frac{5}{c})$ with flow in veh/h and c in seconds. (1 point)

Suppose traffic intensity from direction 1 is 500 veh/h and traffic from direction 2 is 800 veh/h, and suppose a uniform arrival pattern.

(c) **What is the minimum cycle time to ensure that no queue remains at the end of the cycle? Does an approach with vertical queuing models yield the same result as shockwave theory? Why?** (3)

> **Solution:** Both methods will yield the same answer, since not the queue length, but the flow is asked. (1) Most vehicles come from direction 2, so the green

time (still equal) should be based on direction 2 (1). Using the equation from the last question, we have $1800(\frac{1}{2} - \frac{5}{c}) = 800$ (0.5). Solving this for c yields $\frac{1}{2} - \frac{5}{c} = \frac{8}{18} = \frac{4}{9}$. So $\frac{5}{c} = \frac{1}{10}$, so $c = 50$ seconds(0.5).

This cycle time is fixed now at 120 seconds. We relax the assumption of uniform distribution to a more realistic exponential arrival pattern.

(d) **To which distribution function for the number of arrivals per cycle (N) does this lead? Name this distribution (1 pt).** (1)

Solution: Poisson distribution

The probability distribution function of X is given by:

$$f(k;\lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \tag{1}$$

with $e \sim 2.71828...$) and $k!$ is the factorial of k. The positive real number λ is equal to the expected value of X, which also equals the variance.
Assume there is no traffic waiting when the traffic light turns red at the beginning of the cycle.

(e) **Express the probability p that there are vehicles remaining in the queue for direction 2 when the traffic light turns red at the end of the cycle in an equation ($p = ...$). Write your answer as mathematical expression in which you specify the variables. Avoid infinite series. There is no need to calculate the final answer as a number** (3)

> **Solution:** The cycle time is 120 seconds. The expected number of vehicles in a cycle is $120/60 * 800/60 = 160/6 = 26.7$ (0.5) so $\lambda = 26.7$ (0.5) 120 seconds cycle time, so 60 seconds per direction. 5 seconds are lost (2 s clearance time and 3 s startup loss), so 55 seconds leading to floor(55/2)=27 vehicles at maximum through a green phase (0.5). The probability of an overflow queue is the probability that the number of cars arriving is 28 or larger, $P(X >= 28)$ (0.5); this can be calculated by $1 - P(X <= 27)$ (0.5). This is calculated as $p = \sum_{k=0}^{27} \frac{26.7^k e^{-26.7}}{k!}$ (0.5)

(f) **Argue whether this probability is higher or lower than if a uniform arrival process is assumed** (1)

> **Solution:** The spread of a Poisson arrival process is larger(0.5), so the probability of having overflow queues is larger(0.5; only with correct reasoning)

4. Car-following model

Total for Question 4: 10

Consider the IDM car-following model, prescribing the following acceleration:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a_0 \left(1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v,\Delta v)}{s}\right)^2\right) \tag{2}$$

with the desired distance s^* as function of speed v and speed difference Δv:

$$s^*(v, \Delta v) = s_0 + v\,T + \frac{v\,\Delta v}{2\sqrt{a\,b}} \tag{3}$$

(a) **Explain in words the working of this car-following; i.e. comment on the acceleration.** (3)

Solution: Vehicles accelerate in principle with acceleration a, but this reduces when they approach their desired speed (1) or if they approach their desired distance (1). The desired distance increases with speed.

Another car-following model is the relatively simple car-following of Newell.

(b) **How does the Newell car-following model work? (indication: 50 words)** (1)

Solution: It is a translation of the leaders' trajectory (0.5), translated forward in time by a fixed value τ and back in space by Δx (0.5).

(c) **Name two weak points of both car-following models (IDM and Newell)** (2)

Solution: Choose two:

- Drivers are considering multiple vehicles ahead
- Drivers are unable to judge perfectly speed and gap
- Drivers are not changing acceleration continuously
- There is also interaction with lane-changing

(d) **Give two reasons to choose the IDM model over Newell's model** (2)

Solution: Choose 2:

- It better captures the non-equilibrium conditions.
- There is a possibility that stop-and-go waves form (instabilities).
- Drivers have a finite acceleration

(e) **Give two reasons to choose Newell's model over the IDM model** (2)

Solution: Choose 2 (1 point per good answer):

- In the IDM, drivers will, unrealistically, never reach their desired speed, not even if the distance is larger than the desired distance
- It has less parameters to calibrate – more realistic
- It is better understandable

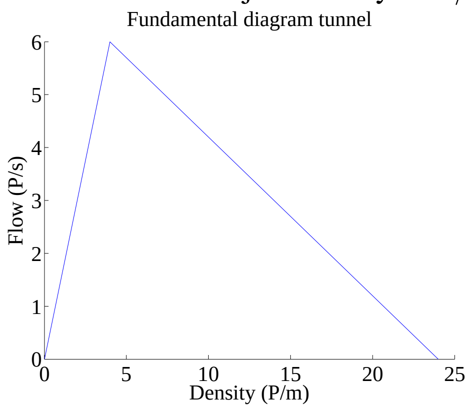
5. Pedestrians in a narrow tunnel

Total for Question 5: 4

We consider a pedestrian flow through a narrow tunnel, which forms the bottleneck in the network under high demand. The tunnel is 4 meters wide and 20 meters long. You may assume that the pedestrians are distributed evenly across the width of the tunnel. The pedestrian flow characteristics are described by a triangular fundamental diagram, with free speed v_0 = $1.5m/s$, critical density k_c = $1P/m^2$ and jam density $k_j = 6P/m^2$ (P stands for pedestrian).

(a) **Draw the fundamental diagram for the tunnel. What is the capacity of the tunnel expressed in P/min?** (2)

Solution: Convert the densities to tunnel-wide properties. The critical density then is 4P/m and the jam density $24P/m$. (1) The capacity is vk=6 P/s. (0.5)



(0.5)

Between 11-12 am, the average flow through the tunnel is q = 180 P/min.

(b) **Assuming stationary conditions, what is the density in the tunnel expressed in P/m^2 ?** (2)

Solution: 180P/min = 3 P/s (unit conversion: 0.5). Undercritical (of course, the tunnel is the bottleneck!), so v=vf. $k = q/v$=3/1.5=2 P/m (0.5). That is 2 pedestrians per meter length, so 0.5 pedestrians per m^2 (1)

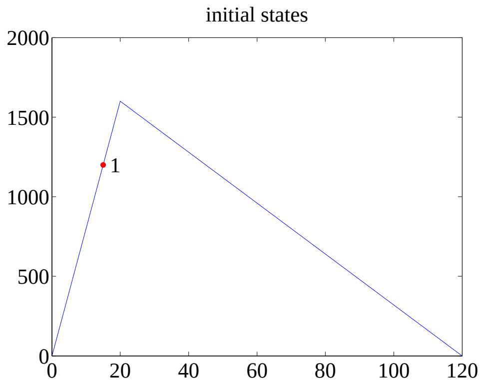
6. Moving bottleneck with different speeds

Total for Question 6: 16

Consider a two-lane road with traffic in opposing directions. Assume a triangular fundamental diagram with a free speed of 80 km/h, a critical density of 20 veh/km and a jam density of 120 veh/h. At t=0 there is a platoon of 30 vehicles on the road with equal spacing in the section x=0 to x=2km. There are no other vehicles on the road. In this question, we will consider the effect of a speed reduction to 15 km/h.

(a) **What is the density on the road in the platoon? Indicate the conditions in the fundamental diagram. Make a clear, large drawing of the fundamental diagram such that you can reuse it for indicating states in the remaining of the question.** (1)

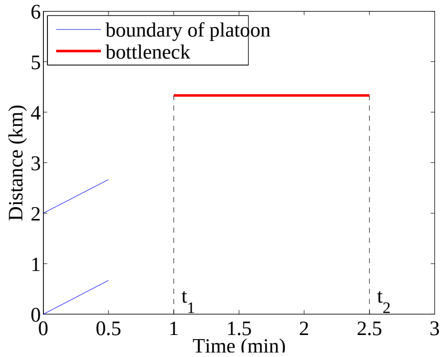
Solution: 30 vehicles in a 2 km section, so the density is 15 veh/km. (0.5)



(figure: 0.5);

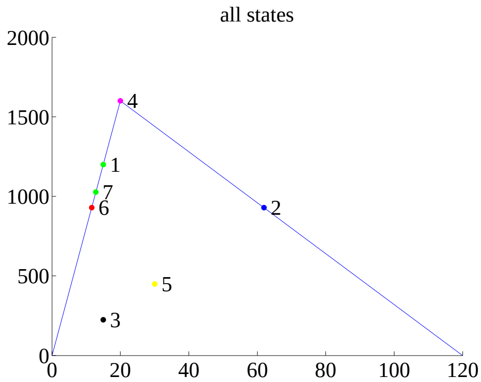
In each of the following subquestions, consider these initial conditions and no other bottlenecks than introduced in that subquestion – i.e., there never is more than one bottleneck.

Suppose there is a local, stationary bottleneck where drivers have to pass at 15 km/h from time $t1$ to $t2$. The figure below shows the position and the duration compared to the platoon in the space time plot.



(b) **Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style. *Indicate how you construct the diagram* (i.e., indicate how you get the lines, angles, and distances in the plot). There is no need to calculate the exact traffic states. Additional information: the combination of speeds and the fundamental diagram will lead to congestion.** (4)

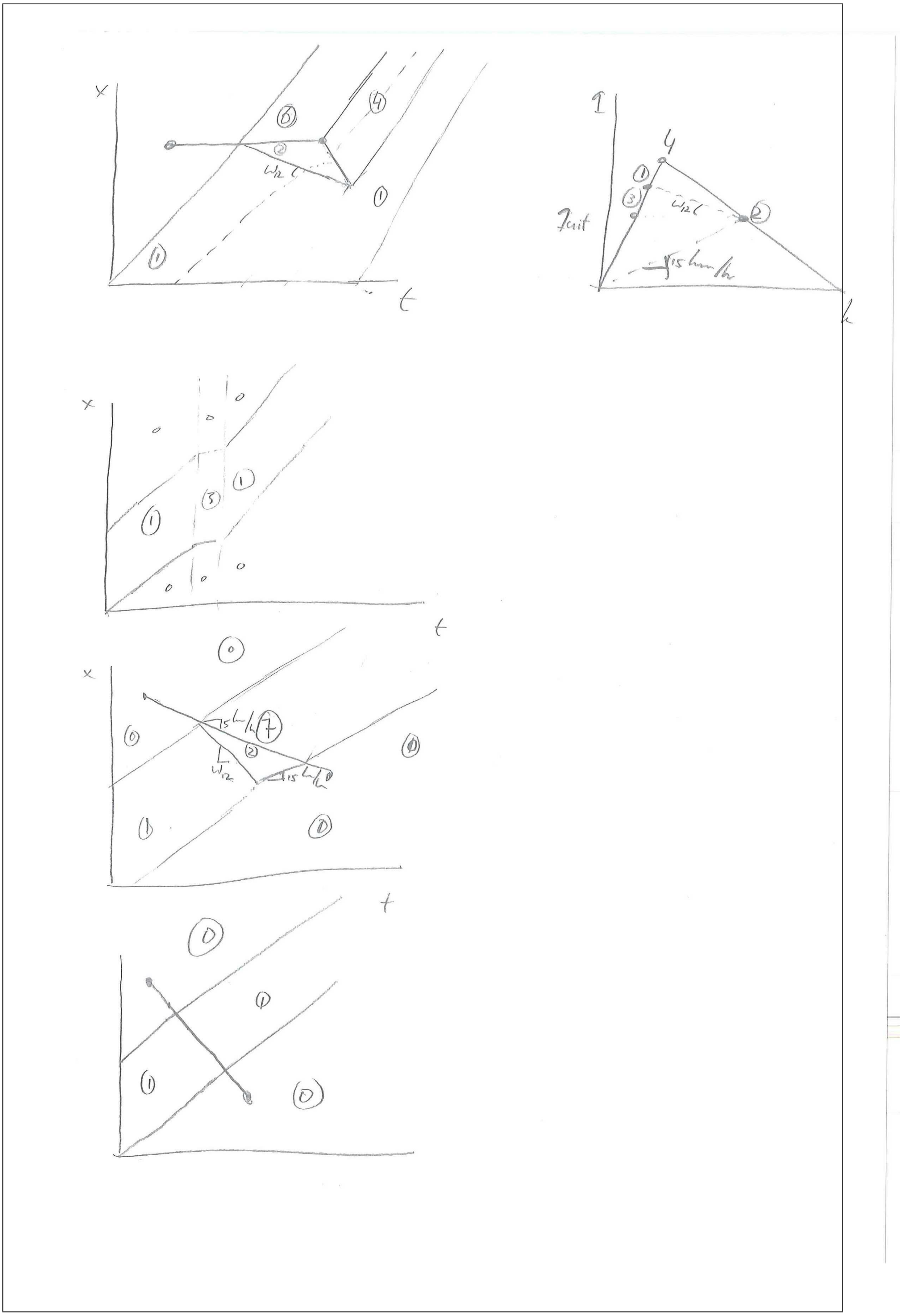
Solution: Draw a line with an slope of 15 km/h up in the fundamental diagram. You find the intersection with the congested branch at point 2. (0.5 point) The flow is lower than the initial conditions, so congestion occurs. (given)



(0.5 points)

For the xt-plot we start with the bottleneck at a constant x from t1 to t2. This causes traffic state 2 to occur. The boundary moves backwards with $w_{1,2}$, which is the same as the angle in the fundamental diagram between state 1 and 2. (0.5). Downstream, the flow is the same as the capacity in 2, but in free flow (state 6) (0.5). After removal of the bottleneck, there are capacity conditions (0.5) and the shock waves propagate with speeds equal to the angle 2-4 and 1-4 (0.5 combined). From $t1$ there also is a shock wave separating state 1 and 6, moving with a speed equalling the slope of line 1-6 in the FD (0.5). Trajectories: 0.5.

All xt-plots are found in the next figure



Suppose there is a temporal speed reduction to 15 km/h for a short period of time over the whole length of the road at the same time. You may assume the platoon to be completely on the road.

(c) **Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style. *Indicate how you construct the diagram* (i.e., indicate how you get the lines, angles, and distances in the plot).** (2)

> **Solution:** Since it is temporal, the density remains constant (1 point). This leads to the construction of point 3 a the intersection of the line with a slope of 15 km/h and the density of 15 veh/km (0.5, including showing this point in the graph). Trajectories 0.5.

Suppose there is a large tractor moving in the opposite direction of the traffic at 5 km/h from x1 to x2. This wide vehicle causes vehicles that are next to it to reduce speed to 15 km/h. The speed (5 km/h) is lower than the speed of the shock wave at the tail of the queue in question b

(d) **Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style. *Indicate how you construct the diagram* (i.e., indicate how you get the lines, angles, and distances in the plot).** (4)

> **Solution:** It is given that the tractor moves back very slowly, so we start at the situation comparable to b/ That means there will be a congestion state at the fundamental diagram where people drive 15 km/h (0.5), state 2 in the graph. The moving vehicle forms a boundary in shockwave theory (1), separating state 2 from another state where traffic moves out of the queue. By using the speed of the tractor, we find state 7 (0.5) as outflow state. After the tractor leaves the road, traffic is at capacity (0.5) and restores just like in question c (0.5).
> fundamental diagram (0.5) & trajectories: (0.5)

Suppose there is a large tractor moving in the opposite direction of the traffic at 50 km/h. This wide vehicle causes vehicles that are next to it to reduce speed to 15 km/h. The tractor speed (50 km/h) is higher than the speed of the shock wave of the tail of the queue in question b.

(e) **Sketch the resulting traffic operations in a space-time plot. Also sketch some trajectories in the plot, using a different color or style. *Indicate how you construct the diagram* (i.e., indicate how you get the lines, angles, and distances in the plot). (Hint: it can help to consider a finite tractor length, but it is not necessary)** (5)

> **Solution:** The tractor forms a moving bottleneck, and the boundary between two traffic states (0.5). It separates the initial traffic state 1 from another traffic state where vehicles travel at 50 km/h (1). Constructing this in the fundamental diagram is done by drawing a line with a slope of 50 km/h down from point 1. We find the traffic state next to the tractor at this point (0.5), drawing point 5 in fundamental

diagram: 0.5. The end of the tractor has a similar effect of a boundary (0.5) moving with 50 km/h. Constructing this in the fundamental diagram gives back point 1 (0.5). So at both ends, we have traffic state 1 (1), and next to the truck state 5 (no points for that). There are no other shock waves. Trajectories move either straight on, or are delayed a bit next to the tractor. (0.5)